# МАТЕМАТИЧНЕ ТА КОМП'ЮТЕРНЕ МОДЕЛЮВАННЯ

# MATHEMATICAL AND COMPUTER MODELING

# МАТЕМАТИЧЕСКОЕ И КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ

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## ON THE KOLMOGOROV-WIENER FILTER FOR RANDOM PROCESSES WITH A POWER-LAW STRUCTURE FUNCTION BASED ON THE WALSH FUNCTIONS

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### ABSTRACT

**Context.** We investigate the Kolmogorov-Wiener filter weight function for the prediction of a continuous stationary random process with a power-law structure function.

**Objective.** The aim of the work is to develop an algorithm of obtaining an approximate solution for the weight function without recourse to numerical calculation of integrals.

**Method.** The weight function under consideration obeys the Wiener-Hopf integral equation. A search for an exact analytical solution for the corresponding integral equation meets difficulties, so an approximate solution for the weight function is sought in the framework of the Galerkin method on the basis of a truncated Walsh function series expansion.

**Results.** An algorithm of the weight function obtaining is developed. All the integrals are calculated analytically rather than numerically. Moreover, it is shown that the accuracy of the Walsh function approximations is significantly better than the accuracy of polynomial approximations obtained in the authors' previous papers. The Walsh function solutions are applicable in wider range of parameters than the polynomial ones.

**Conclusions.** An algorithm of obtaining the Kolmogorov-Wiener filter weight function for the prediction of a stationary continuous random process with a power-law structure function is developed. A truncated Walsh function expansion is the basis of the developed algorithm. In opposite to the polynomial solutions investigated in the previous papers, the developed algorithm has the following advantages. First of all, all the integrals are calculated analytically, and any numerical calculation of the integrals is not needed. Secondly, the problem of the product of very small and very large numbers is absent in the framework of the developed algorithm. In our opinion, this is the reason why the accuracy of the Walsh function solutions is better than that of the polynomial solutions for many approximations and why the Walsh function solutions are applicable in a wider range of parameters than the polynomial ones. The results of the paper may be applied, for example, to practical traffic prediction in telecommunication systems with data packet transfer.

**KEYWORDS:** Kolmogorov-Wiener filter weight function, continuous random process, Walsh functions, Galerkin method, power-law structure function.

### NOMENCLATURE

$T$ is a time interval on which the input process data are observed;

$z$ is a time interval for which the forecast should be made;

$h(t)$ is a Kolmogorov-Wiener filter weight function;

$H$ is a Hurst exponent;

$\sigma^2$ is a process variance;

$\alpha$ is a proportionality constant between the process structure function and the power-law term;

$H^{(2^m)}$ are the Hadamard matrices;

$W^{(2^m)}$ are the Walsh matrices;

$W_{kl}^{(2^m)}$ are the components of the Walsh matrices;


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$\mathrm{wal}_k(t)$ are the Walsh functions;

$g_m$ are the coefficients multiplying the Walsh functions;

$g$ is a column vector of the coefficients $g_m$;

Left(t) is a left-hand side of the Wiener-Hopf integral equation

Right(t) is a right-hand side of the Wiener-Hopf integral equation;

$G_{mk}$ are the integral brackets;

$G$ is a matrix of the integral brackets;

$B_k$ are the free terms in the linear set of algebraic equations in $g_m$;

$B$ is a column vector of the free terms $B_k$;

$V_{\beta\delta}$ are auxiliary integrals needed for the calculation of the integral brackets;

$V$ is a matrix of the integrals $V_{\beta\delta}$;

$X_\beta$ are auxiliary integrals needed for the calculation of the function Left(t);

$N$ is a number of points in the numerical integration.

## INTRODUCTION

Random processes with a power-law structure function are widely used in different fields of knowledge. For example, they are used in plasma physics, statistical physics, in the study of financial markets (see corresponding references in [1]), in astrophysics [2, 3], in the description of turbulent flows [4] and so on. In particular, random processes with a power-law structure function may be used for the telecommunication traffic description (see, for example, [5–7]). The problem of telecommunication traffic prediction is very important for telecommunications because of its applications to power saving, optimal use of network resources, and detection of security attacks (see, for example, [8, 9]). There are plenty of telecommunication traffic models [10]. In the simplest models, telecommunication traffic in systems with data packet transfer is considered as a self-similar stationary random process [10]. One of the known telecommunication traffic models is the model where the traffic is considered to be a stationary random process with a power-law structure function [5]. In the case of a large amount of data, traffic may be treated as continuous random process [5].

As is known [11], the Kolmogorov-Wiener filter maybe applied to the prediction of stationary processes. The Kolmogorov-Wiener filter is a linear stationary filter, so this filter is rather simple and in our opinion it is quite natural to apply it to the prediction of stationary random process with a power-law structure function. There are plenty of rather complicated approaches to the prediction of random processes (see, for example, approaches to traffic prediction [8, 9]). However, as far as we know, a rather simple approach based on the use of the Kolmogorov-Wiener filter has not been sufficiently developed in the literature, and so the results devoted to this approach have scientific novelty.

Our previous papers [5–7] were devoted to obtaining the Kolmogorov-Wiener filter weight function for the prediction of a random process with the power-law structure function. Papers [5–7] were based on the truncated polynomial expansion method, which has some drawbacks. To overcome these drawbacks, in the framework of this paper we propose to derive the unknown filter weight function on the basis of a truncated Walsh function expansion.

**The object of study** is the Kolmogorov-Wiener filter for the prediction of a random process with a power-law structure function.

**The subject of study** is the weight function of the corresponding filter.

**The aim of the work** is to obtain the weight function on the basis of expansion into a truncated Walsh function series.

## 1 PROBLEM STATEMENT

As is known [5], the Kolmogorov-Wiener weight function for the prediction of a random process with a power-law structure function is the solution of the following Wiener-Hopf integral equation:

$$\int_0^T d\tau h(\tau)\left(\sigma^2 - \frac{\alpha}{2}|t-\tau|^{2H}\right) = \sigma^2 - \frac{\alpha}{2}(t+z)^{2H}. \quad (1)$$

The problem statement is as follows: to obtain the unknown filter weight function as an approximate solution of the integral equation (1) on the basis of a truncated Walsh function expansion.

## 2 REVIEW OF THE LITERATURE

Our previous papers [5–7] were devoted to a search for the unknown weight function on the basis of the integral equation (1) with the help of the truncated polynomial expansion method. In paper [5] the polynomials orthogonal without weight were used, and papers [6, 7] are based on the Chebyshev polynomials of the first and second kind, respectively. The behavior of polynomial solutions in [5–7] is almost identical.

It should be stressed that nowadays the truncated polynomial expansion method is rather popular for the solution of integral equation in different fields of knowledge (see, for example, [12–14]). However, in the framework of the problem under consideration polynomial solutions have several drawbacks. Some polynomial approximations in [5–7] indeed give good results, but some approximations absolutely fail. In our opinion, the fact that polynomial solutions lead to the product of very large and very small numbers may cause such failures. Moreover, the analytical calculation of the integral brackets in the framework of the polynomial method meets difficulties. Exact analytical expressions for the integral brackets may be obtained, but they are too cumbersome and, in fact,


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they are not applicable in the framework of approximations of a rather large number of polynomials.

The truncated polynomial expansion method is a special case of the Galerkin method [15], in the framework of which the solution of an integral equation is sought in the form of a truncated orthogonal function series. As mentioned above, the use of orthogonal polynomials has some drawbacks. So, in order to avoid such drawbacks, we propose to use the orthogonal Walsh functions [16] instead of polynomials. The Walsh functions are step ones [16], thus allowing one to obtain analytical expressions for the integral brackets which are also applicable in the case of a rather large number of Walsh functions. Moreover, the use of the Walsh functions does not lead to the product of very large and very small numbers, so Walsh function solutions are applicable in a wider range of parameters than polynomial ones.

## 3 MATERIALS AND METHODS

As is known, the Hadamard matrices are introduced by the following recursive definition [16]:

$$H^{\left(2^{m+1}\right)}=\begin{pmatrix} H^{\left(2^m\right)} & H^{\left(2^m\right)} \\ H^{\left(2^m\right)} & -H^{\left(2^m\right)} \end{pmatrix},\ H^{(2)}=\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{2}$$

The set of rows of the Walsh matrices coincides with that of the Hadamard matrices, but the numeration of the sets is not the same. The rows of the Walsh matrices are numerated as follows [16]: the smaller the number of sign changes in a row, the smaller the number of the row. The Walsh functions in the Walsh numeration are defined on the time interval $t\in[0,T]$ as follows:

$$\mathrm{wal}_k\left(t\right)=\begin{cases} W_{k1}^{\left(2^m\right)}, t\in\left[0,\dfrac{T}{2^m}\right] \\ W_{k2}^{\left(2^m\right)}, t\in\left(\dfrac{T}{2^m},\dfrac{2T}{2^m}\right] \\ \vdots \\ W_{k,2^m}^{\left(2^m\right)}, t\in\left(\dfrac{\left(2^m-1\right)T}{2^m},T\right] \end{cases}. \tag{3}$$

where $m$ is the least natural number for which the inequality $2^m \ge k$ holds. As can be seen, the Walsh functions (3) are step ones, they may take only the values +1 or –1. The Walsh functions form a complete orthogonal function system on the time interval $t\in[0,T]$.

The unknown weight function $h(\tau)$ in the approximation of $n=2^m$ is sought in the form

$$h\left(\tau\right)=\sum_{m=1}^{n} g_m\,\mathrm{wal}_m\left(\tau\right). \tag{4}$$

On substituting (4) into (1) one can obtain

$$\sum_{m=1}^{n} g_m \int_0^T d\tau\,\mathrm{wal}_m\left(\tau\right)\left(\sigma^2-\frac{\alpha}{2}\left|t-\tau\right|^{2H}\right)= \\ =\sigma^2-\frac{\alpha}{2}\left(t+z\right)^{2H}, \tag{5}$$

which after multiplying by $\mathrm{wal}_k(t)$ and integrating over $t$ leads to the following system of linear equations:

$$\sum_{m=1}^{n} G_{mk}\,g_m = B_k,\ k=\overline{1,n} \tag{6}$$

where

$$G_{mk}=\int_0^T\int_0^T dt d\tau\,\mathrm{wal}_m\left(\tau\right)\mathrm{wal}_k\left(t\right)\left(\sigma^2-\frac{\alpha}{2}\left|t-\tau\right|^{2H}\right) \tag{7}$$

and

$$B_k=\int_0^T dt\,\mathrm{wal}_k\left(t\right)\left(\sigma^2-\frac{\alpha}{2}\left(t+z\right)^{2H}\right). \tag{8}$$

The matrix form of the system (6) is as follows:

$$Gg=B, \tag{9}$$

where

$$G=\begin{pmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n1} & G_{n2} & \cdots & G_{nn} \end{pmatrix},\ B=\begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix},\ g=\begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{pmatrix}. \tag{10}$$

The Walsh functions obey the property

$$\mathrm{wal}_k\left(\frac{T}{2}-t\right)=\begin{cases} \mathrm{wal}_k\left(\dfrac{T}{2}+t\right), k\,\vdots\,2 \\ -\mathrm{wal}_k\left(\dfrac{T}{2}+t\right), k\,\not\vdots\,2 \end{cases}, \tag{11}$$

which with account for the fact that the absolute value is sign-independent leads to the following properties of the integral brackets:

$$G_{ks}=G_{sk}; \\ G_{ks}=0 \text{ if } k,s \text{ are of opposite parities,} \tag{12}$$

see a similar derivation of (12) in [5]. So, a straightforward calculation is needed only for the integral brackets $G_{km}$ where $k\ge m$ and $k,m$ are of the same parity. Analytical expressions for the integral brackets may be obtained on the basis of the fact that the Walsh functions are step ones:

$$G_{mk}=\sum_{\beta,\delta=1}^{n} W_{m\beta}^{(n)} W_{k\delta}^{(n)} V_{\beta\delta} \tag{13}$$


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where

$$V_{\beta\delta} = \int_{(\beta-1)T/n}^{\beta T/n} \int_{(\delta-1)T/n}^{\delta T/n} dt d\tau \left( \sigma^2 - \frac{\alpha}{2} |t-\tau|^{2H} \right). \quad (14)$$

The integrals $V_{\beta\delta}$ obey the properties

$$V_{\beta\delta} = V_{\beta+1,\delta+1}\,;\; V_{\beta\delta} = V_{\delta\beta}\,. \quad (15)$$

This can be demonstrated as follows:

$$V_{\beta\delta} = \int_{(\beta-1)T/n}^{\beta T/n} \int_{(\delta-1)T/n}^{\delta T/n} dt d\tau \left( \sigma^2 - \frac{\alpha}{2} |t-\tau|^{2H} \right) = \\ = \left\{ x = t + \frac{T}{n}, y = \tau + \frac{T}{n} \right\} = \quad (16) \\ = \int_{\beta T/n}^{(\beta+1)T/n} \int_{\delta T/n}^{(\delta+1)T/n} dx dy \left( \sigma^2 - \frac{\alpha}{2} |x-y|^{2H} \right) = V_{\beta+1,\delta+1}\,.$$

and

$$V_{\beta\delta} = \int_{\frac{(\beta-1)T}{n}}^{\frac{\beta T}{n}} \int_{\frac{(\delta-1)T}{n}}^{\frac{\delta T}{n}} dt d\tau \left( \sigma^2 - \frac{\alpha}{2} |t-\tau|^{2H} \right) = \\ = \left\{ t \leftrightarrow \tau, |t-\tau| = |\tau - t| \right\} = \quad (17) \\ = \int_{\frac{(\delta-1)T}{n}}^{\frac{\delta T}{n}} \int_{\frac{(\beta-1)T}{n}}^{\frac{\beta T}{n}} dt d\tau \left( \sigma^2 - \frac{\alpha}{2} |t-\tau|^{2H} \right) = V_{\delta\beta}\,.$$

So the matrix $V$ takes the form

$$V = \begin{pmatrix} V_{11} & V_{12} & \cdots & V_{1n} \\ V_{21} & V_{22} & \cdots & V_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ V_{n1} & V_{n2} & \cdots & V_{nn} \end{pmatrix} = \\ = \begin{pmatrix} V_{11} & V_{12} & V_{13} & \cdots & V_{1n} \\ V_{12} & V_{11} & V_{12} & \cdots & V_{1,n-1} \\ V_{13} & V_{12} & V_{11} & \cdots & V_{1,n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ V_{1n} & V_{1,n-1} & V_{1,n-2} & \cdots & V_{11} \end{pmatrix}. \quad (18)$$

and a straightforward calculation is needed only for the first row in (18). Let us obtain analytical results for the quantities $V_{1\beta}$. First of all, from (14) one can see that

$$V_{1\beta} = \left( \sigma \frac{T}{n} \right)^2 - \frac{\alpha}{2} \int_0^{\frac{T}{n}} dt \int_{\frac{(\beta-1)T}{n}}^{\frac{\beta T}{n}} d\tau |t-\tau|^{2H} = \\ = \{ y = t - \tau \} = \left( \sigma \frac{T}{n} \right)^2 - \frac{\alpha}{2} \int_0^{\frac{T}{n}} dt \int_{t-\frac{\beta T}{n}}^{t-\frac{(\beta-1)T}{n}} dy |y|^{2H}\,. \quad (19)$$

For the quantity $V_{11}$ the integrand is of alternating sign and

$$V_{11} = \left( \sigma \frac{T}{n} \right)^2 - \frac{\alpha}{2} \int_0^{\frac{T}{n}} dt \int_{t-\frac{T}{n}}^{t} dy |y|^{2H} = \\ = \left( \sigma \frac{T}{n} \right)^2 - \frac{\alpha}{2} \int_0^{\frac{T}{n}} dt \left( \int_{t-\frac{T}{n}}^{0} dy (-y)^{2H} + \int_0^{t} dy y^{2H} \right) = \\ = \{ x = -y \} = \\ = \left( \sigma \frac{T}{n} \right)^2 - \frac{\alpha}{2} \int_0^{\frac{T}{n}} dt \left( \int_0^{\frac{T}{n}-t} dx x^{2H} + \int_0^{t} dy y^{2H} \right) \quad (20)$$

which after a straightforward calculation leads to

$$V_{11} = \left( \sigma \frac{T}{n} \right)^2 - \frac{\alpha}{(2H+1)(2H+2)} \left( \frac{T}{n} \right)^{2H+2}. \quad (21)$$

For $\beta > 1$ the integrand in (19) does not change the sign:

$$V_{1\beta} = \left( \sigma \frac{T}{n} \right)^2 - \frac{\alpha}{2} \int_0^{\frac{T}{n}} dt \int_{t-\frac{\beta T}{n}}^{t-\frac{(\beta-1)T}{n}} dy (-y)^{2H}\,. \quad (22)$$

which after a straightforward calculation leads to

$$V_{1\beta} = \left( \sigma \frac{T}{n} \right)^2 - \frac{\alpha}{2(2H+1)(2H+2)} \left[ \left( \frac{\beta T}{n} \right)^{2H+2} + \right. \\ \left. + \left( \frac{(\beta-2)T}{n} \right)^{2H+2} - 2 \left( \frac{(\beta-1)T}{n} \right)^{2H+2} \right]. \quad (23)$$

The free terms $B_k$ may be calculated by a similar idea:

$$B_k = \sum_{\beta=1}^{n} W_{k\beta}^{(n)} \int_{\frac{(\beta-1)T}{n}}^{\frac{\beta T}{n}} dt \left( \sigma^2 - \frac{\alpha}{2} (t+z)^{2H} \right) \quad (24)$$

which after a straightforward calculation leads to

$$B_k = \sum_{\beta=1}^{n} W_{k\beta}^{(n)} \left\{ \sigma^2 \frac{T}{n} - \frac{\alpha}{2(2H+1)} \left[ \left( \frac{\beta T}{n} + z \right)^{2H+1} - \right. \right. \\ \left. \left. - \left( \frac{(\beta-1)T}{n} + z \right)^{2H+1} \right] \right\}. \quad (25)$$

So the algorithm of obtaining the coefficients multiplying the Walsh functions may be formulated as follows:

1. Calculate $V_{11}$ by formula (21).
2. Calculate $V_{1\beta}$, $\beta \geq 2$ by formula (23).
3. Form the matrix $V$ by formula (18).
4. Calculate the integral brackets $G_{km}$ by formula (13) where $k \geq m$ and $k,m$ are of the same parity


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5. Calculate all the other integral brackets on the basis of the properties (12) and form the matrix $G$ of the integral brackets (see (10)).

6. Calculate the free terms $B_k$ by formula (25) and form the column vector $B$ of the free terms

7. Calculate the column vector $g$ of the unknown coefficients by the following formula (26):

$$g = G^{-1}B . \qquad (26)$$

In order to verify the obtained solutions, the left-hand side and the right-hand side of (1) should be numerically compared for the obtained solutions:

$$\mathrm{Left}(t) = \int_0^T d\tau h(\tau)\left(\sigma^2 - \frac{\alpha}{2}|t-\tau|^{2H}\right), \qquad \mathrm{Right}(t) = \sigma^2 - \frac{\alpha}{2}(t+z)^{2H} . \qquad (27)$$

The obtained weight function $h(\tau)$ is a step one, so the function $\mathrm{Left}(t)$ may be calculated as

$$\mathrm{Left}(t) = \sum_{\beta=1}^{n} h_\beta X_\beta(t) \qquad (28)$$

where

$$h_\beta = h\left(\frac{1}{2}\left(\frac{(\beta-1)T}{n} + \frac{\beta T}{n}\right)\right) = h\left(\frac{(2\beta-1)T}{2n}\right), \qquad (29)$$

and

$$X_\beta(t) = \int_{\frac{(\beta-1)T}{n}}^{\frac{\beta T}{n}} d\tau\left(\sigma^2 - \frac{\alpha}{2}|t-\tau|^{2H}\right) = \sigma^2\frac{T}{n} - \frac{\alpha}{2}\int_{\frac{(\beta-1)T}{n}}^{\frac{\beta T}{n}} d\tau |t-\tau|^{2H} \qquad (30)$$

which yields

$$X_\beta(t) = \begin{cases} \sigma^2\frac{T}{n} - \frac{\alpha}{2(2H+1)}\left[\left(t-\frac{(\beta-1)T}{n}\right)^{2H+1} - \left(t-\frac{\beta T}{n}\right)^{2H+1}\right], t > \frac{\beta T}{n} \\ \sigma^2\frac{T}{n} - \frac{\alpha}{2(2H+1)}\left[\left(\frac{\beta T}{n}-t\right)^{2H+1} - \left(\frac{(\beta-1)T}{n}-t\right)^{2H+1}\right], t < \frac{(\beta-1)T}{n} \\ \sigma^2\frac{T}{n} - \frac{\alpha}{2(2H+1)}\left[\left(\frac{\beta T}{n}-t\right)^{2H+1} + \left(t-\frac{(\beta-1)T}{n}\right)^{2H+1}\right], \frac{(\beta-1)T}{n} \le t \le \frac{\beta T}{n} \end{cases} . \qquad (31)$$

The mean average error (MAE) may be calculated in order to estimate the accuracy of the obtained solutions:

$$\mathrm{MAE} = \frac{1}{T}\int_0^T \left|\mathrm{Left}(t) - \mathrm{Right}(t)\right| . \qquad (32)$$

The integral (32) may be numerically calculated, for example, as follows:

$$\mathrm{MAE} \approx \frac{1}{T}\sum_{j=1}^{N}\left|\mathrm{Left}\left(\frac{(2j-1)T}{2N}\right) - \mathrm{Right}\left(\frac{(2j-1)T}{2N}\right)\right| \cdot \frac{T}{N} = \frac{1}{N}\sum_{j=1}^{N}\left|\mathrm{Left}\left(\frac{(2j-1)T}{2N}\right) - \mathrm{Right}\left(\frac{(2j-1)T}{2N}\right)\right| . \qquad (33)$$

In this paper, the value $N = 10^{-4}$ is used.

So the algorithm of the MAE obtaining is as follows:

1. Calculate the quantities $h_\beta$ by formulas (29).
2. Introduce the functions $X_\beta(t)$ by formulas (30).
3. Introduce the function $\mathrm{Left}(t)$ by formula (28).
4. Introduce the function $\mathrm{Right}(t)$ by formula (27).
5. Calculate the MAE by formula (33).

It should be stressed that the only integral that should be numerically calculated is the integral (33) for the MAE calculation. For all the other integrals, applicable analytical expressions are obtained.

## 4 EXPERIMENTS

The numerical calculations are made in the Wolfram Mathematica package. The results for the following set of parameters are investigated:

$$T = 100,\ z = 3,\ H = 0.8,\ \sigma = 1.2,\ \alpha = 3 \cdot 10^{-3} . \qquad (34)$$

The results for the polynomial solutions for the parameters (34) were investigated in [5–7]. It was shown that the approximations of a number of polynomials from 9 to 15 absolutely fail (the corresponding MAE is greater than $10^2$, the graphs of $\mathrm{Left}(t)$ and $\mathrm{Right}(t)$ are totally different). On the basis of the Walsh functions the following MAE are obtained, see Table 1.

As can be seen, the approximations of $n = 2^m$ Walsh functions are rather accurate, and the accuracy increases with $m$. For graphical visualization, the graphs of $\mathrm{Left}(t)$ and $\mathrm{Right}(t)$ are given for the approximation of 256 Walsh functions, see Fig. 1.

As can be seen from Fig. 1, an almost ideal coincidence of the curves takes place.


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Table 1 – MAE for the parameters (34) for the approximations of $n$ Walsh functions rounded off to two significant digits

| $n$ | MAE |
|---|---|
| 2 | $3.2\cdot10^{-2}$ |
| 4 | $2.4\cdot10^{-2}$ |
| 8 | $9.2\cdot10^{-3}$ |
| 16 | $7.4\cdot10^{-4}$ |
| 32 | $1.1\cdot10^{-4}$ |
| 64 | $2.0\cdot10^{-5}$ |
| 128 | $3.8\cdot10^{-6}$ |
| 256 | $7.6\cdot10^{-7}$ |

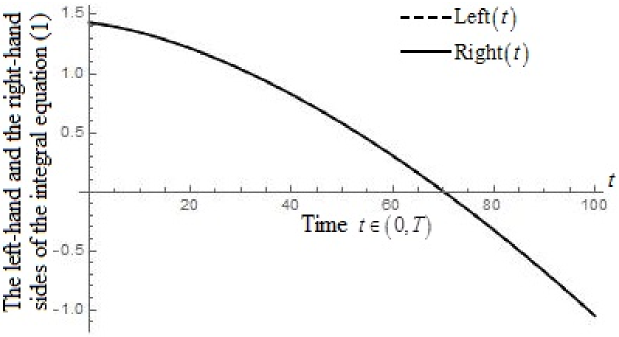


Figure 1 – Comparison of the left-hand and right-hand sides of eq. (1) for parameters (34) for the approximation of 256 Walsh functions.

The following set of parameters is also investigated:

$$T = 1000,\ z = 3,\ H = 0.8,\ \sigma = 1.2,\ \alpha = 8\cdot10^{-5}. \qquad (35)$$

As indicated in [5–7], in fact only the approximation of two polynomials is valid for the parameters (35). As for the Walsh solutions, we have the following, see Table 2:

Table 2 – MAE for the parameters (35) for the approximations of $n$ Walsh functions rounded off to two significant digits

| $n$ | MAE |
|---|---|
| 2 | $3.2\cdot10^{-2}$ |
| 4 | $3.8\cdot10^{-2}$ |
| 8 | $4.1\cdot10^{-3}$ |
| 16 | $4.1\cdot10^{-4}$ |
| 32 | $6.0\cdot10^{-5}$ |
| 64 | $9.8\cdot10^{-6}$ |
| 128 | $1.7\cdot10^{-6}$ |
| 256 | $3.2\cdot10^{-7}$ |

So the solutions based on the Walsh functions give good results for the parameters (35). For graphical visualization, the graphs of $\mathrm{Left}(t)$ and $\mathrm{Right}(t)$ are given for the approximation of 256 Walsh functions, see Fig. 2. As can be seen from Fig. 2, an almost ideal coincidence of the curves takes place.

It should also be stressed that even for the parameters (34) the Wolfram Mathematica is not able to build a graph for $\mathrm{Left}(t)$ for a number of polynomials greater than 18. The method based on the Walsh functions allows one to treat several hundreds of functions.

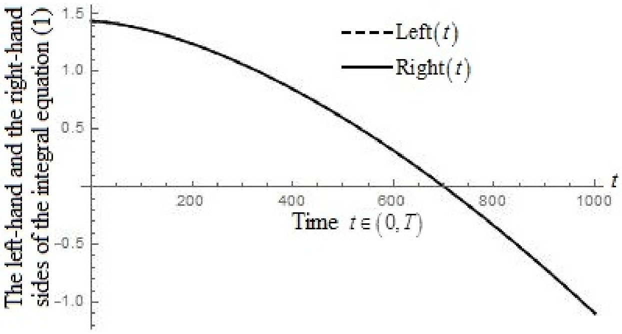


Figure 2 – Comparison of the left-hand and right-hand sides of eq. (1) for parameters (35) for the approximation of 256 Walsh functions

So one can conclude that the method based on a truncated Walsh function expansion works very well and the accuracy of the approximations of rather high numbers of Walsh functions are very accurate.

## 5 RESULTS

The Kolmogorov-Wiener filter weight function for the prediction of a random process with a power-law structure function is investigated. The investigation is based on the Galerkin method, in the framework of which the unknown weight function is sought in the form of a truncated Walsh function series. An algorithm of the weight function derivation is developed, this algorithm does not require a numerical calculation of the integrals.

In order to verify he results, the mean average error (MAE) of the residual (the difference of the left-hand side and the right-hand side) of the integral equation (1) is introduced. The sets of parameters (34) and (35) are investigated (the corresponding sets were also investigated for polynomial solutions [5–7]). It is shown that the Walsh function approximations give good results and the approximations of a rather large number of the Walsh functions are very accurate (they lead to a very small MAE).

It is found that the Walsh function solutions are much better than the polynomial ones in the framework of the problem under consideration.

## 6 DISCUSSION

In this paper we propose to realize the Galerkin method for obtaining the weight function on the basis of a truncated Walsh function expansion. The corresponding algorithm of the weight function derivation is developed. In contrast to the truncated polynomial expansion method, the proposed algorithm have the following advantages:

1. The numerical calculation of the integrals is not needed for the weight function obtaining (the only numerical calculation of the integral is needed for MAE obtaining, not for the weight function obtaining). In particular, applicable analytical expressions for the integral brackets are obtained.


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2. The calculation of the left-hand side of the Wiener-Hopf integral equation does not require numerical calculation of the integral, the corresponding analytical expression for the function $\mathrm{Left}(t)$ is obtained.

3. The use of the Walsh functions does not lead to the products of very small and very large numbers, which can hardly be calculated numerically.

It is shown that for the same parameters the solutions based on the Walsh functions are much better than the polynomial ones. They are applicable in a wider range of parameters, and they lead to far smaller values of the MAE (the MAE for the 18-polynonial approximation for the parameters (34) is of the order $10^{-3}$, the MAE for the 256 Walsh function approximation is of the order $10^{-7}$). The use of the Walsh functions allows one to numerically treat the approximations of several hundreds of Walsh functions (in contrast to only several tens of polynomials). Moreover, there are no Walsh function approximations that absolutely fail, in contrast to polynomial ones.

## CONCLUSIONS

The Kolmogorov-Wiener filter weight function for the prediction of a random process with a power-law structure function is investigated on the basis of a truncated Walsh function expansion. It is shown that in the framework of the problem under consideration the Walsh function solution is much better than the polynomial solutions investigated in the authors' previous papers.

Random processes with a power-law structure function are widely used indifferent fields of knowledge (in particular, for the telecommunication traffic description). So the results of this paper may be useful for practical prediction of stationary random processes with a power-law structure function in various fields of knowledge (in particular, for telecommunication traffic prediction in systems with data packet transfer).

**The scientific novelty** of the paper is the fact that for the first time the weight function under consideration is found on the basis of a truncated Walsh function expansion. The proposed algorithm of the weight function derivation does not lead either to numerical calculation of integrals or to the product of very large and very small numbers. It is shown that for the problem under consideration the Walsh function solutions are much better than the polynomial ones.

**The practical significance** is that the obtained results may be applied for the practical prediction of stationary random processes with a power-law structure function in various fields of knowledge.

**Prospects for further research** are to obtain a practical prediction on the basis of the obtained results.

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## ДО ФІЛЬТРУ КОЛМОГОРОВА-ВІНЕРА ДЛЯ ВИПАДКОВОГО ПРОЦЕСУ ЗІ СТЕПЕНЕВОЮ СТРУКТУРНОЮ ФУНКЦІЄЮ НА ОСНОВІ ФУНКЦІЙ УОЛША

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### АНОТАЦІЯ

**Актуальність**. Розглянуто вагову функцію фільтра Колмогорова-Вінера для прогнозування неперервного стаціонарного випадкового процесу зі степеневою структурною функцією.

**Мета роботи.** Метою роботи є розробити алгоритм отримання наближеного розв’язку для вагової функції, який не містить числового обчислення інтегралів.

**Метод.** Вагова функція, що розглядається, підпорядковується інтегральному рівнянню Вінера-Хопфа. Пошук точного аналітичного розв’язку відповідного інтегрального рівняння стикається з труднощами, тож шукається наближений розв’язок для вагової функції в рамках методу Галеркіна, який базується на основі обірваного розвинення в ряд за функціями Уолша.

**Результати.** Розроблено відповідний алгоритм отримання вагової функції. Усі інтеграли обчислено аналітично, а не чисельно. Більше того, показано, що точність отриманих наближень, що базуються на функціях Уолша, є значно кращою за точність поліноміальних розв’язків, отриманих у попередніх роботах авторів. Розв’язки, що базуються на функціях Уолша, є застосовними у ширшому діапазоні параметрів, ніж поліноміальні розв’язки.

**Висновки.** Розроблено алгоритм отримання вагової функції фільтра Колмогорова-Вінера для прогнозування неперервного стаціонарного випадкового процесу зі степеневою структурною функцією. Основою алгоритму є розвинення за функціями Уолша. На відміну від поліноміальних розв’язків, досліджених у минулих статтях, розроблений алгоритм має наступні переваги. По-перше, усі інтеграли обчислено аналітично, і немає потреби в числовому розрахунку інтегралів. По-друге, проблема добутку дуже малих та дуже великих чисел відсутня в рамках запропонованого алгоритму. На наш погляд, це є причиною того, що точність розв’язків, що базуються на функціях Уолша, є кращою за точність поліноміальних розв’язків для багатьох наближень, і це є причиною того, що розв’язки на основі функцій Уолша є застосовними у ширшому діапазоні параметрів, ніж поліноміальні розв’язки. Результати роботи можуть бути застосовані до, наприклад, прогнозування на практиці трафіку в телекомунікаційних системах з пакетною передачею даних.

**КЛЮЧОВІ СЛОВА:** вагова функція фільтра Колмогорова-Вінера, неперервний випадковий процес, функції Уолша, метод Галеркіна, степенева структурна функція.

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## К ФИЛЬТРУ КОЛМОГОРОВА-ВИНЕРА ДЛЯ СЛУЧАЙНОГО ПРОЦЕССА СО СТЕПЕННОЙ СТРУКТУРНОЙ ФУНКЦИЕЙ НА ОСНОВЕ ФУНКЦИЙ УОЛША

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### АННОТАЦИЯ

**Актуальность.** Рассмотрена весовая функция фильтра Колмогорова-Винера для прогнозирования непрерывного стационарного случайного процесса со степенной структурной функцией.


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**Цель работы.** Целью работы является разработать алгоритм получения приближенного решения для весовой функции, который не содержит численного вычисления интегралов.

**Метод.** Рассматриваемая весовая функция, подчиняется интегральному уравнению Винера-Хопфа. Поиск точного аналитического решения соответствующего интегрального уравнения затруднен, так что ищется приближенное решение для весовой функции в рамках метода Галеркина, основываемого на оборванном разложении в ряд по функциям Уолша.

**Результаты.** Разработан соответствующий алгоритм получения весовой функции. Все интегралы вычислены аналитически, а не численно. Более того, показано, что точность полученных приближений, базируемых на функциях Уолша, значительно лучше, чем точность полиномиальных решений, полученных в предыдущих работах авторов. Решения, которые базируются на функциях Уолша, применимы в более широком диапазоне параметров, чем полиномиальные решения.

**Выводы.** Разработан алгоритм получения весовой функции фильтра Колмогорова-Винера для прогнозирования стационарного непрерывного случайного процесса со степенной структурной функцией. Основной алгоритма есть разложение по функциям Уолша. В отличие от полиномиальных решений, исследованных в предыдущих статьях, разработанный алгоримт имеет следующие преимущества. Во-первых, все интегралы вычислены аналитически, и нет необходимости в численном вычислении интегралов. Во-вторых, проблема произведения очень малых и очень больших чисел отсутствует в рамках предложенного алгоритма. На наш взгляд, это является причиной того, что точность решений, основывающихся на функциях Уолша, лучше, чем точность полиномиальных решений, и это является причиной того, что решения на основе функций Уолша применимы в более широком диапазоне параметров, чем полиномиальные решения. Результаты работы могут быть применимы к, например, прогнозированию на практике траффика в телекоммуникационных системах с пакетной передачей данных.

**КЛЮЧЕВЫЕ СЛОВА:** весовая функция фильтра Колмогорова-Винера, непрерывный случайный процесс, функции Уолша, метод Галеркина, степенная структурная функция.

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