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РАДІОЕЛЕКТРОНІКА ТА ТЕЛЕКОМУНІКАЦІЇ

РАДИОЭЛЕКТРОНИКА И ТЕЛЕКОММУНИКАЦИИ

RADIO ELECTRONICS AND TELECOMMUNICATIONS

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INVESTIGATION OF TEMPERATURE MODES IN THERMOSENSITIVE NON-UNIFORM ELEMENTS OF RADIOELECTRONIC DEVICES

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ABSTRACT

Context. The non-linear boundary value problem of heat conduction for a thermosensitive non-homogeneous strip-shaped element of a radio-electronic system with a through inclusion has been solved whose analytical-numerical solution enables us to analyze temperature regimes in the element.

Objective. Is to develop such a method of linearization of mathematical model of heat conduction which enables us to obtain analytical numerical solution of the corresponding non-linear boundary value problem for determination of temperature field in elements of radio electronic devices, which are geometrically represented by a thermosensitive plate with a through inclusion.

Method. A linearizing function which enables us to partially linearize the initial non-linear mathematical model of heat conduction for a thermosensitive non-homogeneous element of a radio electronic system in the form of “plate-inclusion” structure has been suggested. The introduced piece-wise linear approximation of temperature on plate-inclusion interfaces has enabled us to completely linearize the corresponding partially linearized boundary value problem relative to the linearizing function. After this, it became possible to apply Fourier’s integral transformation to the obtained linear problem with respect to one of the spatial coordinates, as well as to determine the linearizing function. The linear dependence of the coefficient of heat conductivity on temperature for structure materials with the use of the linearizing function has been considered. By solving the boundary value problem, the formulae for determination of temperature field in the “plate-inclusion” thermosensetive structure have been obtained.

Results. The obtained formulae for determination of temperature field in a thermosensitive non-homogeneous element of radio electronic system were used to create the software which enables us to obtain distribution of value of temperature and to analyze temperature regimes.

Conclusions. A mathematical model for the calculation for the temperature field in a “plate-inclusion” thermosensitive structure is adequate to the actual physical process, because no jump of temperature at “plate-inclusion” interfaces is observed. The numerical results for the chosen materials under linear dependence of the coefficient of thermoconductivity on temperature differ by 7% from the results which are obtained for constant coefficient of heat conductivity. Prospect of further investigation will consider more complicated geometric representation of elements of radio electronic systems.

KEYWORDS: temperature, heat conduction, nonlinear boundary-value problem, isotropic infinite thermosensitive plate with insulated faces, through inclusion, perfect thermal contact, heat flow.

NOMENCLATURE

2δ is the thickness of a heat-sensitive isotropic with respect to its thermophysical parameters plate ;

$2h$ is the length of a foreign through inclusion;

Ω_0 is the domain of the inclusion;

Ω_1 is the domain of the plate (beyond the inclusion);

K_- is the boundary surface of the plate, the system is heated by a heat stream; heat stream, whose surface density is $q_0 = const$;

K_+ is the part of the surface of the plate beyond the inclusion and is thermo-insulated;

$K_{\pm h}$ is the boundary surfaces of the inclusion;

$t(x, y)$ is the distribution of temperature with respect to the coordinates of space;

$\lambda(x, t)$ is the coefficient of heat conductivity of the eterogeneous heat-sensitive plate;

$\lambda_0(t)$ is the coefficient of het conductivity of the material of the plate;

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$\lambda_1(t)$ is the coefficient of het conductivity of the material of the inclusion;

$S_{\pm}(\zeta)$ are the asymmetric unit functions;

$\vartheta(x, y)$ is the linearizing function;

Δ is the Laplace operator in the Cartesian system of coordinates;

t_j are the unknown approximated values;

$\delta_{\pm}(\zeta)$ is the asymmetric Dirac delta-function;

$\overline{\vartheta}(\xi, y)$ is the transform formula $\vartheta(x, y)$;

ξ is the parameter of Fourier integral transformation;

n is the number of intervals in the partition $]-l;l[$;

λ_m^0 are basic coefficients of heat conductivity of the materials of the inclusion $(m = 0)$; and the plate $(m = 1)$;

k_m is the temperature coefficients of heat conductivity of the materials of the inclusion $(m = 0)$; and the plate $(m = 1)$;

t^* is the dimensionleess temperature;

x^*, y^* are the dimensionless coordinates of space;

$\lambda(x)$ is the coefficient of heat conductivity of the heterogeneous plate for the corresponding linear model.

INTRODUCTION

Composite materials, development of which is a liding problem of modern materials science, have become especially valuable in radioelectronic device's. The emergence of new composite materials with improved physic-mechanical performance will contribute to creation of new technologies in air-space, naval, power-generating, and electronic branches, in mechanical engineering, and transport. Among the composite materials an important place occupy structures with foreign inclusions, which are widely used structures of microelectronic devices, in particular, in integral sensors for monitoring of temperature and humidity, in light emitting elements for dynamic LED-backlighting etc. Because the aforesaid structures operate in a wide range of temperatures, their high performances cause the necessity of considering and solving of the problems which are non-linear due to the dependence of thermophysical characteristics of the materials on the temperature of the structure and on the conditions of heat exchange, on the temperature of their surfaces; because the calculations of the temperature fields which are performed on the basis of the linear mathematical models of heat conduction processes do not always give us satisfactory results [1]. Therefore, for creation of the most adequate to actual process mathematical model, it is necessary to take into account the dependencies of thermophysical characteristics of materials on temperature, on density of surface heat fluxes, and on intensity of internal heat sources; change of the shape of the body and possible phase and structural transformations are also to be taken into account [2, 3].

The object of study is the process of heat conduction for an element of radio electronic devices presented by a thermosensitive plate with a foreign inclusion.

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In order to create a non-linear mathematical model of the process of heat-conduction in a thermosensitive non-homogeneous element of radio electronic systems, theory of generalized function has been used; and in order to construct a numerical-analytical solution of the corresponding boundary value problem, integral Fourier transformation and methods of solving partial differential equations were used.

The subject of study consists of non-linear mathematical models of heat-conduction process and methods of construction of numerical analytical solution of the corresponding boundary value problems for thermosensitive elements of radio electronic devices of piecewise-homogeneous structure.

The purpose of the work is the development of such a method of linearization of mathematical model of heat conduction which enables us to obtain the analytical-numerical solution of the corresponding non-linear boundary value problem for determination of temperature field in elements of radio electronic devices, which are geometrically represented by a thermosensitive plate with a through inclusion.

1 PROBLEM STATEMENT

Let us consider a heat-sensitive isotropic with respect to its thermophysical parameters plate of 2δ thickness with thermo-insulated faces $|z| = \pm\delta$. The plate contains a foreign through inclusion of $2h$ length. The plate is referenced to the Cartesian system of coordinates (*Oxyz*) with whose origin is in the centre of the inclusion. In the domain of the inclusion $\Omega_0 = \{(x, -l, z) : |x| \le h, |y| \le l, |z| \le \delta\}$ of the boundary surface $K_- = \{(x, -l, z) : |x| < \infty, |z| \le \delta\}$ of the plate, the system is heated by a heat stream, whose surface density is $q_0 = \text{const}$, and the other part of the surface of the plate beyond the inclusion and the surface $K_+ = \{(x, l, z) : |x| < \infty, |z| \le \delta\}$ are thermo-insulated. At the boundary surfaces of the inclusion $K_{\pm h} = \{(\pm h, y, z) : |y| \le l, |z| \le \delta\}$ perfect heat contact takes place $t_0 = t_1$, $\lambda_0(t)\frac{\partial t_0}{\partial x} = \lambda_1(t)\frac{\partial t_1}{\partial x}$ for $|x| = h$ (0 – for inclusion, 1 – for plane) (fig. 1).

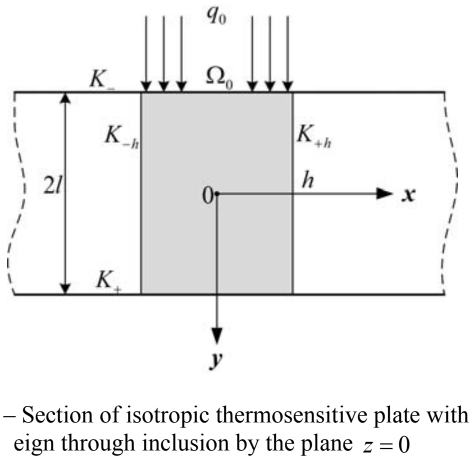

Figure 1 – Section of isotropic thermosensitive plate with foreign through inclusion by the plane $z = 0$

In the given structure, it is necessary to determine the distribution of temperature $t(x, y)$ with respect to the coordinates of space.

2 REVIEW OF THE LITERATURE

The determination of temperature conditions, both of homogeneous and non-homogeneous structures, draws attention of many researchers [4, 10].

In the work [11], a mathematical model of quasi-stationary temperature field in a continuous cylinder of rotation made of a composite material with non-linear boundary conditions is developed; the dependence of thermophysical parameters of material on temperature being taken into account. The obtained analytical expressions for determination of temperature fields enable us to select the composition of composite materials for cylinder-shaped parts in order to increase there are service life.

One-dimentional (plane, cylindrically symmetric, and spherically symmetric) non-linear problems of heat-conduction are considered for a given flux of heat at the origin of coordinate systems in the form of exponential function depending on time. Approximation solutions of the aforesaid problems have been obtained, and there are convergence has been analyzed [12].

An analytical solution of the non-linear problem of heat-conduction has been constructed on the basis of integral method of heat balance [13]. In order to improve the accuracy of the solution, the temperature function is approximated by high-degree polynomials. To determine the coefficients of the polynomials, additional boundary conditions have been introduced. It is shown that the introduction of additional boundary conditions, as early as in the second approximation, leads to considerable improvement of the accuracy of the solution of the problem.

In the work [14], numerical-analytical solution of the non-stationary problem of heat-conduction for a hollow sphere has been constructed; thermophysical parameters of materials of the sphere being variable with temperature. In the particular case, a solution for a continuous sphere has been obtained. On the basis of variational approach, a non-linear mathematical model of heat-conduction process have been constructed for 2D-space with a thin inclusion, this anable us to take into account small thickness of a thick inclusion. For linearization of the stated problem, the Newton-Raphson's methods has been applied. Discreatization with respect to temporal variable has been carried out in accordance with intermediate point scheme. The Discreatization statement of the problem is presented in the form of minimization of a functional [15].

Investigations of temperature regimes for structural thermosensitive elements of a piece-wise homogeneous structure have been carried out in the works [16–19].

Article [20] solved a non-stationary problem on thermal conductivity and thermoelasticity for functional-gradient thick-wall spheres. Thermal-physical and thermoelastic parameters of materials, except for Poisson coefficient, are arbitrary functions of the radial coordinate.

Axisymmetric stationary problem on thermal conductivity and thermoelasticity of the hollow functionally gradient areas relative to the heat source was considered. The solutions are obtained as functions from spatial coordinates for temperature, the displacement component vector and stress tensor by using boundary conditions for radial and angular coordinates [21].

In paper [22], a thermoelasticity solution for steady state response of thick cylinders which are subjected to pressure and external heat flux in inner surface is presented.

A general solution for the one-dimensional steady-state thermal and mechanical stresses in a hollow thick sphere made of porous functionally graded material (FGPM) is presented in [23].

The boundary value non-linear problem of heat conduction is determined below, the technique of its linearization and calculation formulae for determination of the temperature field in a heat-sensitive plate with a through inclusion which is heated by a heat stream concentrated at a surface of the inclusion is given. Numerical analysis under the condition of linear dependence of the coefficient heat conductivity on temperature is performed.

3 MATERIALS AND METHODS

In the given structure (in problem statement), it is necessary to determine the distribution of temperature $t(x, y)$ with respect to the coordinates of space which is obtained from solving the nonlinear equation of heat conduction [24, 25]

$$\frac{\partial}{\partial x}[\lambda(x,t)\frac{\partial t}{\partial x}]+\frac{\partial}{\partial y}\left[\lambda(x,t)\frac{\partial t}{\partial y}\right]=0 \qquad (1)$$

with the boundary condition

$$t\big|_{|x|\to\infty}=0,\quad \left.\frac{\partial t}{\partial x}\right|_{|x|\to\infty}=0,\quad \left.\frac{\partial t}{\partial y}\right|_{y=l}=0,$$
$$\lambda_0(t)\left.\frac{\partial t}{\partial y}\right|_{y=-l}=-q_0 S_-(h-|x|), \qquad (2)$$

where $\lambda(x,t)=\lambda_1(t)+[\lambda_0(t)-\lambda_1(t)]\,S_-(h-|x|)$ is the coefficient of heat conductivity of the heterogeneous heat-sensitive plate; $\lambda_0(t)$, $\lambda_1(t)$ – are the coefficients of hat conductivity of the materials of the plate and the inclusion, respectively;

$S_\pm(\zeta)=\begin{cases}1, & \zeta>0,\\ 0,5\mp 0,5, & \zeta=0,\\ 0, & \zeta<0\end{cases}$ is the asymmetric unit functions [26].

Let us introduce the linearizing function

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$$\vartheta = \int_0^{t(x,y)} \lambda_1(\zeta)d\zeta + S_-(x+h)\int_{t(-h,y)}^{t(x,y)} \Big[\lambda_0(\zeta) - \lambda_1(\zeta)\Big] + S_+(x-h)\int_{t(h,y)}^{t(x,y)} \Big[\lambda_0(\zeta)-\lambda_1(\zeta)\Big]d\zeta, \quad (3)$$

having differentiated which with respect to the variables x and y, we obtain

$$\lambda(t,x)\frac{\partial t}{\partial x} = \frac{\partial \vartheta}{\partial x},$$
$$\lambda(t,x)\frac{\partial t}{\partial y} = \frac{\partial \vartheta}{\partial y} + \left\{\Big[\lambda_0(t)-\lambda_1(t)\Big]\frac{\partial t}{\partial y}\right\}\Bigg|_{x=-h} S_-(x+h) - \left\{\Big[\lambda_0(t)-\lambda_1(t)\Big]\frac{\partial t}{\partial y}\right\}\Bigg|_{x=h} S_+(x-h). \quad (4)$$

Taking into account the expression (4), the original equation (1), assumes the following form:

$$\Delta\vartheta + \frac{\partial}{\partial y}\left\{\Big[\lambda_0(t)-\lambda_1(t)\Big]\frac{\partial t}{\partial y}\right\}\Bigg|_{x=-h} S_-(x+h) - \frac{\partial}{\partial y}\left\{\Big[\lambda_0(t)-\lambda_1(t)\Big]\frac{\partial t}{\partial y}\right\}\Bigg|_{x=h} S_+(x-h) = 0, \quad (5)$$

where $\Delta = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ is the Laplace operator in the Cartesian system of coordinates.

The boundary conditions with the use of the relation (3), are written as follows:

$$\frac{\partial\vartheta}{\partial y}\bigg|_{y=l} = 0, \ \ \vartheta\Big|_{|x|\to\infty} = 0, \ \ \frac{\partial\vartheta}{\partial x}\bigg|_{|x|\to\infty} = 0, \quad (6)$$

$$\frac{\partial\vartheta}{\partial y}\bigg|_{y=-l} = -\left\{q_0 S_-(h-|x|) + \left[\Big(\lambda_0(t)-\lambda_1(t)\Big)\frac{\partial t}{\partial y}\right]\Bigg|_{\substack{x=-h\\y=-l}} S_-(x+h) - \left[\Big(\lambda_0(t)-\lambda_1(t)\Big)\frac{\partial t}{\partial y}\right]\Bigg|_{\substack{x=h\\y=-l}} S_+(x-h)\right\}. \quad (7)$$

The linearizing function (3) has enabled us to reduce the nonlinear boundary value problem (1), (2) to the partially linearized equation with discontinuous coefficients (5) and to the boundary conditions (6), (7).

Let us approximate the function $t(\pm h, y)$ in the form of Fig. 2 to the expression

$$t(\pm h, y) = t_1 + \sum_{j=1}^{n-1}\Big(t_{j+1}-t_j\Big)S_-(y-y_j), \quad (8)$$

where $y_j \in]-l;l[$; $y_1 \le y_2 \le \cdots \le y_{n-1}$; n is the number of intervals in the partition $]-l;l[$; $t_j\ (j=\overline{1,n})$ are the unknown approximated values of temperature.

Having substituted the expression (8) into the relations (6), (7), we obtain the following partial linear differential equation relating the linearizing function

$$\Delta\vartheta = -\sum_{j=1}^{n-1}\Big(t_{j+1}-t_j\Big)\Big[\lambda_0(t_{j+1})-\lambda_1(t_{j+1})\Big]S_-(h-|x|)\delta'_-\Big(y-y_j\Big) \quad (9)$$

with the boundary condition

$$\frac{\partial\vartheta}{\partial y}\bigg|_{y=-l} = -q_0 S_-(h-|x|). \quad (10)$$

Here $\delta_\pm(\varsigma) = \dfrac{dS_\pm(\varsigma)}{d\varsigma}$ is the asymmetric Dirac delta-function [26].

Having applied Fourier integral transformation with respect to the coordinate x to the equation (9) and to the boundary condition (6), (10), we obtain the following ordinary differential equation with constant coefficients

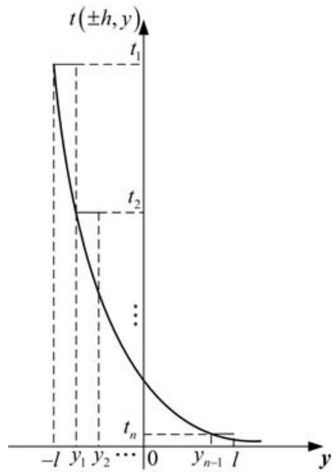

Figure 2 – Approximation of the function $t(\pm h, y)$

$$\frac{d^2\overline{\vartheta}}{dy^2} - \xi^2\overline{\vartheta} = -\sqrt{\frac{2}{\pi}}\frac{\sin\xi h}{\xi}\sum_{j=1}^{n-1}\Big(t_{j+1}-t_j\Big)\Big[\lambda_0(t_{j+1}) - \lambda_1(t_{j+1})\Big]\delta'_-\Big(y-y_j\Big). \quad (11)$$

with the boundary conditions

$$\frac{d\overline{\vartheta}}{dy}\bigg|_{y=l} = 0, \frac{d\overline{\vartheta}}{dy}\bigg|_{y=-l} = -\sqrt{\frac{2}{\pi}}\frac{q_0}{\xi}\sin h\xi, \quad (12)$$

where $\overline{\vartheta} = \dfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\vartheta e^{i\xi x}dx$ is the transform formula $\vartheta(x,y)$; ξ is the parameter of Fourier integral transformation; $i^2 = -1$.

Having solved the problem (11), (12), then having applied the inverse to its solution Fourier integral transformation, we obtain the expression of the function ϑ

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$$\vartheta = \frac{2}{\pi}\int_0^\infty \frac{1}{\xi}\sin h\xi \cos x\xi \left\{ \sum_{j=1}^{n-1}(t_{j+1}-t_j)\left[\lambda_0(t_{j+1}) - \right.\right.$$
$$\left. -\lambda_1(t_{j+1})\right][\frac{ch\xi(y+l)}{sh2\xi l}sh\xi(l-y_j) -$$
$$\left. -ch\xi(y-y_j)S_-(y-y_j)] + \frac{q_0}{\xi}\frac{ch\xi(y-l)}{sh2\xi l}\right\} d\xi. \quad (13)$$

Having substituted the temperature dependence of the coefficient of heat conductivity of the materials of the plate and that of the inclusion into the relations (3), (13), after some transformations, we obtain the system of nonlinear equations for determining the unknown approximated values of the temperature $t_j\,(j=\overline{1,n})$.

For the given system, the unknown temperature field we determine by means of the obtained nonlinear equation with the use of the relations (3), (13) after substitution into them the concrete expressions of the dependence of coefficient of heat conductivity of the structural materials on temperature.

4 EXPERIMENTS

To solve many practical problems, the following dependence of the coefficient of thermal conductivity on temperature is used [27, 28]:

$$\lambda = \lambda_m^0\left(1-k_m t\right), \quad (14)$$

where λ_m^0, k_m are basic and temperature coefficients of heat conductivity of the materials of the inclusion (m=0) and the plate (m=1).

Numerical analysis of the dimensionless temperature $t^* = \lambda_0 t/(q_0 h)$ is carried out for the following initial data: material of the layer is VK94-I ceramics, the material of the inclusion is silver, n=10 is the number of intervals in the partition $]-l;l[;l/h=1$. In the temperature interval $[20°C;1230°C]$, the temperature dependencies of the the coefficient of heat conductivity for the given materials are expressed as follows:

$$\lambda_1(t) = 13{,}67\frac{\text{W}}{\text{Km}}(1-0{,}00064\frac{1}{\text{K}}t),$$
$$\lambda_0(t) = 422{,}54\frac{\text{W}}{\text{Km}}(1-0{,}00031\frac{1}{\text{K}}t), \quad (15)$$

which is a partial case of the relation (14).

Let us express the temperature field for the corresponding linear model from the following relation

$$\lambda(x)\,t(x,y) = \frac{1}{\pi}\int_0^\infty \frac{1}{\xi}\{(\lambda_0-\lambda_1)[t_1 +$$
$$+\sum_{j=1}^{n-1}(t_{j+1}-t_j)(\frac{ch\xi(l+y)}{sh2\xi l}sh\xi(l-y_j) +$$
$$+(1-ch\xi(y-y_j))S_-(y-y_j))][\sin\xi(x+h)-\sin\xi(x-h)] +$$
$$+\frac{2q_0 ch\xi(y-l)}{\xi sh2\xi l}\cos\xi x\,\sin\xi h\}d\xi.$$

The given expression enables us to obtain numerical results represented in Fig. 3b for the considered structure materials with the constant coefficients of their heat conductivities

$$\lambda_1 = 13{,}4\text{W}/(\text{Km}),\ \lambda_0 = 419\text{W}/(\text{Km}).$$

The dependence of the dimensionless temperature t on the special dimensionleess coordinate $x^* = x/h$ and $y^* = y/h$ is shown in Fig. 3. Note, that the maximal value of temperature is reached in the domain of the action of the concentrated heat stream, and at the edge $K_{\pm h}$ of the inclusion, the satisfaction of the conditions of perfect heat contact (no temperature jump) is observed; this is in accordance to the considerate mathematical model.

The change in dimensionleess temperature t^* depending on the dimensionless coordinates y^* for $x^*=0$ (Fig. 4a) and on x^* for $y^*=0$ (Fig. 4b) is shown in Fig. 4. The behavior of the curves indicates the adequacy of the mathematical model to the actual physical process, since at the surface of the inclusion the perfect contact is observed (no jump in temperature).

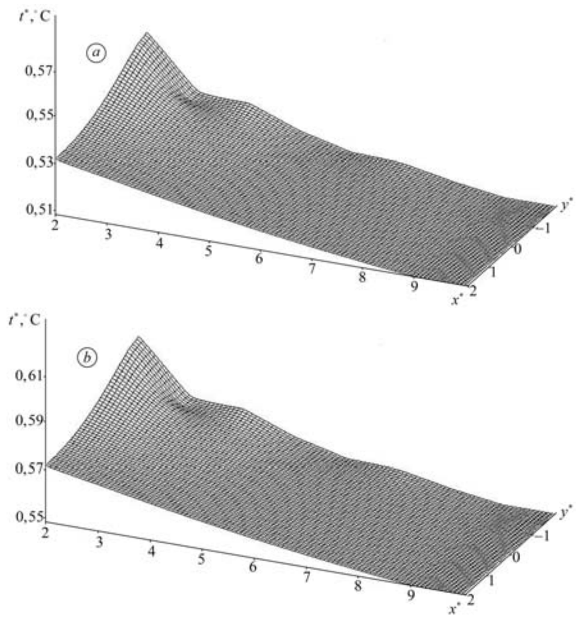

Figure 3 – Dependence of dimensionless temperature t^* on dimensionless coordinates x^* and y^* for linearly variable (a) and constant (b) coefficient of heat conductivity of materials of structure

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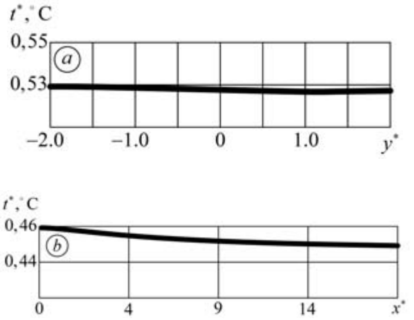

Figure 4 – Dependence of dimensionless temperature t^* on dimensionless coordinates y^* for $x^* = 0$ (*a*) and x^* for $y^* = 0$ (*b*).

5 RESULTS

Having taken into account the relationship (14), from the expressions (3), (13), we obtain the formulae for determining the temperature t in the domain of the inclusion

$$t = \frac{1}{k_0}\left(1 - \sqrt{1 - k_0\left(\frac{2\vartheta}{\lambda_0^0} + \vartheta_1\right)}\right), \qquad (16)$$

in the domain $\Omega_1 = \{(x, y, z) : |x| > h, |y| \le l, |z| \le \delta\}$ of the plate (beyond the inclusion)

$$t = \frac{1}{k_1}\left(1 - \sqrt{1 - \frac{2k_1\vartheta}{\lambda_1^0}}\right). \qquad (17)$$

Here

$$\vartheta_1 = \left\{ t\left[2 - k_0 t - \frac{\lambda_1^0}{\lambda_0^0}(2 - k_1 t)\right]\right\}\Bigg|_{x=h};$$

$$t\Big|_{x=h} = \frac{1}{k_1}\left(1 - \sqrt{1 - \frac{2k_1}{\lambda_1^0}\,\vartheta\Big|_{x=h}}\right).$$

The formulae (16), (17) completely describe the temperature field in a heat-sensitive layer with a foreign-through inclusion (fig. 1).

The number of intervals n=10 in the partition $]-l;l[$ for the given thermophysical (basic and temperature coefficients of heat conductivity for the materials of the plate and that of the inclusion) and geometric (length and width of the inclusion, width of the plate) parameters of the structure enables us to perform calculations accurate to $\varepsilon = 10^{-6}$.

6 DISCUSSION

In the course of development and investigation of linear and non-linear mathematical models of heat conduction process for structures which are geometrically described by means of the presented piece-wise homogeneous structures, it is detected that the numerical results of temperature field for the considered materials in the case of constant coefficient of thermal conductivity differ by 7% from those for linear variable. This indicates that the consideration of the dependence of thermophysical parameters on temperature of materials of structural elements of complex systems are important, since results which are obtained with the use of nonlinear models are more accurate. The conscideration of piece-wise homogeneous structure of elements of the object is also important in the investigations presented; this makes the solving of corresponding linear and non-linear boundary value problems considerably more complicated; but solution of such problems describes temperature distribution in more adequate way to the actual process.

CONCLUSIONS

The introduced linearizing function enables us to partially linearize the original nonlinear boundary value problem, and the suggested piecewise-linear approximation of temperature at the boundary surfaces of the inclusion to completely linearize the equation and the boundary condition. Due to this it became possible to apply Fourier integral transformation to the obtained linear problem relating the suggested function and to obtain its analytical-numerical solution. The linear temperature dependence of coefficient of heat conductivity of the materials of the inclusion and that of the plate is considered. On the basis of this, the formulae for the calculation of the values of the temperature in the considered “plate-inclusion” structure are given. The obtained results for the chosen materials under linear dependence of coefficient of heat conductivity on temperature (Fig. 3a) differ from the results obtained under the condition of constant coefficient (Fig. 3b) by 7%. Their inconsiderable difference is accounted for by the fact that the actual values of the temperature coefficient of the heat conductivity for the considered materials are small.

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ДОСЛІДЖЕННЯ ТЕМПЕРАТУРНИХ РЕЖИМІВ У НЕОДНОРІДНИХ ТЕРМОЧУТЛИВИХ ЕЛЕМЕНТАХ РАДІОЕЛЕКТРОННИХ ПРИСТРОЇВ

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АНОТАЦІЯ

Актуальність. Розв'язано нелінійну крайову задачу теплопровідності для термочутливого неоднорідного елемента радіоелектронної системи у вигляді смуги з наскрізним включенням, аналітично-числовий розв'язок якої дає змогу аналізувати у ньому температурні режими. Мета роботи – розробка методу лінеаризації нелінійної математичної моделі теплопровідності, який дає змогу отримати аналітично-числовий розв'язок відповідної нелінійної крайової задачі для визна-

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чення температурного поля в елементі радіоелектронних пристроїв, які геометрично зображено термочутливою пластиною з наскрізним включенням.

Метод. Запропоновано лінеаризуючу функцію, яка дає змогу частково лінеаризувати вихідну нелінійну математичну модель теплопровідності для термочутливого неоднорідного елемента радіоелектронної системи у вигляді конструкції «пластина-включення». Введена кусково-лінійна апроксимація температури на поверхнях спряження пластини з включенням дозволила повністю лінеаризувати відповідну частково лінеаризовану крайову задачу відносно лінеаризуючої функції. Після цього стало можливим застосувати інтегральне перетворення Фур'є за однією з просторових координат до отриманої лінійної задачі та визначити лінеаризуючу функцію. Розглянуто лінійну залежність коефіцієнта теплопровідності від температури для конструкційних матеріалів з використанням лінеаризуючої функції. Шляхом розв'язання крайової задачі отримано формули для визначення температурного поля в термочутливій конструкції «пластина-включення».

Результати. Із використанням отриманих формул для визначення температурного поля у термочутливому неоднорідному елементі радіоелектронної системи створено обчислювальні програми, які дають змогу отримати числові значення розподілу температури та аналізувати температурні режими.

Висновки. Розроблена математична модель розрахунку температурного поля в термочутливій конструкції «пластина-включення» є адекватною до реального фізичного процесу так як не спостерігається стрибка температури на поверхнях спряження пластини з включенням. Числові результати для вибраних матеріалів за лінійної залежності коефіцієнта теплопровідності від температури відрізняються від результатів, отриманих для сталого коефіцієнта теплопровідності, на 7%. Перспективи подальших досліджень полягатимуть у розгляді складніших геометричних зображень елементів радіоелектронних систем.

КЛЮЧОВІ СЛОВА: температура, теплопровідність, нелінійна крайова задача, ізотропна безмежна термочутлива пластина з теплоізольованими лицевими поверхнями, чужорідне наскрізне включення, ідеальний тепловий контакт, тепловий потік.

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ИССЛЕДОВАНИЕ ТЕМПЕРАТУРНЫХ РЕЖИМОВ В НЕОДНОРОДНЫХ ТЕРМОЧУВСТВИТЕЛЬНЫХ ЭЛЕМЕНТАХ РАДИОЭЛЕКТРОННЫХ УСТРОЙСТВ

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АННОТАЦИЯ

Актуальность. Получено решение нелинейной краевой задачи теплопроводности для термочувствительного неоднородного элемента радиоэлектронной системы в виде полосы со сквозным включением, численно-аналитическое решение которой позволяет анализировать в нем температурные режимы. Цель работы – разработка метода линеаризации нелинейной математической модели теплопроводности, который позволяет получить численно-аналитическое решение соответствующей нелинейной краевой задачи для определения температурного поля в элементе радиоэлектронных устройств, которые геометрически изображены термочувствительной пластиной со сквозным включением.

Метод. Предложено линеаризирующую функцию, которая позволяет частично линеаризировать исходную нелинейную математическую модель теплопроводности для термочувствительного неоднородного элемента радиоэлектронной системы в виде конструкции «пластина-включение». Введеная кусочно-линейная аппроксимация температуры на поверхностях сопряжения пластины с включением позволила полностью линеаризировать соответствующую частично линеаризованную краевую задачу относительно линеаризирующей функции. После этого стало возможным применить интегральное преобразование Фурье по одной из пространственных координат к полученной линейной задаче и определить линеаризирующую функцию. Рассмотрено линейную зависимость коэффициента теплопроводности от температуры для конструкционных материалов с использованием линеаризирующей функции. Путем решения краевой задачи получены формулы для определения температурного поля в термочувствительной конструкции «пластина-включение».

Результаты. С использованием полученных формул для определения температурного поля в термочувствительном неоднородном элементе радиоэлектронной системы созданы вычислительные программы, которые позволяют получить числовые значения распределения температуры и анализировать температурные режимы.

Выводы. Разработана математическая модель расчета температурного поля в термочувствительной конструкции «пластина-включение» является адекватной к реальному физическому процессу так как не наблюдается скачка температуры на поверхностях сопряжения пластины с включением. Числовые результаты для выбранных материалов по линейной зависимости коэффициента теплопроводности от температуры отличаются от результатов, полученных для постоянного коэффициента теплопроводности, на 7%. Перспективы дальнейших исследований будут заключаться в рассмотрении сложных геометрических изображений элементов радиоэлектронных систем.

КЛЮЧЕВЫЕ СЛОВА: температура, теплопроводность, нелинейная краевая задача, изотропная термочувствительная пластина с теплоизолированными лицевыми поверхностями, инородное сквозное включения, идеальный тепловой контакт, тепловой поток.

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