## РАДІОФІЗИКА

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# RADIOPHYSICS

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### SCATTERING MATRIX OF A JUMP OF MEDIUM PARAMETERS IN A RECTANGULAR WAVEGUIDE

For a rectangular waveguide, a generalized scattering matrix of a jump of medium parameters has been obtained under condition that the field has one variation along the direction perpendicular to *E*-plane. The configuration is considered as a model of a plane interface in the theory of piecewise homogeneous *E*-plane structures.

Key words: waveguide discontinuities, plane interface, scattering matrix.

#### INTRODUCTION

The theory of homogeneous E-plane waveguide junctions is similar in many respects to that of H-plane structures. Under assumption that excitation is the  $TE_{10}$ mode, a scalar formulation in terms of  $H_z$  (the component of the magnetic field perpendicular to the plane of the junction) is possible. The analysis becomes more complicated for configurations involving dielectric obstacles. Even in the case of the material parameters invariable in the z direction, the same component of the electric field arises, despite the fact that  $E_{z}$  is not present in the incident wave [1, 2]. The problem remains twodimensional (the z dependence is known), but the field in the structure is expressed through two coupled scalar functions. There are only a few works (e.g. [2, 3]) giving the complete E-plane analysis of discontinuities of this type.

A particular class of E-plane junctions with dielectric inclusions is presented by configurations consisting of homogeneous parts separated by the planes parallel to the z axis or, what is same, connected by waveguides of zero length containing interfaces. The method of generalized scattering matrices [4] can give certain advantages when dealing with this category of problems. In regions between the planes, the field is representable as a

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superposition of *LSE* and *LSM* modes. Since the *LSE* and *LSM* waves do not interact inside a homogeneous cell, the *S*-matrix of such unit is a simple union of scattering matrices obtained in solving two uncoupled scalar problems. That allows not treating a vector problem for complicated geometry. The *S*-matrix of the interface, situated in a connecting waveguide, can be got analytically. The objective of the present paper is to determine the said matrix. In spite of the simplicity of the problem we have not found its solution in previous publications.

#### **PROBLEM FORMULATION**

It is assumed that the medium is isotropic, boundaries of the junction are perfectly conducting and the fields have only one variation along *a*, the broad dimension of the waveguides. Then, omitting the time dependence  $\exp(i\omega t)$ , we can represent vectors **E** and **H** in an *E*plane structure as

$$\mathbf{E} = \mathbf{E}_{\mathbf{t}}(x, y) \sin \frac{\pi z}{a} + \hat{\mathbf{z}} E_z(x, y) \cos \frac{\pi z}{a}, \qquad (1)$$

$$\mathbf{H} = \mathbf{H}_{t}(x, y)\cos\frac{\pi z}{a} + \hat{\mathbf{z}}H_{z}(x, y)\sin\frac{\pi z}{a}, \qquad (2)$$

where  $\hat{\mathbf{z}}$  is the unit vector in the *z* direction. With the help of Maxwell equations, the transverse with respect

to z components (denoted as t) are expressed through  $E_z$  and  $H_z$ :

$$\mathbf{E}_{t} = (E_{x}, E_{y}) =$$

$$= -\frac{1}{\chi^{2}} \left[ i\omega\mu\mu_{0} \left( \hat{\mathbf{x}} \frac{\partial H_{z}}{\partial y} - \hat{\mathbf{y}} \frac{\partial H_{z}}{\partial x} \right) + \frac{\pi}{a} \left( \hat{\mathbf{x}} \frac{\partial E_{z}}{\partial x} + \hat{\mathbf{y}} \frac{\partial E_{z}}{\partial y} \right) \right], \quad (3)$$

$$\mathbf{H}_{t} = (H_{x}, H_{y}) =$$

$$= \frac{1}{\chi^{2}} \left[ i\omega\varepsilon\varepsilon_{0} \left( \hat{\mathbf{x}} \frac{\partial E_{z}}{\partial y} - \hat{\mathbf{y}} \frac{\partial E_{z}}{\partial x} \right) + \frac{\pi}{a} \left( \hat{\mathbf{x}} \frac{\partial H_{z}}{\partial x} + \hat{\mathbf{y}} \frac{\partial H_{z}}{\partial y} \right) \right], \quad (4)$$

with

$$\chi^2 = k^2 - \left(\frac{\pi}{a}\right)^2, \qquad (5)$$

$$k = k_0 \sqrt{\varepsilon \mu}, \tag{6}$$

$$k_0 = \omega \sqrt{\varepsilon_0 \mu_0} = \frac{2\pi}{\lambda}, \qquad (7)$$

 $\varepsilon$ ,  $\mu$  being the relative dielectric constant and the relative permeability, and  $\lambda$  the free space wave length. Thus, once  $H_z$  and  $E_z$  have been obtained, the rest of the electromagnetic field can be found from them.

The configuration in question is shown in Fig. 1. At the left and to the right of the interface, the field is a superposition of the *LSE* and *LSM* modes. The component  $H_z$  of the mode  $LSE_{1r}$  ( $r = \overline{0, N}$ ), propagating along the waveguide, can be written as

 $H_z = a_r N_r^H \cos \frac{r\pi (x+b/2)}{b} e^{-\gamma_r y},$  (8)

where

$$\gamma_r = \sqrt{\left(\frac{r\pi}{b}\right)^2 - \chi^2},\tag{9}$$

 $a_r$  is the wave amplitude, and  $N_r^H$  is a normalizing constant. The superscript *H* indicates, that we deal with a *LSE* wave ( $H_z \neq 0$ ) The scattered wave amplitudes present elements of the generalized scattering matrix on assumption that the amplitude of the incident wave is unity. At that, normalizing factor is taken usually so, that for an undamped mode the quantity  $\frac{|a_r|^2}{2}$  equals to the transmitted power [5]. Conventional computations result in

$$N_r^H = \sqrt{\frac{4i\chi^2}{e_r ab\,\omega\mu\mu_0\gamma_r}},\tag{10}$$

where

$$e_r = \begin{cases} 2 \text{ for } r = 0, \\ 1 \text{ for } r \ge 1. \end{cases}$$
(11)



Fig. 1. Geometry of the problem

In the case of the mode  $LSM_{1r}$   $(r = \overline{1, N})$ ,  $E_z$ -component is of the form

$$E_z = b_r N_r^E \sin \frac{r\pi (x+b/2)}{b} e^{-\gamma_r y}, \qquad (12)$$

$$N_r^E = \sqrt{\frac{4i\chi^2}{ab\omega\varepsilon\varepsilon_0\gamma_r}}.$$
 (13)

The above representations ensure the realization of the boundary conditions at the conducting walls. We supply the used quantities with the indices 1 and 2 respectively at the left and to the right of the interface and note that modes with different values of r do not interact inside the geometry under study. After choice of r, the problem is reduced to finding sets of values of amplitudes on different sides of the plane y = 0, which guarantee the continuity of the tangential components of the electromagnetic field across the interface for all possible types of excitation.

#### SCATTERING MATRIX

In order a little bit to simplify the subsequent calculations, we will take the normalizing factors in (8) and (12) equal to one and consider their actual values in the final stage. Let us assume that the geometry is excited by the wave  $LSE_{1r}$  ( $r = \overline{0, N}$ ) from the left. Then

$$H_{z}^{(1)} = \left(e^{-\gamma_{r}^{(1)}y} - a_{r}^{(1)}e^{\gamma_{r}^{(1)}y}\right)\cos\frac{r\pi(x+b/2)}{b}, \quad (14)$$

$$E_z^{(1)} = b_r^{(1)} e^{\gamma_r^{(1)} y} \sin \frac{r\pi(x+b/2)}{b},$$
 (15)

$$H_z^{(2)} = a_r^{(2)} e^{-\gamma_r^{(2)} y} \cos \frac{r\pi (x+b/2)}{b}, \qquad (16)$$

$$E_z^{(2)} = b_r^{(2)} e^{-\gamma_r^{(2)} y} \sin \frac{r\pi (x+b/2)}{b}, \qquad (17)$$

where  $a_r^{(1)}$ ,  $b_r^{(1)}$ ,  $a_r^{(2)}$  and  $b_r^{(2)}$  are to be found. Substituting (14) and (15) in (3) and (4), we obtain

$$H_{x}^{(1)} = \left[\frac{i\omega\varepsilon_{1}\varepsilon_{0}\gamma_{r}^{(1)}}{\chi_{1}^{2}}b_{r}^{(1)}e^{\gamma_{r}^{(1)}y} - \frac{r\pi^{2}}{ab\chi_{1}^{2}}\left(e^{-\gamma_{r}^{(1)}y} - a_{r}^{(1)}e^{\gamma_{r}^{(1)}y}\right)\right] \times \\ \times \sin\frac{r\pi(x+b/2)}{b}, \qquad (18)$$

$$E_x^{(1)} = \left[\frac{i\omega\mu_1\mu_0\gamma_r^{(1)}}{\chi_1^2} \left(e^{-\gamma_r^{(1)}y} + a_r^{(1)}e^{\gamma_r^{(1)}y}\right) - \frac{r\pi^2}{ab\chi_1^2}b_r^{(1)}e^{\gamma_r^{(1)}y}\right] \times \\ \times \cos\frac{r\pi(x+b/2)}{b}.$$
(19)

With the help of (16) and (17), components  $H_x^{(2)}$  and  $E_x^{(2)}$  are got analogously:

$$H_{x}^{(2)} = -\left[\frac{i\omega\varepsilon_{2}\varepsilon_{0}\gamma_{r}^{(2)}}{\chi_{2}^{2}}b_{r}^{(2)}e^{-\gamma_{r}^{(2)}y} + \frac{r\pi^{2}}{ab\chi_{2}^{2}}a_{r}^{(2)}e^{-\gamma_{r}^{(2)}y}\right] \times \\ \times \sin\frac{r\pi(x+b/2)}{b}, \qquad (20)$$

$$E_{x}^{(2)} = \left[\frac{i\omega\mu_{2}\mu_{0}\gamma_{r}^{(2)}}{\chi_{2}^{2}}a_{r}^{(2)}e^{-\gamma_{r}^{(2)}y} - \frac{r\pi^{2}}{ab\chi_{2}^{2}}b_{r}^{(2)}e^{-\gamma_{r}^{(2)}y}\right] \times \\ \times \cos\frac{r\pi(x+b/2)}{b}.$$
 (21)

After equating at y = 0 the similar tangential components of the electric and magnetic fields and some simple algebra, the amplitudes are as follows

$$(a_r^{(2)}, b_r^{(2)})^T = M^{-1}C,$$
 (22)

$$a_r^{(1)} = 1 - a_r^{(2)}, \quad b_r^{(1)} = b_r^{(2)}.$$
 (23)

Here, T denotes the transposition and the matrices M and C are defined by formulas

$$M_{11} = M_{22} = \frac{r\pi^2}{ab} \left( \frac{1}{\chi_1^2} - \frac{1}{\chi_2^2} \right), \qquad (24)$$

$$M_{12} = -\frac{ik_0}{Z_0} \left( \frac{\varepsilon_1 \gamma_r^{(1)}}{\chi_1^2} + \frac{\varepsilon_2 \gamma_r^{(2)}}{\chi_2^2} \right),$$
(25)

$$M_{21} = ik_0 Z_0 \left( \frac{\mu_1 \gamma_r^{(1)}}{\chi_1^2} + \frac{\mu_2 \gamma_r^{(2)}}{\chi_2^2} \right), \qquad (26)$$

$$C = \left(0, 2ik_0 Z_0 \frac{\mu_1 \gamma_r^{(1)}}{\chi_1^2}\right)^2, \qquad (27)$$

where  $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$  is the intrinsic impedance of the free space.

The knowledge of the amplitudes allows writing down the respective elements of the normalized scattering matrix:

$$S_{rr}^{H_1H_1} = a_r^{(1)}, \quad r = \overline{0, N},$$
 (28)

$$S_{rr}^{E_{1}H_{1}} = \frac{N_{r}^{H_{1}}}{N_{r}^{E_{1}}} b_{r}^{(1)} = \frac{1}{Z_{0}} \sqrt{\frac{\varepsilon_{1}}{\mu_{1}}} b_{r}^{(1)}, \ r = \overline{1, N}, \quad (29)$$

$$S_{rr}^{H_2H_1} = \frac{N_r^{H_1}}{N_r^{H_2}} a_r^{(2)} = \sqrt{\frac{\mu_2 \gamma_r^{(2)} \chi_1^2}{\mu_1 \gamma_r^{(1)} \chi_2^2}} a_r^{(2)}, r = \overline{0, N}, \quad (30)$$

$$S_{rr}^{E_{2}H_{1}} = \frac{N_{r}^{H_{1}}}{N_{r}^{E_{2}}} b_{r}^{(2)} = \frac{1}{Z_{0}} \sqrt{\frac{\varepsilon_{2} \gamma_{r}^{(2)} \chi_{1}^{2}}{\mu_{1} \gamma_{r}^{(1)} \chi_{2}^{2}}} b_{r}^{(2)}, \ r = \overline{1, N}.$$
(31)

In more detail,

$$S_{rr}^{E_{1}H_{1}} = \frac{2ik_{0}\sqrt{\varepsilon_{1}\mu_{1}}\gamma_{r}^{(1)}M_{11}}{\chi_{1}^{2}\det M}, \ r = \overline{1,N}, \qquad (32)$$

and

$$S_{rr}^{E_{2}H_{1}} = \frac{2ik_{0}\sqrt{\varepsilon_{2}\mu_{1}\gamma_{r}^{(1)}\gamma_{r}^{(2)}}M_{11}}{\chi_{1}\chi_{2}\det M}, \ r = \overline{1,N}.$$
 (33)

Thus, as it is seen from (32) and (33) for r > 0, the incoming  $LSE_{1r}$  mode excites also the nonzero LSM modes.

Now, let the  $LSM_{1r}$   $(r = \overline{1, N})$  mode be incident upon the junction from the left. Then

$$H_{z}^{(1)} = -a_{r}^{(1)} e^{\gamma_{r}^{(1)} y} \cos \frac{r\pi(x+b/2)}{b}, \qquad (34)$$

$$E_{z}^{(1)} = \left(e^{-\gamma_{r}^{(1)}y} + b_{r}^{(1)}e^{\gamma_{r}^{(1)}y}\right)\sin\frac{r\pi(x+b/2)}{b}, \quad (35)$$

and components  $H_z^{(2)}$  and  $E_z^{(2)}$  keep form (16) and (17). After determining  $H_x^{(1)}$ ,  $E_x^{(1)}$ ,  $H_x^{(2)}$ ,  $E_x^{(2)}$  with the help of (3), (4) and taking into account the continuity of the tangential field components, we arrive again at the relation (22), where column matrix *C* is given by

$$C = \left(-2\frac{ik_0\varepsilon_1\gamma_r^{(1)}}{Z_0\chi_1^2}, 0\right)^T.$$
 (36)

Instead of (23), the following equations occur

$$a_r^{(1)} = -a_r^{(2)}, \quad b_r^{(1)} = b_r^{(2)} - 1.$$
 (37)

The respective scattering matrix elements are of the form

$$S_{rr}^{H_{1}E_{1}} = \frac{N_{r}^{E_{1}}}{N_{r}^{H_{1}}}a_{r}^{(1)} = Z_{0}\sqrt{\frac{\mu_{1}}{\epsilon_{1}}}a_{r}^{(1)}, \ r = \overline{1, N}, \quad (38)$$

$$S_{rr}^{E_1E_1} = b_r^{(1)}, \ r = \overline{1, N},$$
 (39)

$$S_{rr}^{H_2 E_1} = \frac{N_r^{E_1}}{N_r^{H_2}} a_r^{(2)} = Z_0 \sqrt{\frac{\mu_2 \gamma_r^{(2)} \chi_1^2}{\epsilon_1 \gamma_r^{(1)} \chi_2^2}} a_r^{(2)}, r = \overline{1, N},$$
(40)

$$S_{rr}^{E_{2}E_{1}} = \frac{N_{r}^{E_{1}}}{N_{r}^{E_{2}}} b_{r}^{(2)} = \sqrt{\frac{\varepsilon_{2}\gamma_{r}^{(2)}\chi_{1}^{2}}{\varepsilon_{1}\gamma_{r}^{(1)}\chi_{2}^{2}}} b_{r}^{(2)}, \ r = \overline{1, N}.$$
(41)

When exciting the geometry by the  $LSE_{1r}$   $(r = \overline{0, N})$ mode from the right, the  $H_z^{(1)}$  is defined by (34),  $E_z^{(1)}$  by (15),  $E_z^{(2)}$  by (17), and

$$H_{z}^{(2)} = \left(-e^{\gamma_{r}^{(2)}y} + a_{r}^{(2)}e^{-\gamma_{r}^{(2)}y}\right)\cos\frac{r\pi(x+b/2)}{b}.$$
 (42)

Boundary conditions at y = 0 yield

$$C = \left(\frac{r\pi^2}{ab}\left(\frac{1}{\chi_1^2} - \frac{1}{\chi_2^2}\right), ik_0 Z_0\left(\frac{\mu_1 \gamma_r^{(1)}}{\chi_1^2} - \frac{\mu_2 \gamma_r^{(2)}}{\chi_2^2}\right)\right)^T, \quad (43)$$

$$a_r^{(1)} = 1 - a_r^{(2)}, \quad b_r^{(1)} = b_r^{(2)},$$
 (44)

$$S_{rr}^{H_1H_2} = \sqrt{\frac{\mu_1 \gamma_r^{(1)} \chi_2^2}{\mu_2 \gamma_r^{(2)} \chi_1^2}} a_r^{(1)}, \ r = \overline{0, N},$$
(45)

$$S_{rr}^{E_1H_2} = \frac{1}{Z_0} \sqrt{\frac{\varepsilon_1 \gamma_r^{(1)} \chi_2^2}{\mu_2 \gamma_r^{(2)} \chi_1^2}} b_r^{(1)}, \ r = \overline{1, N},$$
(46)

$$S_{rr}^{H_2H_2} = a_r^{(2)}, \ r = \overline{0, N},$$
 (47)

$$S_{rr}^{E_2H_2} = \frac{1}{Z_0} \sqrt{\frac{\varepsilon_2}{\mu_2}} b_r^{(2)}, \ r = \overline{1, N}.$$
 (48)

In the case of excitation from the right by the wave  $LSM_{1r}$   $(r = \overline{1, N})$ , there hold formulas (34), (15), (16) and  $E_z^{(2)}$  takes on form

$$E_{z}^{(2)} = \left(e^{\gamma_{r}^{(2)}y} + b_{r}^{(2)}e^{-\gamma_{r}^{(2)}y}\right)\sin\frac{r\pi(x+b/2)}{b}.$$
 (49)

The enforcement of the boundary conditions leads to relations

$$C = \left(\frac{ik_0}{Z_0} \left(\frac{\varepsilon_1 \gamma_r^{(1)}}{\chi_1^2} - \frac{\varepsilon_2 \gamma_r^{(2)}}{\chi_2^2}\right), \frac{r\pi^2}{ab} \left(\frac{1}{\chi_2^2} - \frac{1}{\chi_1^2}\right)\right)^T, \quad (50)$$

$$a_r^{(1)} = -a_r^{(2)}, \quad b_r^{(1)} = b_r^{(2)} + 1,$$
 (51)

$$S_{rr}^{H_1 E_2} = Z_0 \sqrt{\frac{\mu_1 \gamma_r^{(1)} \chi_2^2}{\epsilon_2 \gamma_r^{(2)} \chi_1^2}} a_r^{(1)}, \ r = \overline{1, N},$$
(52)

$$S_{rr}^{E_{1}E_{2}} = \sqrt{\frac{\varepsilon_{1}\gamma_{r}^{(1)}\chi_{2}^{2}}{\varepsilon_{2}\gamma_{r}^{(2)}\chi_{1}^{2}}}b_{r}^{(1)}, \quad r = \overline{1, N}, \quad (53)$$

$$S_{rr}^{H_2 E_2} = Z_0 \sqrt{\frac{\mu_2}{\epsilon_2}} a_r^{(2)}, \ r = \overline{1, N},$$
 (54)

$$S_{rr}^{E_2 E_2} = b_r^{(2)}, \ r = \overline{1, N}.$$
 (55)

In whole, the generalized scattering matrix S is characterized by the expression

$$S = \begin{pmatrix} S^{H_1H_1} S^{H_1E_1} S^{H_1H_2} S^{H_1H_2} S^{H_1E_2} \\ S^{E_1H_1} S^{E_1E_1} S^{E_1H_2} S^{E_1E_2} \\ S^{H_2H_1} S^{H_2E_1} S^{H_2H_2} S^{H_2E_2} \\ S^{E_2H_1} S^{E_2E_1} S^{E_2H_2} S^{E_2E_2} \end{pmatrix}.$$
 (56)

Here, the submatrices are of a diagonal form with nonzero elements given by formulas (28)–(31), (38)–(41), (45)–(48) and (52)–(55). At that,  $S^{HH} = (S_{mn}^{HH})$  ( $m, n = \overline{0, N}$ ),  $S^{EH} = (S_{mn}^{EH})$  ( $m = \overline{1, N}$ ,  $n = \overline{0, N}$ ),  $S^{HE} = S_{mn}^{HE}$ ( $m = \overline{0, N}$ ,  $n = \overline{1, N}$ ),  $S^{EE} = (S_{mn}^{EE})$  ( $m, n = \overline{1, N}$ ). Let us also notice, that S-matrices of the homogeneous Eplane sections mentioned in Introduction have formally form (56) too, but the blocks of the type  $S^{EH}$  or  $S^{HE}$  consist completely of zeros and the remaining blocks are dense.

#### CONCLUSIONS

For a rectangular waveguide, the problem of determining the scattering characteristics of the jump of medium parameters is considered under condition that the field has one variation along the direction perpendicular to *E*-plane. It is shown that, when exciting the structure by one of the modes of type  $LSE_{1m}$  or  $LSM_{1m}$ , the scattered field consists of the similar modes both nonzero (except for excitation by  $LSE_{10}$ ). The calculation formulas for elements of the generalized scattering matrix of the unit have been derived. The examined configuration can be useful in the analysis of piecewise homogeneous *E*-plane structures as a model of a plane interface between sections of constant permittivity and permeability.

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Ващенко В. В., Чумаченко В. П. МАТРИЦЯ РОЗСНОВАННЯ СТРИБКА ПАРАМЕТРІВ СЕРЕДОВИЩА В ПРЯМОКУТНОМУ ХВИЛЕВОДІ

Отримана узагальнена матриця розсіювання стри-бка параметрів середовища в прямокутному хвилеводі при умові, що поле має одну варіацію в напрямі перпендикулярному площині *E*. Конфігурація розглядається в якості моделі плоского інтерфейсу в теорії кусково-однорідних Еплощинних структур.

**Ключові слова:** хвилеводні неоднорідності, плоский інтерфейс, матриця розсіювання.

Ващенко В. В., Чумаченко В. П. МАТРИЦА РАССЕЯНИЯ СКАЧКА ПАРАМЕТРОВ СРЕДЫ В ПРЯМОУГОЛЬНОМ ВОЛНОВОДЕ

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Получена обобщенная матрица рассеяния скачка параметров среды в прямоугольном волноводе при условии, что поле имеет одну вариацию в направлении перпендикулярном плоскости *E*. Конфигурация рассматривается в качестве модели плоского интерфейса в теории кусочно-однородных *E*-плоскостных структур.

**Ключевые слова:** волноводные неоднородности, плоский интерфейс, матрица рассеяния.

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### ВЗАЄМНА ФАЗОВА СИНХРОНІЗАЦІЯ ДВОХ НАНОРОЗМІРНИХ МАГНІТНИХ СТРУКТУР З УРАХУВАННЯМ ЗАПІЗНЕННЯ СИГНАЛУ

Розглянуто можливість взаємної фазової синхронізації двох взаємодіючих нанорозмірних магнітних структур (HMC) з урахуванням часового запізнення сигналу. Показано, що залежність ширини смуги синхронізації HMC від відстані між ними має нелінійний, осцилюючий характер. Визначено оптимальне значення відстані між HMC, що може бути використано для створення масивів синхронізованих HMC.

**Ключові слова:** нанорозмірна магнітна структура, взаємна фазова синхронізація, запізнення сигналу, смуга синхронізації.

#### вступ

Було теоретично передбачено Слончевським [1-2] та Берже [3], що спін-поляризований електричний струм I, який протікає крізь намагнічений феромагнітний шар, може передавати цьому шару спіновий магнітний момент, що, у свою чергу, призводить до зміни напрямку вектора намагніченості  $\vec{M}$  у феромагнітному шарі. Залежно від величини електричного струму I, більшого за деякий критичний (пороговий) струм I<sub>th</sub>, цей ефект, названий spin-torque effect (дослівно - ефект спінового крутильного моменту), може призводити до появи двох явищ [4-5]: 1) «переключення» феромагнітного шару за рахунок суттєвої зміни напрямку вектора намагніченості; 2) появи у феромагнітному шарі прецесії намагніченості з частотою  $\omega/2\pi$ , близькою до частоти феромагнітного резонансу (ФМР). Оскільки, як правило, частота ФМР тонких магнітних плівок лежить у мікрохвильовому діапазоні, для таких систем з'являється новий фізичний механізм генерації мікрохвильових сигналів, заснований на ефекті спінового крутильного моменту. Підкреслимо, що цей механізм генерації мікрохвильових сигналів може бути використаний для створення нелінійних активних нанорозмірних магнітних структур (НМС) [1-5].

На сьогоднішній день найбільш поширеними серед НМС є структури двох типів: 1) НМС типу «наностовпчик» (nanopillar) та 2) НМС типу «наноконтакт» (nano-contact). У найпростішому випадку ці структури складаються з трьох шарів (рис. 1). Один з цих шарів є магнітожорстким шаром (шар 1 на рис. 1) – у зовнішньому магнітному полі  $\vec{H}_{ext}$  вектор намагніченості  $\vec{M}_{FXL}$  в цьому шарі має цілком визначений



**Рис. 1.** Схематичне зображення тришарової нанорозмірної магнітної структури (HMC):

1 – магнітожорсткий шар, 2 – немагнітний прошарок товщиною  $1 \times 10$  нм, 3 – магнітом'який шар товщиною  $d_0 \approx 5 \div 20$  нм;  $R_0 \approx 5 \div 500$  нм – радіус НМС,  $h_0 \approx 20 \div 50$  нм – її товщина