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TELETRAFFIC MODEL BASED ON $HE_2/H_2/1$ SYSTEMS WITH ORDINARY AND WITH SHIFTED INPUT DISTRIBUTIONS

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ABSTRACT

Context. The problem of deriving a solution for the average waiting time in a closed form queue for an ordinary system with second-order hyper-Erlang and hyperexponential input distributions and a system with shifted hyper-Erlang and hyperexponential input distributions is considered.

Objective. Obtaining a solution for the main characteristic of the system – the average waiting time for requirements in the queue for a queuing system of type G/G/1 with conventional and shifted second-order hyper-Erlang and hyperexponential input distributions.

Method. To solve this problem, we used the classical method of spectral decomposition of the solution of the Lindley integral equation, which allows us to obtain a solution for the average waiting time for the systems in question in a closed form. The spectral decomposition method for solving the Lindley integral equation occupies an important part of the theory of G/G/1 systems. For the practical application of the results obtained, the well-known method of moments of probability theory is used.

Results. For the first time, spectral decompositions of the solution of the Lindley integral equation for both systems were obtained, with the help of which calculation formulas for the average waiting time in the queue for the above systems in closed form are derived. This approach allows you to calculate the average waiting time for these systems in mathematical packages for a wide range of traffic parameters. All other system characteristics are derived from the average waiting time.

Conclusions. It is shown that the hypererlang second-order distribution law, as well as the hyperexponential one, which is three-parameter, can be determined by both the first two moments and the first three moments. The choice of this law of probability distribution is because its coefficient of variation covers a wider range than for hyperexponential distribution. For shifted hypererlang and hyperexponential distribution laws, the coefficients of variation decrease and cover an even wider range than for conventional distributions. The introduction of time-shifted distributions expands the scope of QS taking into account the well-known fact from the queuing theory that the average waiting time is associated with the coefficients of variation of the intervals of arrivals and the service time by a quadratic dependence. The spectral decomposition method for solving the Lindley integral equation for a queuing system with second-order hyper-Erlang and hyperexponential input distributions allows us to obtain a solution in a closed form and this solution is published for the first time. The resulting solution complements and extends the well-known queuing theory formula for the average queue waiting time for queuing systems of type G/G/1.

KEYWORDS: hypererlangian and hyperexponential distribution laws, Lindley integral equation, spectral decomposition method, Laplace transform.

ABBREVIATIONS

LIE is a Lindley integral equation;

G/G/1 is a QS with arbitrary laws of distribution of intervals between receipt of requirements and service time;

QS is a queuing system;

PDF is a probability distribution function.

NOMENCLATURE

$a(t)$ is a density function of the distribution of time between arrivals;

$A^*(s)$ is a Laplace transform of the function $a(t)$;

$b(t)$ is a density function of the distribution of service time;

$B^*(s)$ is a Laplace transform of the function $b(t)$;

c_λ is a coefficient of variation of time between arrivals;

c_μ is a coefficient of variation of service time;

D_λ is a variance of a random interval between arrivals;

D_μ is a service time variance;

M is an exponential distribution law;

H_2 is a hyperexponential distribution law of the second order;

H_2^- is a shifted hyperexponential distribution law of the second order;

HE_2 is a hypererlangian distribution law of the second order;

HE_2^- is a shifted hypererlangian distribution law of the second order;

$\bar{I}$ is an average idle time;

$\overline{I^2}$ is a second initial moment of the idle period;

p is a parameter of the hyper-Erlang law;

q is a parameter of the hyperexponential law;

$W(y)$ is a PDF of the waiting time in the queue;

$\bar{W}$ is an average waiting time in the queue;

$W^*(s)$ is a Laplace transform of waiting time density function;

z is an any number from the interval (–1, 1);

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λ is an input flow rate;

λ_1, λ_2 are parameters of the hypererlangian distribution law of the input flow;

μ is a service intensity;

μ_1, μ_2 are parameters of the hyperexponential distribution law of service time;

ρ is a system load factor;

$\overline{\tau}_\lambda$ is an average time between arrivals;

$\overline{\tau}_\lambda^2$ is a second initial moment of time between arrivals;

$\overline{\tau}_\mu$ is an average service time;

$\overline{\tau_\mu^2}$ is a second initial moment of service time;

$\Phi_+(s)$ is a Laplace transform of the PDF of waiting time;

$\psi_+(s)$ is a first component of spectral decomposition;

$\psi_-(s)$ is a second component of spectral decomposition;

$\chi(t)$ is a characteristic function of a random variable ξ.

INTRODUCTION

The article is devoted to the analysis of QS of type G/G/1 with arbitrary laws of the distribution of the input flow of requirements and the time of their servicing, for which, in the general case, a solution cannot be found for the main characteristic – the average waiting time of the requirements in the queue. Therefore, systems of the type G/G/1 can be studied only with specific laws of the distributions of the input flow of service time [1–3]. As is known, for example, from [1], for the G/G/1 system, the average waiting time is determined by the expression

$$\overline{W} = \frac{D_\lambda + D_\mu + (1-\rho)^2/\lambda^2}{2(1-\rho)/\lambda} - \frac{\overline{I^2}}{2\overline{I}}. \quad (1)$$

Since expression (1) is associated with the coefficients of variation of the intervals of receipt and service by a quadratic dependence, the role of the latter for the value of the average waiting time is significant. The second term on the right-hand side of (1) remains unknown and it is likely that it may depend on the moments of the arrival intervals and the service time of a higher order than the first two. Because of this, formula (1) will be considered incomplete so far. In teletraffic theory, using average waiting time, packet delays in packet switching networks are estimated. In this paper, we restrict ourselves to the second-order hyper-Erlang distribution of the intervals between the input flow requirements and the second-order hyperexponential distribution of the service time because, when the order is higher than two are, further calculations become extremely time-consuming.

The object of study is the queueing systems type G/G/1.

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The subject of study is the average waiting time in systems $HE_2/H_2/1$ and $HE_2^-/H_2^-/1$.

The purpose of the work is obtaining a solution for the average waiting time of requirements in the queue in closed form for the above-mentioned systems.

In the queueing theory, the studies of G/G/1 systems are relevant because they are actively used in modern teletraffic theory, moreover, one cannot obtain solutions for such systems in the final form for the general case. The laws of the Weibull or Gamma distributions of the most general form, which provide the range of variation of the coefficients of variation from 0 to ∞ depending on the value of their parameters, are not applicable in the spectral decomposition method. This is because the Laplace transform of the density function for these distributions cannot be expressed in elementary functions. Therefore, it is necessary to use other private laws of distributions.

In the study of G/G/1 systems, an important role is played by the method of spectral decomposition of the solution of the Lindley integral equation and most of the results in the theory of mass service are obtained using this method.

1 PROBLEM STATEMENT

The article poses the problem of finding a solution for the waiting time of requirements in a queue in QS $HE_2/H_2/1$ with ordinary and shifted with distributions and constructing a mechanism for approximating arbitrary distribution laws with hyper-Erlang and hyperexponential ones. To study the G/G/1 systems, as is known, for example, from [1–4], the Lindley integral equation is used. One form of the Lindley integral equation (LIE) looks like this:

$$W(y) = \begin{cases} \int\limits_{-\infty}^{y} W(y-u)\,\mathrm{d}C(u), & y \ge 0; \\ 0, & y < 0. \end{cases}$$

Here is $C(u)$ is PDF of the random variable $\tilde{u} = \tilde{x} - \tilde{t}$, where, in turn, $\tilde{x}$ is the random service time and the random variable $\tilde{t}$ is the interval between arrivals.

When applying the method for solving the LIE, we will adhere to the author's approach and symbolism [1], as was done in the author's early works. For this, by $A^*(s)$ and $B^*(s)$ we denote the Laplace transforms of the density functions of the distribution of intervals between arrivals and service time, respectively.

We need to find the law of waiting time distribution in the system through the spectral decomposition of the form: $A^*(-s)\cdot B^*(s) - 1 = \psi_+(s)/\psi_-(s)$, where $\psi_+(s)$ and $\psi_-(s)$ are some rational functions of s that can be factorized. Functions $\psi_+(s)$ and $\psi_-(s)$ must satisfy the following conditions according to [1]. To solve the problem, it is necessary first to construct for these systems spectral decompositions of the form $A^*(-s)\cdot B^*(s)-1=\psi_+(s)/\psi_-(s)$.

2 REVIEW OF THE LITERATURE

The method of spectral decomposition of the solution of the Lindley integral equation was first presented in detail in the classic queueing theory [1], and was subsequently used in many papers, including [2,3]. A different approach to solving Lindley's equation has been used in [4]. That work used factorization instead of the term "spectral decomposition" and instead of the functions $\psi_+(s)$ and $\psi_-(s)$ it used factorization components $\omega_+(z,t)$ and $\omega_-(z,t)$ of the function $1-z\cdot\chi(t)$, where $\chi(t)$ is the characteristic function of a random variable ξ with an arbitrary distribution function $C(t)$, and z is any number from the interval $(-1, 1)$.

In the scientific literature, including the web resources of specialized queue theory journals, the author was not able to find the results on the waiting time for QS with hyper-Erlang and hyperexponential input distributions of the 2nd order of the general form.

In the field of systems with delay in time, the authors published the following works. In [5] the results on systems with delay $H_2/H_2/1$, $H_2/M/1$, $M/H_2/1$ are given, in [6] – on system with delay $HE_2/HE_2/1$, in [7] – on systems with a delay based on the QS $E_2/E_2/1$, $E_2/M/1$, $M/E_2/1$, and in [8] – on systems with a delay based on the QS $HE_2/M/1$. Article [9] presents the results for a system with a delay $M/HE_2/1$, and article [10] summarizes the results for eight systems with a delay in time.

In [11] presents the results of the approach of queues to the Internet and mobile services as queues with a delay in time. Approximate methods with respect to the laws of distributions are described in detail in [3, 13–15], and similar studies in queuing theory have recently been carried out in [16–24].

3 MATERIALS AND METHODS

For the $HE_2/H_2/1$ system, the distribution laws of the input flow intervals and the service time are given by the density functions of the form:

$$a(t)=4p\lambda_1^2te^{-2\lambda_1 t}+4(1-p)\lambda_2^2te^{-2\lambda_2 t}, \tag{2}$$

$$b(t)=q\mu_1e^{-\mu_1 t}+(1-q)\mu_2e^{-\mu_2 t}. \tag{3}$$

Distributions (2) and (3) in the scientific literature are denoted by HE_2 and H_2. They contain three parameters $0<p<1$, $\lambda_1,\lambda_2>0$ and $0<q<1$, $\mu_1,\mu_2>0$ respectively, therefore, they allow you to approximate arbitrary input distributions at the level of the first three moments using the well-known method of moments. It was shown in [6] that the distribution of HE_2 as well as H_2 can be unambiguously described using both the two and three first moments.

We write the Laplace transform functions (2) and (3):

$$A^*(s)=p\left(\frac{2\lambda_1}{s+2\lambda_1}\right)^2+(1-p)\left(\frac{2\lambda_2}{s+2\lambda_2}\right)^2,$$

$$B^*(s)=q\frac{\mu_1}{s+\mu_1}+(1-q)\frac{\mu_2}{s+\mu_2}.$$

The expression $A^*(-s)\cdot B^*(s)-1=\psi_+(s)/\psi_-(s)$ for the spectral decomposition of the solution of the LIE for the $HE_2/H_2/1$ system takes the form:

$$\frac{\psi_+(s)}{\psi_-(s)}=\left[p\left(\frac{2\lambda_1}{2\lambda_1-s}\right)^2+(1-p)\left(\frac{2\lambda_2}{2\lambda_2-s}\right)^2\right]\times \\ \times\left[q\frac{\mu_1}{\mu_1+s}+(1-q)\frac{\mu_2}{\mu_2+s}\right]-1. \tag{4}$$

The first factor in (4) on the right side in square brackets is:

$$\left[p\left(\frac{2\lambda_1}{2\lambda_1-s}\right)^2+(1-p)\left(\frac{2\lambda_2}{2\lambda_2-s}\right)^2\right]= \\ =\frac{p\left(16\lambda_1^2\lambda_2^2-16\lambda_1^2\lambda_2 s+4\lambda_1^2s^2\right)}{(2\lambda_1-s)^2(2\lambda_2-s)^2}+ \\ +\frac{(1-p)\left(16\lambda_1^2\lambda_2^2-16\lambda_1\lambda_2^2 s+4\lambda_2^2s^2\right)}{(2\lambda_1-s)^2(2\lambda_2-s)^2}= \\ =\frac{a_0-a_1s+a_2s^2}{(2\lambda_1-s)^2(2\lambda_2-s)^2},$$

intermediate parameters used here $a_0=16\lambda_1^2\lambda_2^2$, $a_1=16\lambda_1\lambda_2[p\lambda_1+(1-p)\lambda_2]$, $a_2=4[p\lambda_1^2+(1-p)\lambda_2^2]$.

Similarly, imagine the second factor:

$$\left[q\frac{\mu_1}{\mu_1+s}+(1-q)\frac{\mu_2}{\mu_2+s}\right]=\frac{\mu_1\mu_2+\left[q\mu_1+(1-q)\mu_2\right]s}{(\mu_1+s)(\mu_2+s)}= \\ =\frac{b_0+b_1s}{(\mu_1+s)(\mu_2+s)},$$

intermediate parameters used here $b_0=\mu_1\mu_2$, $b_1=q\mu_1+(1-q)\mu_2$.

Continuing the decomposition, we obtain:

$$\frac{\psi_+(s)}{\psi_-(s)}=\frac{(a_0-a_1s+a_2s^2)(b_0+b_1s)}{(2\lambda_1-s)^2(2\lambda_2-s)^2(\mu_1+s)(\mu_2+s)}- \\ -\frac{(2\lambda_1-s)^2(2\lambda_2-s)^2(\mu_1+s)(\mu_2+s)}{(2\lambda_1-s)^2(2\lambda_2-s)^2(\mu_1+s)(\mu_2+s)}= \\ =\frac{-s(s^5-c_4s^4-c_3s^3-c_2s^2-c_1s-c_0)}{(2\lambda_1-s)^2(2\lambda_2-s)^2(\mu_1+s)(\mu_2+s)}= \\ =\frac{-s(s+s_1)(s+s_2)(s-s_3)(s-s_4)(s-s_5)}{(2\lambda_1-s)^2(2\lambda_2-s)^2(\mu_1+s)(\mu_2+s)}. \tag{5}$$

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The polynomial in the numerator on the right-hand side of expansion (5) usually always has one zero $s=0$ [1]. In this case, the free term of the expansion is also equal to 0: $a_0 b_0 - 16\lambda_1^2\lambda_2^2\mu_1\mu_2 \equiv 0$. In the numerator, the fractions on the right side of the expansion obtained a polynomial of the sixth degree $-s(s^5 - c_4 s^4 - c_3 s^3 - c_2 s^2 - c_1 s - c_0)$, whose coefficients are equal to:

$$
\begin{gathered}
c_0 = a_0 b_1 - a_1 b_0 - a_0(\mu_1+\mu_2) + 16\lambda_1\lambda_2\mu_1\mu_2(\lambda_1+\lambda_2), \\
c_1 = -a_1 b_1 + a_2 b_0 - a_0 + 16\lambda_1\lambda_2(\lambda_1+\lambda_2)(\mu_1+\mu_2) - \\
-4\mu_1\mu_2[(\lambda_1+\lambda_2)^2 + 2\lambda_1\lambda_2], \\
c_2 = a_2 b_1 + 4(\lambda_1+\lambda_2)(4\lambda_1\lambda_2 + \mu_1\mu_2) - \\
-4[(\lambda_1+\lambda_2)^2 + 2\lambda_1\lambda_2](\mu_1+\mu_2), \\
c_3 = 4(\lambda_1+\lambda_2)(\mu_1+\mu_2) - 4(\lambda_1^2+\lambda_2^2) - 16\lambda_1\lambda_2 - \mu_1\mu_2, \\
c_4 = 4(\lambda_1+\lambda_2) - \mu_1 - \mu_2 .
\end{gathered}
\tag{6}
$$

These coefficients are obtained by performing symbolic operations of the Mathcad mathematical package on the decomposition numerator (5), because in the numerator of the decomposition, 42 terms are obtained, and it is rather problematic to manually process and bring such terms. Perhaps that is why this problem was solved for the first time. Next, we select the polynomial in the decomposition numerator (5)

$$
s^5 - c_4 s^4 - c_3 s^3 - c_2 s^2 - c_1 s - c_0 , \tag{7}
$$

because determination of its roots and work with them is an important point in the method of spectral decomposition of the solution of LIE.

The study of polynomial (7) with coefficients (6) using the Vietta formulas confirms the presence of two negative real roots as well as three positive real roots or one positive and two complex conjugate roots with positive real parts. The study of the sign of the least coefficient $c_0 > 0$ of the polynomial (9) shows that it is always in the case of a stable system when $0 < \rho = \overline{\tau}_\mu / \overline{\tau}_\lambda < 1$. In the general case, the presence of such roots follows from the existence and uniqueness of the spectral decomposition [1] or factorization [4].

Denoting the roots of the polynomial (7) with negative real parts, for convenience, by $-s_1, -s_2$, and with positive real parts, through s_3, s_4, s_5, the relation $\psi_+(s)/\psi_-(s)$ can finally be decomposed into the following factors:

$$
\frac{\psi_+(s)}{\psi_-(s)} = \frac{-s(s+s_1)(s+s_2)(s-s_3)(s-s_4)(s-s_5)}{(2\lambda_1 - s)^2(2\lambda_2 - s)^2(\mu_1+s)(\mu_2+s)} . \tag{8}
$$

Therefore, taking into account conditions [1], we take for the function $\psi_+(s) = \dfrac{s(s+s_1)(s+s_2)}{(\mu_1+s)(\mu_2+s)}$, because the zeros of the polynomial (7): $s=0$, $s=-s_1$, $s=-s_2$, and the poles $s=-\mu_1$, $s=-\mu_2$ lie in the region $\mathrm{Re}(s) \le 0$, and for the function $\psi_-(s) = -\dfrac{(2\lambda_1 - s)^2(2\lambda_2 - s)^2}{(s-s_3)(s-s_4)(s-s_5)}$, because its zeros and poles lie in the region $\mathrm{Re}(s) < D$ defined by condition [1].

The fulfillment of conditions for the constructed functions also confirms the figure 1 where the zeros and poles of the relation are displayed on the complex s-plane to eliminate errors in constructing the spectral decomposition. In Figure 1, the poles are marked with crosses, and zeros are indicated by circles.

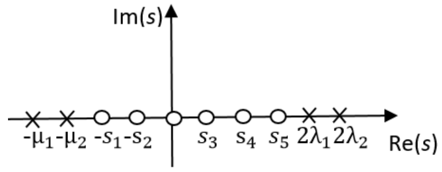

Figure 1 – Zeros and poles of the function $\psi_+(s)/\psi_-(s)$ for the system $HE_2/H_2/1$

Next, by the method of spectral decomposition, we determine the constant $K = \lim\limits_{s\to 0} \dfrac{\psi_+(s)}{s} = \dfrac{s_1 s_2}{\mu_1\mu_2}$, where s_1, s_2 are the absolute values of the negative roots $-s_1, -s_2$. The constant K determines the probability that the demand entering the system finds it free.

Using the function $\psi_+(s)$ and constant K, we define the Laplace transform of the PDF waiting time $W(y)$:

$$
\Phi_+(s) = \frac{K}{\psi_+(s)} = \frac{s_1 s_2 (s+\mu_1)(s+\mu_2)}{s(s+s_1)(s+s_2)\mu_1\mu_2} .
$$

From here, the Laplace transform of the waiting time density function $W^*(s) = s \cdot \Phi_+(s)$ is

$$
W^*(s) = \frac{s_1 s_2 (s+\mu_1)(s+\mu_2)}{(s+s_1)(s+s_2)\mu_1\mu_2} . \tag{9}
$$

To find the average waiting time, we find the derivative of the function $W^*(s)$ with a minus sign at the point s=0:

$$
-\frac{dW^*(s)}{ds}\Big|_{s=0} = -\frac{s_1 s_2 (s+\mu_1)(s+\mu_2)}{(s+s_1)(s+s_2)\mu_1\mu_2}\Big|_{s=0} =
$$
$$
= \frac{1}{s_1} + \frac{1}{s_2} - \frac{1}{\mu_1} - \frac{1}{\mu_2} .
$$

Finally, the average wait time for the $HE_2/H_2/1$ system

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$$\overline{W} = \frac{1}{s_1} + \frac{1}{s_2} - \frac{1}{\mu_1} - \frac{1}{\mu_2}, \quad (10)$$

where s_1, s_2 the absolute values of the negative roots are $-s_1, -s_2$ of the polynomial (9) with the coefficients given above, and μ_1, μ_2 – the distribution parameters (3). Thus, for the average waiting time in the QS $HE_2/H_2/1$, the solution in closed form (10) is obtained.

From the expression (9), if necessary, you can also determine the moments of higher orders of the waiting time, for example, the second derivative of the transformation (9) at the point $s=0$ gives the second initial moment of the waiting time $\overline{W^2} = 2[(\mu_1^{-1}+\mu_2^{-1})(s_1^{-1}+s_2^{-1}) - (s_1^{-1}+s_2^{-1})^2 + (s_1^{-1}s_2^{-1} - \mu_1^{-1}\mu_2^{-1})]$, which allows you to determine the dispersion of the waiting time. Given the definition of jitter in telecommunications as the spread of the waiting time around its average value [12], we will thereby be able to determine jitter through dispersion. This is an important result for delay-sensitive traffic analysis.

The problem of approximating the distribution law (2) using both the first two moments and the first three moments is considered in detail in [6]. To do this, we use the initial moments up to the third order found for the distribution (2) found based on the Laplace transform property of the moment reproductions:

$$\overline{\tau}_\lambda = \frac{p}{\lambda_1} + \frac{(1-p)}{\lambda_2},\ \overline{\tau_\lambda^2} = \frac{3}{2}\left[\frac{p}{\lambda_1^2} + \frac{(1-p)}{\lambda_2^2}\right], \quad \overline{\tau_\lambda^3} = \frac{3p}{\lambda_1^3} + \frac{3(1-p)}{\lambda_2^3}. \quad (11)$$

When approximating using the first two moments, the unknown distribution parameters (2) are determined using the following expressions:

$$\lambda_1 = 2p/\overline{\tau}_\lambda,\ \lambda_2 = 2(1-p)/\overline{\tau}_\lambda,\ p = \frac{1}{2} \pm \sqrt{\frac{2(1+c_\lambda^2)-3}{8(1+c_\lambda^2)}}.$$

In this case, for the probability *p* you can take any of these values. It follows that the coefficient of variation of time between arrivals $c_\lambda \ge 1/\sqrt{2}$. When approximating using the first three moments to find the distribution parameters (2), it is necessary to solve the system of three equations of the method of moments (11) in the Mathcad package. Moreover, a necessary and sufficient condition for the existence of a solution is the fulfillment of the condition: $\overline{\tau_\lambda^3} \cdot \overline{\tau_\lambda} \ge \overline{\tau_\lambda^2}$. From (11) it follows that the square of coefficient of variation of time between arrivals

$$c_\lambda^2 = \frac{\lambda_1^2 - 2p\lambda_2(\lambda_1-\lambda_2) + p(1-2p)(\lambda_1-\lambda_2)^2}{2[(1-p)\lambda_1 + p\lambda_2]^2}. \quad (12)$$

The same problem for distribution law (3) using both the first two moments and the first three moments was considered in detail by the author in [5]. To do this, we write the expressions for the initial moments of distribution (3):

$$\overline{\tau}_\mu = \frac{q}{\mu_1} + \frac{(1-q)}{\mu_2},\ \overline{\tau_\mu^2} = \frac{2q}{\mu_1^2} + \frac{2(1-q)}{\mu_2^2}, \quad \overline{\tau_\mu^3} = \frac{6q}{\mu_1^3} + \frac{6(1-q)}{\mu_2^3}. \quad (13)$$

In this case, to determine the unknown parameters, the following expressions are obtained

$$\mu_1 = 2q/\overline{\tau}_\mu,\ \mu_2 = 2(1-q)/\overline{\tau}_\mu,\ q = \frac{1}{2}[1 \pm \sqrt{(c_\mu^2-1)/(c_\mu^2+1)}].$$

In this case, for the probability *q* you can take any of these values. It follows that the coefficient of variation of the service time $c_\mu \ge 1$.

From (13) it follows that the square of the coefficient of variation of the service time will be equal to

$$c_\mu^2 = \frac{(1-q^2)\mu_1^2 - 2q(1-q)\mu_1\mu_2 + q(2-q)\mu_2^2}{[(1-q)\mu_1 + q\mu_2]^2}. \quad (14)$$

When approximating using the first three moments, in order to find the distribution parameters (3), it is necessary in the Mathcad package to solve the system of three equations (13) obtained by the method of moments. In this case, a necessary and sufficient condition for the existence of a solution is the fulfillment of the condition: $\overline{\tau_\lambda^3} \cdot \overline{\tau_\lambda} \ge 1{,}5\overline{\tau_\lambda^2}$ [13]. Due to the simplicity of calculations, we will dwell on the approximation of the laws of distributions using the first two initial moments.

Thus, the hypererlang law, as well as the hyperexponential distribution law, can be determined completely by the first two moments and cover the entire range of variation of the coefficient of variation from $1/\sqrt{2}$ to ∞, which is wider than for the hyperexponential distribution from 1 to ∞.

Next, we consider a system that is fundamentally different from the QS studied. For the $HE_2/H_2/1$ system with shifted laws of distributions of input flow intervals and service time, these laws are defined by density functions of the form:

$$a(t) = \begin{cases} 4p\lambda_1^2(t-t_0)e^{-2\lambda_1(t-t_0)} + 4(1-p)\lambda_2^2(t-t_0)e^{-2\lambda_2(t-t_0)}, \\ t > t_0, \\ 0,\ 0 \le t \le t_0, \end{cases} \quad (15)$$

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$$b(t)=\begin{cases} q\mu_1 e^{-\mu_1(t-t_0)}+(1-q)\mu_2 e^{-\mu_2(t-t_0)}, \ t>t_0, \\ 0, \ 0\le t\le t_0. \end{cases} \quad (16)$$

The density functions (15) and (16) are shifted to the right from the zero point by the value of $t_0>0$. Thus, we have a QS with a time delay of $t_0>0$. Such a QS, unlike the conventional system, is denoted as $HE_2^-/H_2^-/1$. We are interested in the average wait time for the $HE_2^-/H_2^-/1$ system.

Statement. The spectral expansions $A^*(-s)\cdot B^*(s)-1=\psi_+(s)/\psi_-(s)$ of the LIE solution for systems $HE_2^-/H_2^-/1$ and $HE_2/H_2/1$ completely coincide and have the form (10). Consequently, the Laplace transforms of the waiting time density function for them also coincide.

Proof. The Laplace transforms of functions (15) and (16) will be respectively:

$$A^*(s)=[p\left(\frac{2\lambda_1}{s+2\lambda_1}\right)^2+(1-p)\left(\frac{2\lambda_2}{s+2\lambda_2}\right)^2]\cdot e^{-t_0 s},$$

$$B^*(s)=[q\frac{\mu_1}{s+\mu_1}+(1-q)\frac{\mu_2}{s+\mu_2}]e^{-t_0 s}.$$

The spectral decomposition $A^*(-s)\cdot B^*(s)-1=\psi_+(s)/\psi_-(s)$ of the LIE solution for the $HE_2^-/H_2^-/1$ system will be:

$$\frac{\psi_+(s)}{\psi_-(s)}=\left[p\left(\frac{2\lambda_1}{2\lambda_1-s}\right)^2+(1-p)\left(\frac{2\lambda_2}{2\lambda_2-s}\right)^2\right]e^{t_0 s}\times$$
$$\times\left[q\frac{\mu_1}{\mu_1+s}+(1-q)\frac{\mu_2}{\mu_2+s}\right]e^{-t_0 s}-1=$$
$$=\left[p\left(\frac{2\lambda_1}{2\lambda_1-s}\right)^2+(1-p)\left(\frac{2\lambda_2}{2\lambda_2-s}\right)^2\right]\times$$
$$\times\left[q\frac{\mu_1}{\mu_1+s}+(1-q)\frac{\mu_2}{\mu_2+s}\right]-1.$$

Here, the exponential functions due to the opposite signs of the exponents are zeroed out and thus the shift operation is leveled. We thereby obtained the same expression (8). Therefore, the spectral expansions for the $HE_2^-/H_2^-/1$ and $HE_2/H_2/1$ systems completely coincide and have the form (8). Thus, all the above considerations for the $HE_2/H_2/1$ system are also valid for the system, but already with the changed numerical characteristics of the shifted distributions (15) and (16). The statement is proved. Now we can use for the new $HE_2^-/H_2^-/1$ system the results for the ordinary $HE_2/H_2/1$ system, but with the changed distributions parameters (15) and (16) due to the introduction of the shift parameter $t_0>0$ into them.

We define the numerical characteristics of the interval between the arrivals of requirements and service time for the new $HE_2^-/H_2^-/1$ system. To do this, we use the Laplace transforms of functions (15) and (16). The value of the first derivative of the function $A^*(s)$ with a minus sign at the point s=0 is equal to

$$-\left.\frac{dA^*(s)}{ds}\right|_{s=0}=p\lambda_1^{-1}+(1-p)\lambda_2^{-1}+t_0.$$

Hence, the average value of the intervals between adjacent requirements of the input flow will be equal to

$$\overline{\tau}_\lambda=p\lambda_1^{-1}+(1-p)\lambda_2^{-1}+t_0. \quad (17)$$

The value of the second derivative of the function at s=0 gives the second initial moment of the arrival interval

$$\overline{\tau_\lambda^2}=t_0^2+2t_0[\frac{p}{\lambda_1}+\frac{(1-p)}{\lambda_2}]+\frac{3}{2}[\frac{p}{\lambda_1^2}+\frac{(1-p)}{\lambda_2^2}]. \quad (18)$$

Define the square of the coefficient of variation

$$c_\lambda^2=\frac{\lambda_1^2-2p\lambda_2(\lambda_1-\lambda_2)+p(1-2p)(\lambda_1-\lambda_2)^2}{2[t_0\lambda_1\lambda_2+(1-p)\lambda_1+p\lambda_2]^2}. \quad (19)$$

Similarly, we determine the average service time through the Laplace transform $B^*(s)$ of function (16)

$$\overline{\tau}_\mu=q\mu_1^{-1}+(1-q)\mu_2^{-1}+t_0. \quad (20)$$

The value of the second derivative of the function $B^*(s)$ at s=0 gives the second initial moment of service time

$$\overline{\tau_\mu^2}=t_0^2+2t_0[\frac{q}{\mu_1}+\frac{(1-q)}{\mu_2}]+2[\frac{q}{\mu_1^2}+\frac{(1-q)}{\mu_2^2}]. \quad (21)$$

From here we define the square of the coefficient of variation of the service time:

$$c_\mu^2=\frac{[(1-q^2)\mu_1^2-2\mu_1\mu_2 q(1-q)+q(2-q)\mu_2^2]}{[t_0\mu_1\mu_2+(1-q)\mu_1+q\mu_2]^2}. \quad (22)$$

Note that the coefficients of variation $c_\lambda, c_\mu>0$ for the shift parameter $t_0>0$.

Now we estimate the effect of the shift parameter $t_0>0$ on the numerical characteristics of distributions (15) and (16). We are primarily interested in the square of the coefficient of variation, since the average waiting time in the G/G/1 system is related to the variation coefficients by the quadratic dependence (1).

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Comparing expressions (12) and (19), we see that the time shift operation reduces the coefficient of variation of the intervals of receipts by a factor of $1+\frac{t_0\lambda_1\lambda_2}{[\lambda_1(1-p)+\lambda_2 p]}$. Comparing (14) and (22) we get a decrease in $1+\frac{t_0\mu_1\mu_2}{[\mu_1(1-q)+\mu_2 q]}$ times. Consequently, a delayed system provides shorter waiting times than a conventional system with the same load ρ. This is the essence of introducing a delay parameter $t_0 > 0$ to reduce the coefficients of variation of intervals in the input stream and service time, and because of reducing the waiting time for requests in the queue.

Also in this lies the quantitative and qualitative difference between a delayed system and a conventional system. The results of computational experiments unequivocally confirm these facts.

Considering expressions (17)–(19) as a record of the method of moments, we find the unknown distribution parameters (15) by doing the same with the usual distribution (2), setting

$$\lambda_1 = 2p/(\bar{\tau}_\lambda - t_0),\ \lambda_2 = 2(1-p)/(\bar{\tau}_\lambda - t_0). \tag{23}$$

and demanding the fulfillment of condition (19). Substituting expressions (23) into (19) and solving the obtained fourth-degree equation with respect to the parameter *p* and taking into account the condition $0 < p < 1$, we find $p = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{3(\bar{\tau}_\lambda - t_0)^2}{8[(\bar{\tau}_\lambda - t_0)^2 + c_\lambda^2 \bar{\tau}_\lambda^2]}}$ and then we determine the parameters λ_1 and λ_2 from (23).

By doing the same with expressions (20)–(22), setting

$$\mu_1 = 2q/(\bar{\tau}_\mu - t_0),\ \mu_2 = 2(1-q)/(\bar{\tau}_\mu - t_0). \tag{24}$$

and substituting (24) into (22) we obtain an equation of the fourth degree with respect to the parameter *q*. Having solved it taking into account the conditions $0 < q < 1$, we determine the parameter $q = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{(\bar{\tau}_\mu - t_0)^2}{2[(\bar{\tau}_\mu - t_0)^2 + c_\mu^2 \bar{\tau}_\mu^2]}}$. Substituting the expression for *q* in (24), we find the unknown distribution parameters (16) μ_1, μ_2. Moreover, as *p* and *q*, you can choose any of two values.

Note. The range of applicability of the $HE_2^- / H_2^- / 1$ system will be determined by the no negativity of the expressions under the square root for parameters *p* and *q*.

Thus, the algorithm for calculating the average waiting time for given input parameters $\bar{\tau}_\lambda, \bar{\tau}_\mu, c_\lambda, c_\mu, t_0$ is reduced to a sequential solution of these equations. Next, we determine the coefficients of the polynomial (9) from the above expressions (8) and find the necessary roots with negative real parts $-s_1, -s_2$. Substituting the absolute values of these roots in expression (10), we determine the average waiting time. The presence of such roots is due to the existence and uniqueness of spectral decomposition. Numerous experiments carried out only confirm this fact.

4 EXPERIMENTS

Below in the table. 1 shows the calculation data in Mathcad package for the $HE_2/H_2/1$ system for the cases of low, medium and high loads $\rho = 0,1; 0,5; 0,9$. Note that the $HE_2/H_2/1$ system is applicable for $c_\lambda \geq 1/\sqrt{2}$, $c_\mu \geq 1$. The load factor ρ in all tables is determined by the ratio of average intervals $\rho = \bar{\tau}_\mu / \bar{\tau}_\lambda$. The calculations given in all tables are carried out for the normalized service time $\bar{\tau}_\mu = 1$. Moreover, for comparison, the known results for $H_2/H_2/1$ QS were used [5]. Due to the fact that the $H_2/H_2/1$ system is not applicable in the case $c_\lambda < 1$, in table 1 dashes are given.

Table 1 – Results of experiments for QS $HE_2/H_2/1$ and $H_2/H_2/1$

Input parameters		Average waiting time	
ρ	(c_λ, c_μ)	For QS $HE_2/H_2/1$	For QS $H_2/H_2/1$
0.1	(0.71;1)	0.030	–
	(2.2)	0.335	0.445
	(4.4)	1.666	1.779
	(8.8)	7.10	7.112
0.5	(0.71;1)	0.620	–
	(2.2)	3.974	4.044
	(4.4)	16.392	16.129
	(8.8)	65.967	64.178
0.9	(0.71;1)	6.607	–
	(2.2)	36.271	36.20
	(4.4)	145.465	144.833
	(8.8)	580.822	577.861

Table 2 shows the calculation results for the $HE_2^- / H_2^- / 1$ system. For comparison, table 2 on the right shows the results for the conventional $HE_2/H_2/1$ system.

Table 2 – Results of experiments for QS $HE_2^- / H_2^- / 1$ and $HE_2/H_2/1$

Input parameters		Average waiting time			
ρ	$(c_\lambda; c_\mu)$	For QS $HE_2^- / H_2^- / 1$			For QS $HE_2/H_2/1$
		t_0=0.9	t_0=0.5	t_0=0.01	
0.1	(0.71;0.71)	0.021	0.023	–	0.030
	(2;2)	0.282	0.321	0.334	0.335
	(4;4)	1.200	1.528	1.663	1.666
	(8;8)	4.875	6.400	7.088	7.100
0.5	(0.71;0.71)	0.270	0.313	–	0.620
	(2;2)	2.311	3.118	3.957	3.974
	(4;4)	9.322	12.794	16.323	16.392
	(8;8)	37.366	51.497	65.692	65.967
0.9	(0.71;0.71)	3.052	4.125	–	6.607
	(2;2)	24.313	33.405	36.241	36.271
	(4;4)	97.284	133.291	145.327	145.465
	(8;8)	389.166	532.177	580.254	580.822

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5 RESULTS

With a decrease in the parameter value t_0, the average waiting time in the system tends to the average waiting time in the $HE_2/H_2/1$ system, which confirms the complete adequacy and reliability of the results. In some cases, with t_0=0.01, the system with delay is not defined and there are dashes in the table. This is due to the note made above.

In the work, spectral expansions of the solution of the Lindley integral equation for two systems $HE_2/H_2/1$, $HE_2^- / H_2^- / 1$ are obtained, and it is proved that they completely coincide. Using the spectral decomposition, a formula is derived for the average waiting time in the queue for these systems in closed form. These formula complement and extend the well-known incomplete formula (1) for the average waiting time for G/G/1 systems.

As can be seen from tables 1, the results in both cases are quite close, and the difference is explained by the fact that the distributions of HE_2 and H_2 are still different. The data in table 2 confirm the adequacy and reliability of the above mathematical calculations.

6 DISCUSSION

The article presents an analytical solution for the average waiting time in the queue for the $HE_2/H_2/1$ system using the symbolic operations of the Mathcad package. The same solution allows it to be used for a $HE_2^- / H_2^- / 1$ system with delay. Using the proposed approach, in addition to the average waiting time, one can determine the variance and moments of higher orders of waiting time. The result obtained, on the one hand, supplements the $HE_2/H_2/1$ system, and, on the other hand, expands the range of variation of the coefficients of variation of the intervals of arrivals and service time from $1/\sqrt{2}$ to ∞. To be convincing, the calculation data for the $HE_2/H_2/1$ system are compared with the results for the $H_2/H_2/1$ system, which demonstrates their sufficient proximity.

The time shift operation reduces the variation coefficients of the interval between arrivals and the service time of requirements. Because the average waiting time in the G/G/1 system is related to the coefficients of variation of the arrival intervals and service time by the quadratic dependence, the average waiting time in the delayed system will be less than in a conventional system with the same load factor.

CONCLUSIONS

The article presents the solution to the problem of determining the average waiting time for two queuing systems $HE_2/H_2/1$ and $HE_2^- / H_2^- / 1$ by the classical method of spectral decomposition.

The scientific novelty of the results is that spectral decompositions of the solution of the Lindley integral equation for the systems under consideration were obtained for the first time and, using spectral decompositions, formulas were derived for the average queue waiting time for these systems in closed form. These formulas expand and supplement formula (1) for the average waiting time for G/G/1 systems with arbitrary laws of the distributions of the input stream and service time.

The practical significance of the work lies in the fact that the obtained results can be successfully applied in the modern theory of teletraffic, where the delays of incoming traffic packets play a primary role. For this, it is necessary to know the numerical characteristics of the incoming traffic intervals and the service time at the level of the first two moments, which does not cause difficulties when using modern traffic analyzers.

Prospects for further research are seen in the continuation of the study of systems of type G/G/1 with other common input distributions and in expanding and supplementing the formulas for average waiting time.

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МОДЕЛЬ ТЕЛЕТРАФІКА НА ОСНОВІ СИСТЕМ $HE_2/H_2/1$ ЗІ ЗВИЧАЙНИМИ ТА ЗСУНУТИМИ ВХІДНИМИ РОЗПОДІЛАМИ

Актуальність. Розглянуто задачу виведення рішення для середнього часу очікування в черзі у замкнутій формі для звичайної системи з гіперерлангівськими і гіперекспонеційними вхідними розподілами другого порядку і системи зі зсунутими г гіперерлангівськими і гіперекспонеційними вхідними розподілами.

Мета роботи. Отримання рішення для основної характеристики системи – середнього часу очікування вимог у черзі для двох систем масового обслуговування типу G/G/1.

Метод. Для вирішення поставленого завдання був використаний класичний метод спектрального розкладання рішення інтегрального рівняння Ліндлі. Цей метод дозволяє отримати рішення для середнього часу очікування для розглянутих систем у замкнутій формі. Метод спектрального розкладання рішення інтегрального рівняння Ліндлі грає важливу роль в теорії систем G/G/1. Для практичного застосування отриманих результатів було використано відомий метод моментів теорії ймовірностей.

Результати. Вперше отримано спектральне розкладання рішення інтегрального рівняння Линдли для двох систем, за допомогою якого виведено розрахунковий вираз для середнього часу очікування в черзі у замкнутій формі. Такий підхід дозволяє розрахувати середній час очікування для зазначених систем у математичних пакетах для широкого діапазону зміни параметрів трафіку. Усі інші характеристики систем є похідними від середнього часу очікування.

Висновки. Показано, що гіперерлангівський закон розподілу другого порядку, як і гіперекспонеційний є трипараметричним, може визначатися як двома першими моментами, так і трьома першими моментами. Вибір такого закону розподілу ймовірностей обумовлений тим, що його коефіцієнт варіації охоплює більш широкий діапазон, ніж у гіперекспонеційнего розподілу. Для зсунутих гіперерлангівського і гіперекспонеційнего законів розподілів коефіцієнти варіацій зменшуються і охоплюють ще більш широкий діапазон, ніж у звичайних розподілів. Введення зсунутих в часі розподілів розширює сферу застосування систем масового обслуговування з урахуванням відомого факту з теорії масового обслуговування, що середній час очікування пов'язаний з коефіцієнтами варіацій інтервалів надходжень і часу обслуговування квадратичною залежністю. Метод спектрального розкладання рішення інтегрального рівняння Ліндли для системи масового обслуговування з гіперерлангівськими і гіперекспонеційними вхідними розподілами другого порядку дозволяє отримати рішення в замкнутій формі і це рішення публікується вперше. Отримане рішення доповнює і розширює відому формулу теорії масового обслуговування для середнього часу очікування вимог в черзі для системи масового обслуговування типу G/G/1.

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КЛЮЧОВІ СЛОВА: гіперерлангівський і гіпереспонеційний закони розподілу, інтегральне рівняння Ліндлі, метод спектрального розкладання, перетворення Лапласа.

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МОДЕЛЬ ТЕЛЕТРАФИКА НА ОСНОВЕ СИСТЕМ $HE_2/H_2/1$ С ОБЫЧНЫМИ И СО СДВИНУТЫМИ ВХОДНЫМИ РАСПРЕДЕЛЕНИЯМИ

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АННОТАЦИЯ

Актуальность. Рассмотрена задача вывода решения для среднего времени ожидания в очереди в замкнутой форме для обычной системы с гиперэрланговскими и гиперэкспоненциальными входными распределениями второго порядка и системы со сдвинутыми гиперэрланговскими и гиперэкспоненциальными входными распределениями.

Цель работы. Получение решения для основной характеристики системы – среднего времени ожидания требований в очереди для системы массового обслуживания типа G/G/1 с обычными и со сдвинутыми с гиперэрланговскими и гиперэкспоненциальными входными распределениями второго порядка.

Метод. Для решения поставленной задачи использован классический метод спектрального разложения решения интегрального уравнения Линдли, который позволяет получить решение для среднего времени ожидания для рассматриваемых систем в замкнутой форме. Метод спектрального разложения решения интегрального уравнения Линдли занимает важную часть теории систем G/G/1. Для практического применения полученных результатов использован известный метод моментов теории вероятностей.

Результаты. Впервые получены спектральные разложения решения интегрального уравнения Линдли для обеих систем, с помощью которых выведены расчетные формулы для среднего времени ожидания в очереди для вышеуказанных систем в замкнутой форме. Такой подход позволяет рассчитать среднее время ожидания для указанных систем в математических пакетах для широкого диапазона изменения параметров трафика. Все остальные характеристики систем являются производными от среднего времени ожидания.

Выводы. Показано, что гиперэрланговский закон распределения второго порядка, как и гиперэкспоненциальный, являющийся трехпараметрическим, может определяться как двумя первыми моментами, так и тремя первыми моментами. Выбор такого закона распределения вероятностей обусловлен тем, что его коэффициент вариации охватывает более широкий диапазон, чем у гиперэкспоненциального распределения. Для сдвинутых гиперэрланговского и гиперэкспоненциального законов распределений коэффициенты вариаций уменьшаются и охватывают еще более широкий диапазон, чем у обычных распределений. Введение сдвинутых во времени распределений расширяет область применения систем массового обслуживания с учетом известного факта из теории массового обслуживания, что среднее время ожидания связано с коэффициентами вариаций интервалов поступлений и времени обслуживания квадратичной зависимостью. Метод спектрального разложения решения интегрального уравнения Линдли для системы массового обслуживания с гиперэрланговскими и гиперэкспоненциальными входными распределениями второго порядка позволяет получить решение в замкнутой форме и это решение публикуется впервые Полученное решение дополняет и расширяет известную формулу теории массового обслуживания для среднего времени ожидания требований в очереди для системы массового обслуживания типа G/G/1.

КЛЮЧЕВЫЕ СЛОВА: гиперэрланговский и гиперэкспоненциальный законы распределения, интегральное уравнение Линдли, метод спектрального разложения, преобразование Лапласа.

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