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MATHEMATICAL AND COMPUTER MODELING

МАТЕМАТИЧЕСКОЕ И КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ

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FAST ALGORITHM FOR SOLVING A ONE-DIMENSIONAL UNCLOSED DESIRABLE NEIGHBORS PROBLEM

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ABSTRACT

Contex. The paper formulates a general combinatorial problem for the desired neighbors. Possible areas of practical application of the results of its development are listed. Within the framework of this problem, an analysis of the scientific literature on the optimization of combinatorial problems of practical importance that are close in subject is carried out, on the basis of which the novelty of the formulated problem accepted for scientific and algorithmic development is established.

Objective. For a particular case of the problem, the article formulates a one-dimensional unclosed integer combinatorial problem of practical importance about the desired neighbors on the example of the problem of distributing buyers on land plots, taking into account their recommendations on the desired neighborhood.

Method. A method for solving the mentioned problem has been developed and an appropriate effective algorithm has been created, which for thousands of experimental sets of hundreds of distribution subjects allows to get the optimal result on an ordinary personal computer in less than a second of counting time. The idea of developing the optimization process is expressed, which doubles the practical effect of optimization by cutting off unwanted neighbors without worsening the maximum value of the desirability criterion.

Results. The results of the work include the formulation of a one-dimensional unclosed combinatorial problem about the desired neighbors and an effective algorithm for its solution, which makes it possible to find one, several, and, if necessary, all the options for optimal distributions. The main results of the work can also include the concept and formulation of a general optimization combinatorial problem of desirable neighbors, which may have theoretical and practical prospects.

Conclusions. The method underlying the algorithm for solving the problem allows, if necessary, to easily find all the best placement options, the number of which, as a rule, is very large. It is established that their number can be reduced with benefit up to one by reducing the number of undesirable neighborhoods, which contributes to improving the quality of filtered optimal distributions in accordance with this criterion. The considered problem can receive prospects for evolution and development in various subject areas of the economy, production, architecture, urban studies and other spheres.

KEYWORDS: problem of desirable neighbors, unclosed problem, combinatorial problem, chain of vertices of the graph, optimal distribution.

ABBREVIATIONS

BBM is a method of branches and boundaries;
DNP is a desirable neighbors problem;
MP is a marriages problem;
RMP is a problem of roommates;
TSP is a traveling salesman problem.

NOMENCLATURE

$C(V)$ – desirability criterion;
G – desirability graph;
K – the number of optimal options for placing buyers by plot;
$M(W)$ – chain length W;
n – number of buyers and plots;
V – desirability vector;
$v[i].s$ – i-buyer number;
$v[i].d$ – number of the desirable neighbor for the i-buyer;
W – one-way or two-way chain of vertices of the desirability graph.

INTRODUCTION

A number of algorithms for solving classical optimization combinatorial problems are known, which find practical application in scientific, economic, industrial and other spheres. Examples are algorithms for solving the traveling salesman problem in determining optimal routes [1, 2], the classical and modified problem of roommates [3, 4], marriages problem, satchels, appointments and a number of other problems [5, 6, 7].

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Along with these problems, we can give examples of other equally important modern problems that were not only not solved, but not even put in mathematical and algorithmic plans. These problems can be combined with the theme of the desired neighborhood. These may include problems of

– ensuring the desired neighborhood of wagons in the formation of a railway train that meets some criterion of optimality (for example, fire safety) [8];

– optimal planning of the sequence of technological operations when loading the production capacities of machine-building enterprises, ensuring minimal resource costs for their adjustment [9];

– drawing up calendar schedules and schedules of the sequence of loading of these capacities for the same purpose [10];

– ensuring the desirable order of placing goods in commercial premises for the shortest bypass from the point of view of a potential buyer when buying goods of a certain type or conflict-free of their neighbors [11];

– optimal urbanism in the desired neighborhood of residential buildings and related infrastructure [12];

– in agricultural engineering when determining the desired neighborhood of crops for joint fertilizing or processing of sown areas, providing optimal cost savings [13];

– in decorative design of objects to ensure an optimal aesthetic combination of colors, styling of buildings in architectural terms, minimizing the conflict of neighboring architectural styles in large areas, etc. [14].

Such problems can have a one-dimensional character (for example, the neighbors of houses along one side of the street) or a two-dimensional character (neighborhood on the plane of a residential area, adjacent apartments of the house). Other variants of similar problems are possible.

To solve optimization combinatorial problems, appropriate methods are required. To the listed classical problems are usually applied methods related to the areas of optimization theory in applied mathematics, operations research, algorithm theory and computational complexity theory [14]. All of them are united by the problem of finding the optimal combination of objects on a finite set, provided by the methods of integer programming [15].

It is known that optimal combinatorial problems can be solved by a universal method of simply enumeration of all possible combinations of permutation of objects, on the basis of which the criterion of arrangement is consistently optimized. This method allows you to find one, several or all accurate solutions to the problem, if any. However, these problems belong to the class of so-called NP-complete problems, which, even with a relatively small number of objects, cannot be solved by iterating over options by any computers in an acceptable time [16]. Therefore, to solve each such problem, special methods are developed that take into account their characteristics, which are able to work faster than algorithms based on the brute-machine method.

1 PROBLEM STATEMENT

As an example, consider the problem of the desired neighbors when placing objects, which can be of practical importance. Suppose, there is a rectangular land mass of suburban purposes, which is divided into n plots in one line, and there are n buyers for these plots. The administration allows each buyer to choose another buyer as a desirable neighbor. For simplicity, we assume that each buyer can choose only one desirable neighbor, and his as a desirable neighbor can choose no more than two other buyers. The problem is to find such an option for placing buyers on the sites, in which the number of implemented wishes will be maximum.

At first glance, this problem corresponds to the RMP. The difference is that in the latter there are n rooms and $2n$ students, but the above problem does not contain such conditions. A correct analogy would be a soldier's barracks, which takes into account the wishes of the soldiers about the desired neighborhood. The MP is also close in meaning, but it assumes the presence of a gender of participants in the optimization process, which is not important for the DNP. Thus, the problem has no analogues and this is its novelty.

Obviously, the problem is one-dimensional. Fig. 1 shows an example of the desirability vector V, which contains data on the wishes expressed by buyers.

S	1	2	3	4	5
D	4	5	4	2	3

Figure 1 – Desirability vector V

Each element $v[i]$ of the vector V contains two attributes $v[i].s$ – the buyer number and the $v[i].d$ number of the desired neighbor. In this case, the array is divided into $n = 5$ plots and there are 5 buyers on them. By the vector it is possible to determine that the first buyer $v[1].s = 1$ would like to have a neighbor of the buyer $v[1].d = 4$, 2 – 5, 3 – 4, 4 – 2, 5 – 3. Graphically indicated customer preferences can be visualized using the oriented desirability graph G, which is shown in Fig. 2.

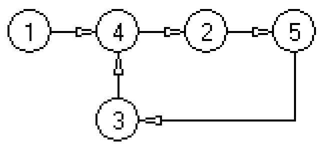

Figure 2 – Desirability graph G

If arrange buyers on the plots in the specified order, then only the desire of the buyer 3, who will receive the desired neighbor 4, will be satisfied. The wishes of other buyers will not be taken into account. For such a vector, the desirability criterion $C(V) = 1$. For the arrangement of 4–2–5–3–1 we have $C(V) = 3$, that is, the result will be better. However, if you arrange buyers in line S in order of 2–5–3–4–1, then the result $C(V) = 4$ will be even better.

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Having performed all the permutations, we find that this result is optimal and, therefore, is the solution to the problem. In this case, it turns out that there are several other options that give the same value of the criterion of optimality *C*(*V*), (for example, such an option is 1–4–2–5–3). This analysis shows that the problem is integer, combinatorial and optimization, allowing a solution in the form of many optimal options for arranging buyers by site.

2 REVIEW OF THE LITERATURE

As noted above, the universal method of solving integer combinatorial problems is the method of enumerating the possible variants of the arrangement [17]. The disadvantage of the method is that it can be used for optimization only with a small number of objects. Even for the fastest computers, its capabilities are limited to n = 15, and for the bulk of personal computers used in calculations, this number is limited to n = 12 [18]. Therefore, the brute-put method is unsuitable for solving practical problems for $n > 12$.

The idea of reducing the number of options to go is to consider only promising options [18]. The proposed approach to finding optimal solutions to many problems of optimal combinatorics is called the BBM. Finding a solution by the BBM is associated with the tree of finding the optimal solution. The method assumes the presence of a root from which the branches of the decision tree emanate. The use of boundaries stimulates or inhibits the growth of branches on such a tree. The method uses the procedure for breaking down the permissible solutions of the original problem into non-intersecting subsets of smaller size and evaluating them. Each step of the partitioning algorithm is accompanied by a check of the condition of whether or not contains each such subset of the optimal solution, thereby cutting off a large number of variants that do not need to be checked [1]. The fact of using the root indicates the futility of using BBM in solving the DNP since it is not known from which vertex of the graph *G* the optimal sequence of distribution of plots buyers can proceed.

Two other combinatorial problems are known. This is the problem of RMP and the MP [3, 4, 5]. Both problems are specific. The first is characterized by the fact that in the dormitory students live by two people in a room and there is no restriction used by the second problem, involving the division into two sexes. Both problems are solved by brute-factor method, while unlike the MP, the RMP may not have a solution [3].

There are other optimization combinatorial problems solved by go-over methods, the effectiveness of which is based on cutting off unpromising permutations [19].

The analysis of the methods used to determine the optimal solutions of optimization combinatorial problems allows us to conclude that there is no variant of the method of branches and boundaries for solving the DNP. Methods of solving problems about RMP and MP are also not suitable. Therefore, the approach that guarantees the solution of the above problem is to develop a method that takes into account the specifics of this class of problems.

3 MATERIALS AND METHODS

The DNP has obvious solutions for some special cases.

1. A situation is possible when all buyers do not care who will be their neighbors. An example is shown in Fig. 3. In this case, any arrangement of buyers on the sites will be optimal. For them, $C(V) = 0$.

S	1	2	3	4	5	6
D	0	0	0	0	0	0

Figure 3 – Neutral neighborhoods

2. Obviously, the maximum value of the desirability criterion with the number of subjects $M(V) = n$ is equal to $C(V) = n$, which is possible only with an even number of buyers and a mutual pairwise desirable neighborhood, as shown in Fig. 4.

S	1	2	3	4	5	6
D	4	6	5	1	3	2

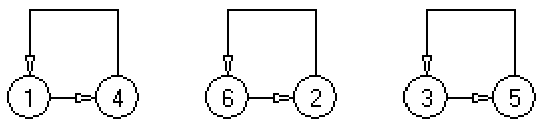

Figure 4 – Reciprocal paired desirable neighborhoods

The initial distribution for n = 6 will be optimal, giving the optimality criterion the maximum possible value.

3. For all other variants of the arrangement, the desirability criterion cannot exceed the value of C(V) = n–1. A typical example of such a distribution is represented by the cyclic arrangement, which is shown in Fig. 5.

S	1	2	3	4	5	6
D	2	3	4	5	6	1

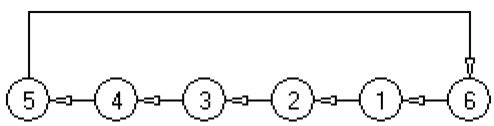

Figure 5 – Cyclical desirable neighborhoods

Any cyclical shift of such an arrangement to the left or right will give a new optimal arrangement. Inverse cyclic arrangements, in which the arrows have opposite directions, will also be optimal.

The data presented in Fig. 4 show that in some cases the set of vertices of a graph *G* can be represented by combining non-intersecting subsets of vertices.

$$V = V_1 \cup V_2 \cup ... \cup V_{k-1} \cup V_k, \quad (1)$$

where k is the number of subsets.

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For example, for the matrix shown in Fig. 4, $k = 3$, $V_1(S) = (1,4)$, $V_2(S) = (6,2)$, $V_3(S) = (3,5)$. Obviously, the value of the desirability criterion of a vertices set is equal to the sum of the values of its subsets criteria

$$C(V) = \sum_{i=1}^{k} C(V_i). \quad (1)$$

It is also obvious that for distributions where all buyers have chosen desirable neighbors, subsets of a graph in which there are at least two vertices always have a vertex cycle. Indeed, if there were no such cycle, there would always be at least one vertex without a desirable neighbor, when there would be a break in ties, which within one subset is impossible. This conclusion extends to a graph represented by one set of vertices connected to each other. An example of such a graph is shown in Fig. 6.

In the above example, the cycle is formed by vertices 1, 6, 4, 7. If there were no way out for any of these vertices in such a graph, it would mean that the relevant buyers are neutral in choosing the desired neighbor. For other vertices, the absence of an arrow would mean, in addition, that the set is divided into at least two non-intersecting subsets, which contradicts the condition that the set of vertices of the graph for this example is unique.

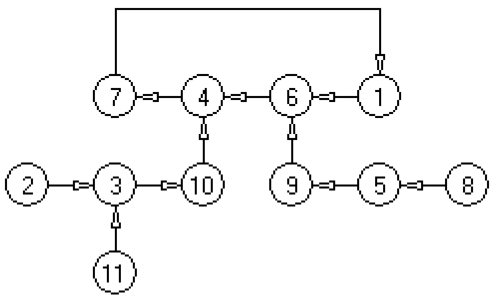

Figure 6 – Graph with cycle

If the original arrangement does not correspond to any of the special cases considered, then the search for a solution begins with the division of the set V into non-intersecting subsets. The first subset includes the first element V, and among the remaining elements is selected that is desirable for the first or the first is desirable for it. If there is such an element, it is added to the first subset. The process continues as long as such elements can be detected. Similarly, we are looking for other subsets until all the elements of the set V are exhausted. Fig. 7 shows an example of two formed subsets V_1 and V_2 of the original set V of 11 objects.

It follows that each buyer and his desired neighbor always belong to one subset. The next step in optimization is to find the optimal distributions for each subset. To do this, divide all the vertices of the graph into three types – those that do not have inputs, have one input and have more than one input. Let's call the first "terminal", the second "transit" and the third "unification". In Fig. 8, the terminal vertices are colored yellow, the transit vertices as azure and the unifying ones are green.

S	1	2	3	4	5	6	7	8	9	10	11
D	3	5	5	8	2	4	2	11	6	4	6

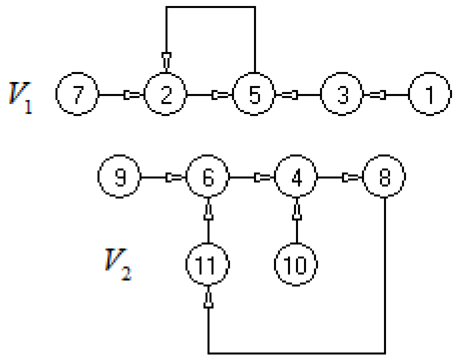

Figure 7 – Graphical representation of a set V of two non-intersecting subsets V_1 and V_2

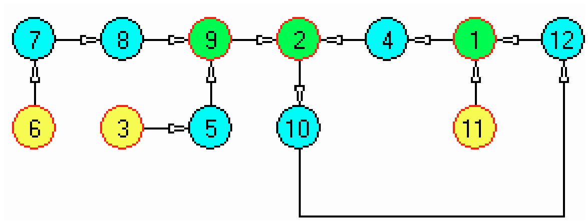

Figure 8 – Three types of vertices of a graph

The chains of vertices that are part of the distributions, including optimal, often begin and end with terminal vertices. Unifying vertices are the branching points of chains, or unifying points, in which the ends of two chains merge into one. An example is the chain of vertices $W_1 = (1,3,2)$, which is shown in Fig. 9. For this chain, the desirability criterion is $C(W_1) = 2$. The sequence W_1 is obtained by combining chains (1,3) and (2,3), where 1 and 2 are terminal, 3 are unification.

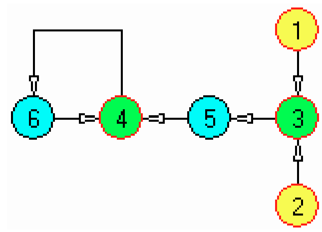

Figure 9 – Combined vertex chain

The key point of the method is the obvious fact that for the chain W_1 it is impossible to find the best option for vertex alternation from the point of view of the desirability criterion, except for its involution, which has the same desirability criterion. Therefore, if we cut off a given chain from its subset, then the optimal value of the criterion of the desirability of the subset will be equal to the sum of the criterion of this chain and the optimal criterion of the subset minus the vertices of the cut chain.

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Subtracting a subset of W_1 from the set W will give a new subset of W_2 = (5,4,6), in which, as can be seen from Fig. 10, a new terminal vertex 5 appears.

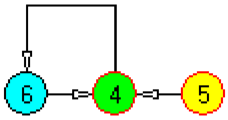

Figure 10 – Graph of vertices W_2 obtained by subtracting chain W_1 from chain W

If you select the combined chain W_2 = (6,4,5) in the graph, then for it $C(W_2)$ = 3.

The total desirability criterion for W is $C(W_2) = C(W_1) + C(W_2)$ = 2 + 3 = 5. The distribution of W = (1,3,2) (6,4,5) = (1,3,2,6,4,5) is optimal. This distribution takes into account the wishes of subjects 1, 2, 6, 4, 5 and does not take into account the wishes of subject 3.

The combined chains can be divided into two types – one-way and two-way. One-way chains have no unifying vertices. Such chains may not have vertices, or have an arbitrary number of them (Fig. 11a). Two-way chains always contain one unifying vertex, two terminal vertices, and an arbitrary number of transit vertices (Fig. 11b).

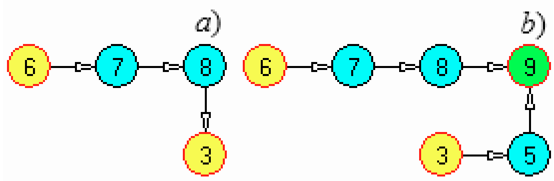

Figure 11 – One- way *a* and two-way *b* vertex chains

Sequential cutting of two-way chains should be carried out until there is not a single unifying vertex left outside the cycle (Fig. 12).

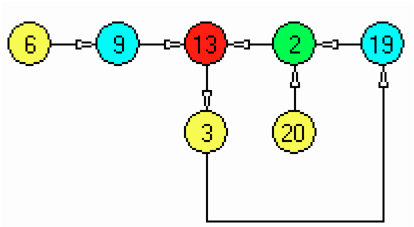

Figure 12 – Unifying vertices remained only on the cycle

At the same time, only one-way chains adjacent to the unifying vertices of the cycle will remain outside the cycle.

If the cycle does not contain two mutually desirable subjects, then the further optimization process deals with the break of the cycle along its vertices.

The break is made at each vertex of the cycle, followed by the cutting off of the two-way chains until a single one-way chain remains. Fig. 13 shows the situation with the gap at vertex 13.

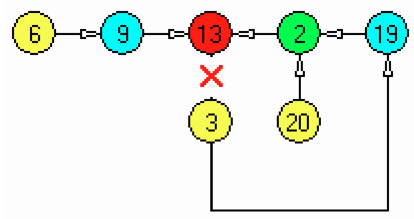

Figure 13 – Cycle with a gap at the vertex 13

There are two options for completing optimization on the current vertex with a gap. In the first case, it is possible to construct a two-way chain W_1 = (6,9,13,2,20), for which $C(W_1)$ = 4, and a one-way chain W2 = (3,19), for which $C(W_2)$ = 1. In this case, $C(W) = C(W_1) + C(W_2)$ = 5. In the second case, W_1 = (3,19,2,13,9,6), for which $C(W_1)$ = 5, and a one-way chain W_2 = (20), for which $C(W_2)$ = 0. In this case, $C(W) = C(W_1) + C(W_2)$ = 5. Both options give the same total desirability and either of these distributions can be chosen as the optimal. By adding the last chains to the previously cut off two-way chains, we get the final distribution for this variant of cycle break. Similarly, a gap is made on other vertices of the cycle and for each such variant from them a distribution is created. Choosing the best of them, we get the optimal distribution, which is a solution for a given subset of vertices. By performing similar optimization for all subsets, we get an optimal solution to the problem for the whole set by combining optimal solutions for all its subsets.

Thus, the enlarged step-by-step algorithm for solving the DNP can be represented in the following form.

Step 1. Divide the set of subjects into non-intersecting subsets.

Step 2. For each subset, cut off the two-way chains as long as possible. If the result is a one-way chain, then after its optimization and inclusion in the union of previously cut off two-way chains, we get the optimal distribution for the current subset. If a loop exists, then you must consistently perform optimization with a gap at each vertex of the cycle, choosing the best of the distributions as the optimal. By adding it to the union of previously cut two-way chains, we get the optimal distribution for this subset.

Step 3. Combine the optimal distributions for each subset. This association will give an optimal distribution for the whole set of subjects.

4 EXPERIMENTS

The developed algorithm was programmed on an ordinary personal computer in the Delphi visual programming environment [20]. Calculations were carried out for $n \in [3,1000]$.

Random sets with one and many subsets, with and without loops (the latter correspond to sets with subjects that have a neutral relation to the neighborhood) were considered. Verification of the algorithm was carried out by comparing the calculated data with the results of

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calculations by brute-factor method at $n < 12$ for thousands of random sets. In all cases, the optimality of the solutions obtained is confirmed.

Verification of the algorithm was carried out for 2500 randomly generated initial distributions. Table 1 shows the main results of the experiments for different values of the parameter n.

Table 1 – Results of computational experiments

n	Average number of subsets	Average number of cut off chains	Average number of vertices in a chain	Average optimization time, s
10	1.41	2.2	3.4	0.01
50	1.63	8.6	6.1	0.21
100	1.97	11.4	8.5	0.43
500	2.65	18.7	16.4	0.65
1000	3.82	44.3	22.6	0.87

As can be seen from Table 1, the average number of subsets of the generated initial distributions selected by the algorithm is weakly dependent on the number n of their subjects. This dependence is much stronger on the random number of cut off chains and the average number of vertices in each such chain. It can also be seen that the optimization time is almost linearly dependent on the number of subjects, and in all cases it does not exceed one second to obtain the optimal distribution. It was found that there is no regularity for the dependence of the average number of vertices in cycles on the number of subjects. Their number may vary significantly from one distribution option to another. For example, in a distribution with $n = 100$, there may be 15 vertices in a cycle, and in the distribution for $n = 1000$ it can be 4.

Thus, in practice, the effectiveness of the developed algorithm for solving the DNP in terms of its optimality and speed is confirmed.

5 RESULTS

The results of the work include the formulation of a one-dimensional unclosed combinatorial problem about the desired neighbors and an effective algorithm for its solution, which allows you to find one, several, and, if necessary, all the options for optimal distributions. In addition, the main results of the work can also include the concept and formulation of a general optimization combinatorial DNP, which, presumably, will have theoretical and practical prospects. This problem can be evoluted and developed in various subject areas of economics, production, architecture, urban studies and other areas.

6 DISCUSSION

The method underlying the algorithm for solving the problem allows, if necessary, to easily find all the best options for placing buyers on the sites. To do this, you need to define all subsets of the vertices of the desirability graph and all the cut off two-way and one-way chains in each of them. The subset arranged in any sequence and the vertex chains belonging to them will give optimal distributions. In addition, the involution of such chains will also give an optimal distribution.

To reduce the number of optimal distributions, it is possible to set an additional problem of cutting off those options that indicate an undesirable neighbors of the terminal vertices of the chains connected to each other that has practical interest. To do this, after the end of the optimization process, it would be possible to conduct a second round of a survey among buyers about which neighbors from the proposed list, on the contrary, are undesirable for them. This procedure seems appropriate, because with the same optimal desirability criterion, the effect of the resulting solution is doubled by cutting off the best options with undesirable neighborhoods, which seems no less important within the framework of the problem under consideration.

For example, the chains $W_1 = (1,3,2)$ and $W_2 = (6,4,5)$ shown in Fig. 9 may be optimally distributed adjacent to each other. In this case, each of the subjects 1 or 2 may be neighbors of subjects 5 or 6. If you conduct a survey on the undesirability of such a neighborhood, where a positive answer will be received, then one of the chains can be moved to another place of distribution without compromising the general criterion of optimality. At the same time, obviously, the quality of the method will increase, because recommendations will be taken into account not only about the desirable, but also about the undesirable neighborhood.

Such surveys can be iterative. They can be carried out until there is the only optimal option for the final distribution of buyers on the sites. Since the algorithm works quickly, this option can be obtained in a short time within each iterative survey session.

CONCLUSIONS

The paper formulates a general combinatorial problem of the desired neighbors. Possible areas of practical application of the results of its development are listed. Within the framework of this problem, an analysis of the scientific literature on the optimization of combinatorial problems of practical importance that are close in subject is carried out. On this basis, the novelty of the formulated problem is established. A one-dimensional unclosed problem is considered on the example of the problem of distribution of buyers on land plots, taking into account their recommendations on the desired neighborhood. An effective method of solving the problem has been developed and an appropriate algorithm has been created, which for hundreds of distribution subjects allows to get

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the optimal result on an ordinary personal computer in less than a second of counting time. The idea of continuing the optimization process is proposed, which doubles the practical effect by cutting off unwanted neighborhoods without worsening the optimal value of the general desirability criterion.

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БЫСТРЫЙ АЛГОРИТМ РЕШЕНИЯ ОДНОМЕРНОЙ НЕЗАМКНУТОЙ ЗАДАЧИ О ЖЕЛАТЕЛЬНЫХ СОСЕДЯХ

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АННОТАЦИЯ

Актуальность. В работе сформулирована общая комбинаторная проблема желательного соседства. Перечислены возможные сферы практического применения результатов ее разработки. В рамках данной проблемы проведен анализ научной литературы по оптимизации близких по тематике комбинаторных задач, имеющих практическое значение, на основе которого установлена новизна сформулированной проблемы, принятой к научной и алгоритмической разработке.

Цель. Для частного случая проблемы в статье сформулирована имеющая практическое значение одномерная незамкнутая целочисленная комбинаторная задача о желательном соседстве на примере проблемы распределения покупателей по земельным участкам с учетом их рекомендаций о желательном соседстве.

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Метод. Разработан метод решения упомянутой задачи и создан соответствующий эффективный алгоритм, который для тысяч экспериментальных множеств из сотен субъектов распределения позволяет на обычном персональном компьютере получить оптимальный результат менее чем за секунду времени счета. Высказана идея развития процесса оптимизации, которая удваивает практический эффект от оптимизации за счет отсечения нежелательных соседств без ухудшения максимальной величины критерия желательности.

Результаты. К результатам работы относятся постановка одномерной незамкнутой комбинаторной задачи о желательных соседях и эффективный алгоритм ее решения, который позволяет найти один, несколько, а при необходимости все варианты оптимальных распределений. К основным результатам работы можно также отнести концепцию и постановку общей оптимизационной комбинаторной проблемы желательных соседей, которая может иметь реальные теоретические и практические перспективы.

Выводы. Метод, лежащий в основе алгоритма решения задачи, позволяет при необходимости без затруднений найти все оптимальные варианты размещения, число которых как правило, весьма велико. Установлено, что их количество может быть уменьшено с пользой вплоть до единицы за счет уменьшения количества нежелательных соседств, что способствует повышению качества отфильтрованных оптимальных распределений в соответствии с данным критерием. Рассмотренная проблема может получить перспективы развития и разработки в различных предметных областях экономики, производства, архитектуры, урбанистики и других сферах.

КЛЮЧЕВЫЕ СЛОВА: задача о желательных соседях, незамкнутая задача, комбинаторная задача, цепочка вершин графа, оптимальное распределение.

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ШВИДКИЙ АЛГОРИТМ ВИРІШЕННЯ ОДНОМІРНОЇ НЕЗАМКНЕНОЇ ЗАДАЧІ ПРО БАЖАНИХ СУСІДІВ

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АНОТАЦІЯ

Актуальність. У роботі сформульовано спільну комбінаторну проблему бажаного сусідства. Наведено можливі сфери практичного застосування результатів її розробки. В рамках цієї проблеми проведено аналіз наукової літератури з оптимізації близьких за тематикою комбінаторних завдань, що мають практичне значення, на основі якого встановлено новизну сформульованої проблеми, прийнятої до наукової та алгоритмічної розробки.

Ціль. Для окремого випадку проблеми у статті сформульовано одномірне незамкнене цілечисленне комбінаторне завдання, що має практичне значення, про бажане сусідство на прикладі проблеми розподілу покупців по земельних ділянках з урахуванням їх рекомендацій про бажане сусідство.

Метод. Розроблено метод вирішення згаданої задачі та створено відповідний ефективний алгоритм, який для тисяч експериментальних множин із сотень суб'єктів розподілу дозволяє на звичайному персональному комп'ютері отримати оптимальний результат менш ніж за секунду часу рахунку. Висловлено ідею розвитку процесу оптимізації, яка подвоює практичний ефект від оптимізації за рахунок відсікання небажаних сусідств без погіршення максимальної величини критерію бажаності.

Результати. До результатів роботи відносяться постановка одновимірної незамкнутої комбінаторної задачі про бажаних сусідів та ефективний алгоритм її вирішення, який дозволяє знайти один, кілька, а за необхідності всі варіанти оптимальних розподілів. До основних результатів роботи можна також віднести концепцію та постановку загальної оптимізаційної комбінаторної проблеми бажаних сусідів, яка може мати реальні теоретичні та практичні перспективи.

Висновки. Метод, що лежить в основі алгоритму розв'язання задачі, дозволяє при необхідності легко знайти всі оптимальні варіанти розміщення, число яких як правило, дуже велике. Встановлено, що їх кількість може бути зменшена з користю до одиниці за рахунок зменшення кількості небажаних сусідств, що сприяє підвищенню якості відфільтрованих оптимальних розподілів відповідно до даного критерію. Розглянута проблема може отримати перспективи розвитку та розробки у різних предметних галузях економіки, виробництва, архітектури, урбаністики та інших сферах.

КЛЮЧОВІ СЛОВА: задача про бажаних сусідів, незамкнена задача, комбінаторна задача, ланцюжок вершин графа, оптимальний розподіл.

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