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МАТЕМАТИЧЕСКОЕ И КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ

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THE STATES' FINAL PROBABILITIES ANALYTICAL DESCRIPTION IN AN INCOMPLETELY ACCESSIBLE QUEUING SYSTEM WITH REFUSALS

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ABSTRACT

Context. There is a problem of forecasting the efficiency of real queuing systems with refusals in the case of incomplete accessibility of service devices for the input flow of requirements. The solution of problem is necessary to create the possibility of more accurate design and control of such systems operation in real time.

Objective. The aim of the research is to obtain an analytical description of the state's final probabilities in a Markov queuing system with refusals and with incomplete accessibility of service devices for the input flow of requirements that is necessary to forecast the values of the queuing system performance indicators.

Method. The probabilities of queuing systems' states with refusals in the case of incomplete accessibility of service devices for the input flow of requirements are described by Kolmogorov differential equations. In a stationary state, these equations are transformed into a linearly dependent homogeneous system of algebraic equations. The number of equations is determined by the set-degree and for modern queuing and communication systems can be in the thousands, millions and more. Therefore, an attempt to predict the efficiency of a system is faced with the need to write down and numerically solve a countable set of algebraic equations systems that is quite difficult.

The key idea of the proposed method for finding an analytical description of final probabilities for a given queuing system was the desire to move from the description of individual states (of 2^n amount) to the description of groups of system states (of n+1 number) and to localize the influence of incomplete accessibility of service devices for the input flow of requirements in multiplicative functions of incomplete accessibility. Such functions allow obtaining the required analytical description and assessing the degree of the final probabilities transformation, in comparison with known systems, as well as assessing the forecasted values of the noted queuing system's efficiency indicators when building a system and choosing the parameters for its controlling.

Results. For the first time analytical expressions are obtained for the final probabilities of the queuing system states with refusals and with incomplete accessibility of service devices for the input flow of requirements, which makes it possible to evaluate as well as forecast values of all known system efficiency indicators.

Conclusions. The resulting description turned out to be a general case for well-known type of Markov queuing systems with refusals. The results of the numerical experiment testify in favor of correctness the obtained analytical expressions for the final probabilities and in favor of possibility for their practical application in real queuing systems when solving problems of forecasting efficiency, as well as analyzing and synthesizing the parameters of real queuing systems.

KEYWORDS: Markov models, queuing systems, incomplete accessibility of queuing devices.

ABBREVIATIONS

QS is a queuing system;

SAMS is the surface-to-air missile system;

No. i, j is a cell address in Table 2: i- row number, j-column number.

NOMENCLATURE

A is the absolute QS capacity;

C_n^m is number of combinations from n to m;

e=2.71… is a second remarkable limit;

$f_1()$ is a density distribution of the requirements flow at the input of the QS;

$f_2()$ is a density distribution of service duration;

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f_k is a function, which deforms the probability p_k of k-th state in incompletely accessible QS with respect to the Erlang model;

I is a flow intensity of requirements at the input of QS;

$i_1, \ldots, i_n$ are the individual numbers of service devices;

M is a designation of the exponential distribution of random time intervals between the requirements of the input flow and the time of servicing the requirements;

m is the number of devices in one group accessible for service to the input requirements;

$M/M/n$ is a designation of QS with refusals in the Kendall-Basharin classification;

$M_{busy.div}$ is the mathematical expectation of busy devices number;

n is a number of identical channels/devices in the QS;

N is the total number of patients per month;

$N_{dif.eq}$ is the number of differential equations and states in incompletely accessible QS with refusals;

n_i is the number of requirements served by i-th device;

N_m is the number of patients accessible for "service" to exactly m medical specialists per month;

N_{missed} is the mathematical expectation of the enemy aircraft number that broke through to the target;

$N_{missed.Erl}$ is the N_{missed} calculated by the Erlang model;

N_{SAMS} is the number of SAM systems in grouping;

N_{total} is the number of requirements that entered the system per month;

$N_{total.En}$ is the total number of enemy aircraft in the air blow;

p_{ijk} is a probability of a QS state at which devices with numbers i, j, k are occupied with servicing;

$\dot{p}_k$ is a probability derivative;

P_k is a probability of a QS state in which exactly k requirements are in the system;

$P_{k.Erl}$ is P_k but for Erlang QS $M/M/n$ model with refusals;

P_{refuse} is a service refusal probability;

$P_{refuse.Erl}$ is a service refusal probability for Erlang QS $M/M/n$ model with refusals;

$P_{service}$ is a service probability of QS;

$P_{service.Erl}$ is a service probability for Erlang QS $M/M/n$ model with refusals;

Q_{ijk} is the area of accessibility for input flow requirements by devices with numbers i, j, k

q_m is a probability of requirements' accessibility for service to a group of m service channels/devices;

$R[S_0, S_1]$ is the edge R that connects the vertex S_0 with the vertex S_1;

R_{SAMS} is the SAM fire zone radius;

$S_{ar.k}$ is the size of area with "k" multiple overlapping of fire zones;

$S_{ar.max}$ is the maximum possible coverage area by means of all SAM systems in grouping;

$S_{ar.tot}$ is the size of the cover area by all SAM systems in grouping on the terrain;

S_{ijk} is a system state, at which devices with numbers i, j, k are occupied with servicing;

S_k is vertex of graph and a system state, at which exactly k requirements are under service;

t is a current time;

T_{avr} is a mathematical expectation of requirement's service duration by the service device;

T_i is the total time spent by the i-th device for servicing;

T_{work} is the time of system operation per month;

v_i is the probability of transferring the requirement for service to one of channels provided that i accessible channels are already busy;

v_i^1 is the intensity of transferring the requirement for service to one of channels provided that i accessible channels are already busy;

γ_m is the probability that a patient will be accessible for "service" to the specialists of one group of m doctors;

ΔP_{serv} is the relative error in the service probability forecast for the Erlang model;

μ is a performance of one service device as the inverse value to the mathematical expectation of service time;

ξ_m is the maximum number of groups of m devices from the total number of n devices;

π is the ratio of a circle length to its diameter;

ρ is a load factor of a QS with a simplest flow of requirements.

INTRODUCTION

In the field of transport, trade, medicine, industry, information networks, control systems and in other areas, there is often appears repeated massive demand (flow of requirements) for various services. To work out such requirements, the corresponding "service" systems are created.

The wide distribution and diversity of such systems has caused the need to develop appropriate models of queuing systems for solving problems of analysis, synthesis and control of real systems. The moments of each requirement occurrence and the duration of its working out (service) are not known in advance (are random). If all service devices are busy, requirements can wait for their turn. "Impatient" requirements may leave the queue at an unknown point in time. Therefore, most models are stochastic

In real systems, as a rule, the conditions of the central limit theorem of A. Ya. Khinchin [1] are satisfied, and an input flow of requirements, that is close to the simplest one, is automatically generated. For such conditions, there are well-known models

However, in some real systems, not every free device can start servicing the next requirement that enters the system.

So, at a gas station, refueling a car with fuel can be done only with a device that has the required type of fuel, which can lead to a refusal to refuel the driver's car even if there are free devices, but with the wrong type of fuel.

In communication systems, there may be load schemes in which some of the options for connecting the sender to the recipient cannot be implemented, and the subscriber may receive a denial of service even if there are free channels, but in a different load group.

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In a polyclinic, not every specialist doctor can consult the next patient who needs medical care.

In a grouping of anti-aircraft missile forces, the next enemy aircraft may be in the zone of fire of a SAM system, which is still busy firing at the previous aircraft, while for other SAM systems this aircraft was outside their zones of fire. Such an aircraft will receive a refusal of service and will be able to attack with impunity and hit a protected object despite the presence of free SAM systems in the grouping.

To control such systems, the problem arises of forecasting the effectiveness of their work, taking into account the incomplete accessibility of service devices.

The models of incompletely accessible queuing systems were studied most deeply in the theory of teletraffic [2], where an analytical description of the probabilities of states for the case of a single-link ideal incomplete switching circuit [2] was obtained (the third Erlang formula).

However, in the general case, an incompletely accessible circuit has 2^n states, which leads to the need to compose and solve a system of 2^n differential and, accordingly, algebraic equations.

For the values $n = 50 \div 100$ and more encountered in practice, it is not possible to solve such a problem which complicates the control of such systems and makes the topic of this article relevant.

The object of research is a steady-state process of servicing in *M*/*M*/*n* queuing system with refusals and with incomplete accessibility of service devices for the input flow of requirements.

The subject of research is the distribution law of the final probabilities of groups of states in queuing system *M*/*M*/*n* with refusals and with incomplete accessibility of service devices for the input flow of requirements.

The research goal is to obtain an analytical description for final probabilities of states groups for the queuing system *M*/*M*/*n* with refusals and with incomplete accessibility of service devices for the input flow of requirements and also checking the correctness of the results by transforming the obtained description into a description of known Erlang system *M*/*M*/*n*.

The noted final probabilities are a complete description of the systems operation and allow estimating the expected values of all known indicators for the queuing systems efficiency.

1 PROBLEM STATEMENT

The queuing system consists of several groups of similar devices. Each device can be included in one or several groups of service devices.

The requirements flow with intensity I and density $f_1(t) = I e^{-It}$ enters the queuing system. The requirement of the input flow gets into service in one of the devices groups. If there are no free devices in this group, then the request is denied service and leaves the system. If there are free devices in this group, then any free device is selected to service the request with the same probability for all free devices. Service duration is random and has exponential distribution $f_2(t) = \mu e^{-\mu t}$. By virtue of the noted distribution densities, a Markov process with continuous time and discrete states arises in the system.

The problem statement of constructing a model of an incompletely accessible queuing system with refusals in theory of teletraffic is known [2]. In the actual area of air defence, the task of formalizing real processes with the transition to a model of an incompletely accessible queuing system with refusals will be considered in section experiments.

In order to visually demonstrate the logic of formalizing physical processes when building a model of an incompletely accessible queuing system, let's consider a simplified example of a city polyclinic work, where n medical specialists see patients. Let's assign an individual number to each doctor (service device): i_1, i_2, …, i_n and conditionally represent the areas of diagnosis inherent in each specialist doctor by circles (Fig. 1).

When making a diagnosis and prescribing treatment, there are mutually overlapping areas of physicians' capabilities (Fig. 1, areas Q_{12}, Q_{23}, Q_{13}, Q_{123}). So, for example, a patient with pain symptoms of the spine can be seen by a surgeon, by a vertebrologist and by a neuropathologist (Fig. 1, area Q_{123}), about which the patient can receive information from a nurse at the polyclinic registry. In this case, the group of doctors available to the patient includes three ($m_1 = 3$) specialist doctors. Let us introduce the necessary concept.

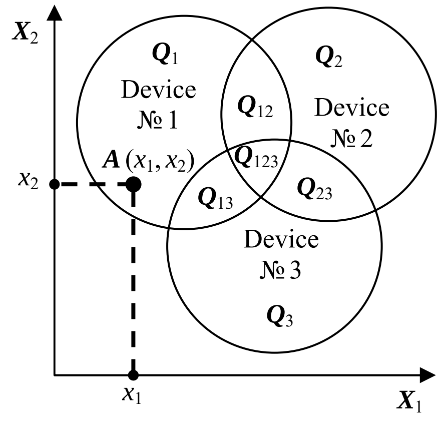

Figure 1 – Visualization of the principles for choosing an affordable device to serve the next requirement

The coverage ratio of the requirements flow by a specific group of m service devices (specialist doctors) is the share of requirements (patients) in the input flow that is available for servicing at once by all m devices (specialist doctors) of a particular group ($m \geq 0$). In the given example, the value of the patients flow share depends on the qualifications of doctors and on the structure of the patients' flow. There are also such symptoms of patients that are available for analysis only by a specific specialist doctor (Fig. 1, areas Q_1, Q_2, Q_3,).

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Let us formulate the noted property: the capabilities of an incompletely accessible system for servicing the input flow of requirements are characterized by the presence of the devices groups' operation areas with m-multiple "coverage coefficient", that is, such areas where the input requirement can be accessible for servicing by all m devices of a particular group.

The probability that the next requirement of the input flow will be accessible for service immediately to m devices of a particular group is denoted by a symbol $Q_{i_1 \dots i_m}$ indicating the numbers of i_j devices.

Such "areas" of coverage (areas of possible service), for example, a neuropathologist, can be not only with a surgeon or vertebrologist, but also with a general practitioner and other specialist doctors. That is, the same doctor-specialist can "participate" in the formation of specialists (devices) groups different in composition and number m of doctors that care patients (service input flow requirements).

A list of such groups of doctors (service devices) in the area of symptoms-complaints of patients can be pre-established, for example, by the head doctor of a polyclinic.

The maximum number of groups ξ_m of m specialists from the total number of n doctors (service devices) exactly coincides with the number of combinations C_n^m from n by m [3]:

$$\xi_m = C_n^m = \frac{n!}{m! \cdot (n-m)!}, \quad 0 \le m \le n . \tag{1}$$

In the noted situation, a vector of probabilities q_m of requirements' accessibility to service channels arises.

In practice, the larger the doctors' group accessible in terms of symptom-complaints, the higher probability that the next patient will be accessible for "service" to any of the doctors in this group. For different groups of doctors within the same size m, this probability turns out to be approximately the same:

$$Q_{i_1 \dots i_m} = \gamma_m . \tag{2}$$

The direct calculation of necessary probability q_m can be difficult. However, an indirect estimate can be made on the basis of the polyclinic registry data, say for a month. For each patient, indicate the number of medical specialists who could successfully see this patient, diagnose and prescribe treatment.

Then you can calculate the total number N of patients per month and the number N_m of patients accessible for "service" to exactly m medical specialists at the same time ($0 \le m \le n$), which is equivalent to the number N_m of "favorable" events in the case scheme [3].

The estimate of the probability that the next requirement (patient) will be accessible for "service" to any group of service devices (specialists) consisting of exactly m devices can be found as the ratio of the favorable cases number to the total number of cases:

$$q_m = \frac{N_m}{N}; \quad 0 \le m \le n . \tag{3}$$

On the other hand, if the probability that a patient will be accessible for "service" to one of the specialists of one group of m doctors is equal to γ_m (2), and the number of such groups is estimated by ξ_m (1), then the probability of being accessible for "service " or by the first or second ... or by ξ_m-th group of specialists can be found as the sum of these probabilities:

$$q_m = \sum_{i=1}^{\xi_m} \gamma_m = \xi_m \cdot \gamma_m = C_n^m \cdot \gamma_m, \quad 0 \le m \le n . \tag{4}$$

Equating expressions (3) and (4), we obtain the probability estimate γ_m:

$$C_n^m \cdot \gamma_m = \frac{N_m}{N} \quad \text{then}$$

$$\gamma_m = \frac{N_m}{N \cdot C_n^m} = \frac{q_m}{C_n^m}; \quad 0 \le m \le n. \tag{5}$$

An estimate of the average service duration T_{avr} of one requirement and the performance μ of the service device, as well as the intensity I of requirements flow into the system can be found taking into account the total number of requirements N_{total} that entered the system, the number of requirements N served during the time of system operation T_{work} and the number n_i of requirements served by i-th device, as well as the total time T_i spent by the i-th device for servicing:

$$\left. \begin{aligned} & T_{aver} = \sum_{i=1}^{n} \frac{n_i}{N} \cdot \frac{T_i}{n_i} = \frac{1}{N} \sum_{i=1}^{n} T_i; \\ & \mu = \frac{1}{T_{avr}}; \qquad I = \frac{N_{total}}{T_{work}}. \end{aligned} \right\} \tag{6}$$

The considered variant of parameter estimation allows us to proceed to the formal problem statement. In a partially accessible queuing system $M/M/n$ with refusals and with the probability vector of the requirements accessibility to groups of service channels (γ_m), a flow of requirements of intensity I enters.

A requirement that met several accessible free channels is transferred for servicing to any of these channels with the same probability. A request that met busy all accessible channels leaves the system unserved (receives a refusal of service). The service time is exponentially distributed with the parameter $\mu = 1/T_{avr}$.

The problem is to find expressions for the final probabilities P_k, ($k = 1, 2, \dots, n$) of group states of the service process, in which there are exactly k requirements in the system with refusals and with incomplete accessibility of service devices, which corresponds to the occupancy of exactly k channels, and to find the service

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probability $P_{service}$ of requirements. In such a way there is a problem of forecasting the efficiency of real queuing systems with refusals in the case of incomplete accessibility of service devices for the input flow of requirements.

2 REVIEW OF THE LITERATURE

The first model for calculating the part of calls that receive service at a telephone station was described by A. K. Erlang [4] in 1909. The process of the telephone station included the receipt and service of applications from subscribers to switch communication channels with other subscribers. The service of each requirement consisted in connecting the subscriber – the source of the application to the free channel of communication with the required subscriber. After the end of this call, the channel was released and could be used to service the next requirement. The requirement that arrived at the telephone station at the time when all channels were busy, received a denial of service. The moments of requirements receipt and the end of their service were random.

The Erlang-developed model of the requirements mass service system at the telephone station turned out to be a universal tool for describing the processes of service in different systems and in different spheres of human activity. Each of these areas and systems has its own peculiarities, which led to the development of more complex models and the appearance of an independent scientific direction – the queuing theory.

Currently, queuing system models are being actively used for analysis, for predicting efficiency and for optimizing decisions made in various areas.

These include the following areas: telecommunication networks [2], socio-economic systems [5], production systems [6] and logistic systems [7], computing systems [8], traffic management systems [9], management systems in medicine [10], as well as systems for the defence of objects from air blows [11].

Therefore, the aim of this research is to obtain an analytical description for final probabilities of states groups for the queuing system *M*/*M*/*n* with refusals and with incomplete accessibility of service devices for the input flow of requirements and also checking the results correctness by transforming the obtained description into a description of known Erlang system *M*/*M*/*n*.

3 MATERIALS AND METHODS

In order to demonstrate the logic of obtaining an analytical description of sought final probabilities, let us consider a relatively easily visible example for the *M*/*M*/3 system (Fig. 2) with incomplete accessibility of service devices for the input flow of requirements. On Fig. 2, the symbol S_{ijk} denotes the states in which are occupied channels (devices) with numbers i, j, k. The possibility of a steady state in the system follows from the formulation of the problem.

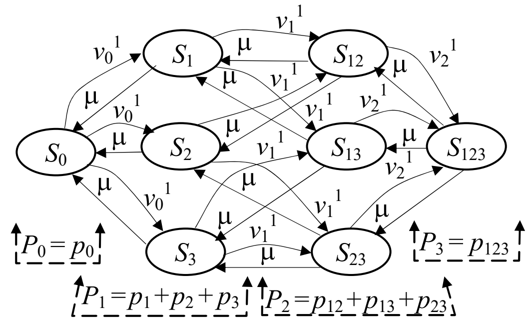

Figure 2 – Graph of M/M/3 QS model with refusals and with incomplete accessibility of service devices

Let us find the intensities of transitions along the edges R of graph (Fig. 2). The transition along the edge $R[S_0, S_1]$, connecting the vertex S_0 with the vertex S_1, can occur only if the next requirement enters the system and is accessible to the first channel, and this channel is selected for service.

The mathematical expectation $\boldsymbol{v}_0^1$ of this event occurrence intensity is equal to the product of the intensity of requirements input flow I and the probability $\boldsymbol{v}_0$ of transferring the requirement for servicing to the first channel:

$$v_0^1 = I \cdot v_0 . \tag{7}$$

To visualize the process of determining the probability (v_0), we assume that each requirement has only two features (x_1, x_2), the values of which determine the choice of the service channel.

For each channel, the area of acceptable feature values has the shape of a circle, in the center of which we indicate the channel number (Fig. 1). The attribute values (x_1, x_2) of the next requirement determine the point $A(x_1, x_2)$ on the plane (X_1, X_2). The channel, accessible for servicing, is selected in accordance with the area in which the requirement points on the plane (X_1, X_2) fell. In any case, in order to transfer a requirement for service to the first channel, the point $A(x_1, x_2)$ of the requirement, that has entered the system, must fall into the range of features accessible for the first channel (Fig. 1). If there are n features, then the area will be n-dimensional.

We'l find the probability v_0 of transferring the requirement to the first channel by listing the possible outcomes of the analysis as the sum of the marked events' probabilities and at the same time taking into account equality (2), we obtain:

$$\begin{aligned} v_0 &= Q_1 + \frac{1}{2}Q_{12} + \frac{1}{3}Q_{123} + \frac{1}{2}Q_{13} = \\ &= \gamma_1 + \frac{1}{2}\gamma_2 + \frac{1}{3}\gamma_3 + \frac{1}{2}\gamma_2 = \gamma_1 + \gamma_2 + \frac{1}{3}\gamma_3 . \end{aligned} \tag{8}$$

The final value of the transferring requirement probability for service to the second v_1 or third v_3 channel will coincide with found value of the probability v_1.

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If the service process is in the state S_1, then the transition along the edge $R[S_1, S_{12}]$ (Fig. 2) from the vertex S_1 to the vertex S_{12} can occur only if the next requirement enters the system, becomes accessible to the second channel and this channel will be selected for service.

The intensity v_1^1 of such events flow will be found taking into account the probability v_2 of transferring the requirement to the second channel:

$$v_1^1 = I \cdot v_1 . \tag{9}$$

To transfer the requirement for servicing to the second channel, the point $A(x_1, x_2)$ of the requirement, that has entered the system, must fall into the range of features accessible for the second channel (Fig. 1, see circle around the second channel).

The probability v_1 of transferring the requirement for service to the second channel, provided that the first one is busy, we find by listing the possible outcomes of the analysis, we get:

$$\begin{aligned} v_1 &= Q_2 + \frac{1}{2}Q_{23} + \frac{1}{2}Q_{123} + Q_{12} = \\ &= \gamma_1 + \frac{1}{2}\gamma_2 + \frac{1}{2}\gamma_3 + \gamma_2 = \gamma_1 + \frac{3}{2}\gamma_2 + \frac{1}{2}\gamma_3 . \end{aligned} \tag{10}$$

The final value of the transition's probability for the remaining edges from states S_i to states S_{ij} will coincide with the found value v_1.

If the service process is in the state S_{12}, (Fig. 2) then the transition along the edge $R[S_{12}, S_{123}]$ from the vertex S_{12} to the vertex S_{123} can occur only if the next requirement enters the system, becomes accessible to the third channel and this channel will be selected for service.

The intensity v_2^1 of such events flow will be found taking into account the probability v_2 of transferring the requirement to the third channel:

$$v_2^1 = I \cdot v_2 . \tag{11}$$

In this case, the point $A(x_1, x_2)$ of the received requirement (Fig. 1) should fall into the range of features values accessible to the third channel

The probability v_2 of transferring the requirement for service to the third channel, provided that the first and second channels are busy, can be found as the sum of the probabilities of hitting the requirement to all areas of the third channel:

$$\begin{aligned} v_2 &= Q_3 + Q_{13} + Q_{123} + Q_{23} = \\ &= \gamma_1 + \gamma_2 + \gamma_3 + \gamma_2 = \gamma_1 + 2\gamma_2 + \gamma_3 . \end{aligned} \tag{12}$$

The final value of the transition's probability for the remaining edges from states S_{ij} to the states S_{123} will coincide with the found value v_2.

States with the same number (k) of busy service channels determine the levels of the graph.

The number of states at each level is equal to the number of combinations C_n^k. The total number of model graph states can be found as a "degree-set" [3]:

$$N_{dif.eq} = \sum_{k=0}^{n} C_n^k = 2^n . \tag{13}$$

Next, we denote the probability of each state of the model graph by a small letter p_{ij} with indices that correspond to the numbers of busy channels in this state: p_{ij} – for the state S_{ij}.

The set of probabilities of the graph k-th level ($k = 0, 1, 2, 3$) determines the required probability P_k of occupancy of exactly k service channels:

$$\left.\begin{aligned} &P_0 = p_0; \quad P_1 = p_1 + p_2 + p_3; \\ &P_2 = p_{12} + p_{13} + p_{23}; \quad P_3 = p_{123}. \end{aligned}\right\} \tag{14}$$

Let's compose the system of Kolmogorov equations for states S_0, S_1, S_2, S_3, S_{123} (Fig. 2):

$$\dot{p}_0 = -3v_0^1 \cdot p_0 + \mu \cdot p_1 + \mu \cdot p_2 + \mu \cdot p_3 ; \tag{15}$$

$$\left.\begin{aligned} &\dot{p}_1 = -(\mu + 2v_1^1)p_1 + v_0^1 \cdot p_0 + \mu \cdot p_{12} + \mu \cdot p_{13} ; \\ &\dot{p}_2 = -(\mu + 2v_1^1)p_2 + v_0^1 \cdot p_0 + \mu \cdot p_{12} + \mu \cdot p_{23} ; \\ &\dot{p}_3 = -(\mu + 2v_1^1)p_3 + v_0^1 \cdot p_0 + \mu \cdot p_{13} + \mu \cdot p_{23} ; \end{aligned}\right\} \tag{16}$$

$$\dot{p}_{123} = -3\mu\, p_{123} + v_2^1 \cdot p_{12} + v_2^1 \cdot p_{13} + v_2^1 \cdot p_{23} . \tag{17}$$

Further, for the conditions of QS operation in stationary mode, we find the sum of the first four equations – (15), (16):

$$\begin{aligned} &2\mu \cdot p_{12} + 2\mu \cdot p_{13} + 2\mu \cdot p_{23} = \\ &= 2v_1^1 \cdot p_1 + 2v_1^1 \cdot p_2 + 2v_1^1 \cdot p_3 . \end{aligned} \tag{18}$$

As a result, for the conditions of the QS operation stationary mode and the model's graph (Fig. 2), we write down the system of algebraic equations:

$$\left.\begin{aligned} &\mu \cdot p_1 + \mu \cdot p_2 + \mu \cdot p_3 = 3v_0^1 \cdot p_0; \\ &2\mu \cdot p_{12} + 2\mu \cdot p_{13} + 2\mu \cdot p_{23} = \\ &= 2v_1^1 \cdot p_1 + 2v_1^1 \cdot p_2 + 2v_1^1 \cdot p_3; \\ &3\mu \cdot p_{123} = v_2^1 \cdot p_{12} + v_2^1 \cdot p_{13} + v_2^1 \cdot p_{23}. \end{aligned}\right\} \tag{19}$$

The resulting equations are transformed taking into account equalities (7), (9), (11) and the dimensionless load factor of service devices ρ:

$$\left.\begin{aligned} &1\mu \cdot P_1 = 3v_0^1 \cdot P_0 ; \\ &2\mu \cdot P_2 = 2v_1^1 \cdot P_1 ; \\ &3\mu \cdot P_3 = 1v_2^1 \cdot P_2 ; \end{aligned}\right\} \text{ then } \left.\begin{aligned} &1P_1 = P_0 \cdot 3 \cdot \rho \cdot v_0 ; \\ &2P_2 = P_1 \cdot 2 \cdot \rho \cdot v_1 ; \\ &3P_3 = P_2 \cdot 1 \cdot \rho \cdot v_2 . \end{aligned}\right\} \tag{20}$$

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The resulting regularity can be represented in general form:

$$k \cdot P_k = P_{k-1} \cdot C^1_{n-k+1} \cdot \rho \cdot v_{k-1}\,, \quad k = 1, 2, ..., n\,. \qquad (21)$$

The expressions for the final probabilities are defined in a form close to the Erlang formulas [4]:

$$\left.\begin{aligned} &P_k = \frac{\rho^k}{k!} \cdot P_0 \cdot f_k, \quad k = 1, ..., n; \\ &P_0 = \left(\sum_{k=0}^{n} \frac{\rho^k}{k!} \cdot f_k \right)^{-1}; \qquad \rho = \frac{I}{\mu}. \end{aligned}\right\} \qquad (22)$$

We find the functions of incomplete accessibility f_k by substituting formulas (22) into (21) and performing equivalent transformations, we obtain:

$$k \cdot \frac{\rho^k}{k!} P_0 \cdot \boldsymbol{f_k} = \frac{\rho^{k-1}}{(k-1)!} P_0 \cdot f_{k-1} \cdot C^1_{n-k+1} \cdot \rho \cdot v_{k-1}; \qquad (23)$$

$$\begin{aligned} \boldsymbol{f_k} &= \frac{\rho^{k-1}}{(k-1)!} \cdot f_{k-1} \cdot C^1_{n-k+1} \cdot \rho \cdot v_{k-1} \left(\frac{k!}{k \cdot \rho^k} \right) = \\ &= f_{k-1} \cdot C^1_{n-k+1} \cdot v_{k-1}\,. \end{aligned} \qquad (24)$$

Note that the probability v_k corresponds to the event of transferring of the requirement to the k-th channel. We find the function f_0 from formula (22) under the condition $k = 0$:

$$P_0 = P_0 \cdot \boldsymbol{f_0} \quad \text{then} \quad f_0 = 1\,. \qquad (25)$$

The expressions for the incomplete accessibility functions f_k turn out to be:

$$\boldsymbol{f_k} = f_{k-1} \cdot C^1_{n-k+1} \cdot v_{k-1}\,; \quad k = 0, 1, ..., n-1\,, \qquad (26)$$

where expressions for (8), (10), (12) generally take the form:

$$v_k = \sum_{j=1}^{n} \left(\gamma_j \sum_{i=0}^{j-1} \frac{1}{C^1_{i+1}} C_k^{j-i-1} C^i_{n-k-1} \right), 0 \le k < n\,. \qquad (27)$$

The probabilities v_k of choosing specific service channels for a three-channel incompletely accessible QS were obtained above and are equal to:

$$\left.\begin{aligned} &v_0 = \gamma_1 + \gamma_2 + \frac{1}{3}\gamma_3\,; \\ &v_1 = \gamma_1 + \frac{3}{2}\gamma_2 + \frac{1}{2}\gamma_3\,; \\ &v_2 = \gamma_1 + 2\gamma_2 + \gamma_3\,. \end{aligned}\right\} \qquad (28)$$

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For a non-fully accessible QS with four ($n = 4$) and five ($n = 5$) channels, the calculation formulas for the probabilities v_k of choosing specific service channels are presented in Table 1.

Table 1 – Probability formulas v_k for choosing the k-th service channel in incompletely accessible QS of $M/M/n$ type

$\boldsymbol{n} = 4$
$v_0 = \gamma_1 + \frac{3}{2}\gamma_2 + \gamma_3 + \frac{1}{4}\gamma_4$
$v_1 = \gamma_1 + 2\gamma_2 + \frac{4}{3}\gamma_3 + \frac{1}{3}\gamma_4$
$v_2 = \gamma_1 + \frac{5}{2}\gamma_2 + 2\gamma_3 + \frac{1}{2}\gamma_4$
$v_3 = \gamma_1 + 3\gamma_2 + 3\gamma_3 + \gamma_4$
$\boldsymbol{n} = 5$
$v_0 = \gamma_1 + 2\gamma_2 + 2\gamma_3 + \gamma_4 + \frac{1}{5}\gamma_5$
$v_1 = \gamma_1 + \frac{5}{2}\gamma_2 + \frac{5}{2}\gamma_3 + \frac{5}{4}\gamma_4 + \frac{1}{4}\gamma_5$
$v_2 = \gamma_1 + 3\gamma_2 + \frac{10}{3}\gamma_3 + \frac{5}{3}\gamma_4 + \frac{1}{3}\gamma_5$
$v_3 = \gamma_1 + \frac{7}{2}\gamma_2 + \frac{9}{2}\gamma_3 + \frac{5}{2}\gamma_4 + \frac{1}{2}\gamma_5$
$v_4 = \gamma_1 + 4\gamma_2 + 6\gamma_3 + 4\gamma_4 + \gamma_5$

For incompletely accessible queuing systems with a large number of channels ($n > 5$), the calculation formulas can be obtained using the expression (27).

When estimating the service probability, one has to take into account the possibility of refuse not only in the case of occupied all channels, but also in any other state of an incompletely accessible QS. Therefore, the service probability should be sought using the expression for the mathematical expectation of the busy channels number $M_{busy.div}$ and the absolute capacity (A) of QS:

$$\left.\begin{aligned} &M_{busy.div} = \sum_{k=0}^{n} k \cdot P_k\,; \;\; A = \mu \cdot M_{busy.div}\,; \\ &P_{service} = \frac{A}{I}; \qquad P_{refuse} = 1 - P_{service}\,. \end{aligned}\right\} \qquad (29)$$

In order to check the correctness of obtained description for incompletely accessible QS, we perform an asymptotic transition from expressions (22), (26), (27) to a description of a fully accessible Erlang queuing system with refusals. In this case, in formulas (3) and (5) all values $N_m = 0$, q_m=0 and $\gamma_m = 0$ are equal to zero for all ($m < n$) groups of devices. For m=n the value $N_n = N$ and $C_n^n = 1$, which, according to (3) and (5), leads to the equalities q_n=1 and $\gamma_{\boldsymbol{n}} = 1$. Then the combinations in expression (27) will be different from zero only for the values $i = n - k$. In this case, from (26) and (27) it follows:

$$\left.\begin{aligned} &C^{k-1}_{k-1} = C^{n-k}_{n-k} = 1, \text{ then } v_k = \frac{1}{C^1_{n-k+1}}, \\ &\text{then } f_k = 1; \quad k = 0, 1, ..., n\,. \end{aligned}\right\} \qquad (30)$$

As a result, formulas (22) are automatically converted into well-known Erlang formulas [4]:

$$\left.\begin{aligned}&P_{k.Erl}=\frac{\rho^k}{k!}P_{0.Erl},\,k=1,..,n\,;\,P_{0.Erl}=\left(\sum_{k=0}^{n}\frac{\rho^k}{k!}\right)^{-1};\\&P_{refuse.Erl}=P_{n.Erl}\,;\,P_{service.Erl}=1-P_{n.Erl}\,;\,\rho=\frac{I}{\mu}.\end{aligned}\right\}\quad(31)$$

which testifies in favor of correctness of obtained final probabilities analytical description in an incompletely accessible multichannel queuing system $M/M/n$ with refusals.

4 EXPERIMENTS

In order to test the operability of analytical description of the incompletely accessible $M/M/n$ model, we use an example from the topical sphere – of important objects air defense (Fig. 3) by a grouping of five single-channel SAM systems (Table 7, No. 1, 3) – "service devices", which should prevent a planned air blow of ten enemy aircraft (Table 7, No. 16, 5) with a duration of 2,5 minutes (Table 7, No. 17, 5).

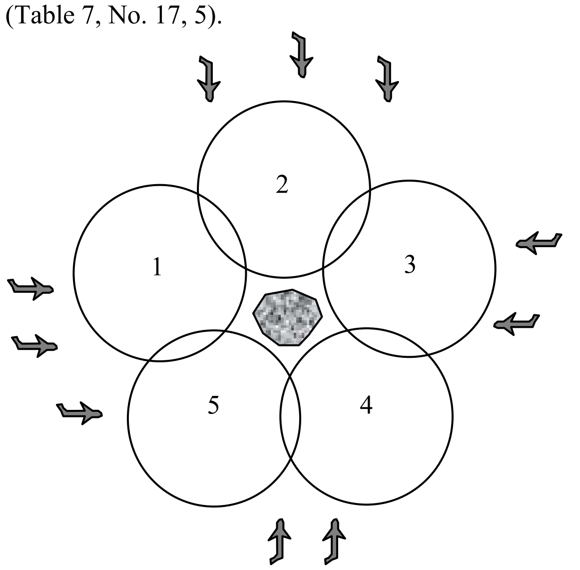

Figure 3 – An example of setting the task of assessing objects' air defense effectiveness using the $M/M/5$ model of an incompletely accessible queuing system with refusals

For each SAM system, the average shooting time on to one aircraft is one minute (Table 7, No. 3, 3), the radius of the affected area is twenty kilometers (Table 7, No. 2, 3). The aircraft that came under fire is destroyed. To destroy an object covered, at least four aircraft is required. Therefore, the task of SAM systems grouping is considered fulfilled in the case when no more than three aircraft can break through to the object.

To apply the incompletely accessible $M/M/n$ model in the field of air defence tasks, we clarify the relationship between the real parameters of the SAM grouping and the parameters of incompletely accessible $M/M/n$ model, expressions (32).

$$\left.\begin{aligned}&S_{ar.\max}=N_{SAMS}\cdot\pi\cdot R_{SAMS}^2;\\&S_{ar.2}=S_{ar.\max}-S_{ar.tot};\\&q_m=\frac{S_{ar.m}}{S_{ar.tot}};\qquad 1\le m\le 2\,.\end{aligned}\right\}\quad(32)$$

Thus, the maximum possible coverage area $S_{ar.\max}$ by means of five SAM systems is calculated as the sum of areas of all SAM systems fire zones in the group and is 6283 km^2 (Table 7, No. 5, 3).

On the terrain (Fig. 3), the cover area $S_{ar.tot}$ is 5202 km^2 (Table 7, No. 6, 3), which makes it possible to estimate the size of the $S_{ar.2}$ area with a double (Table 7, No. 8, 3) and $S_{ar.1}$ with a single (Table 7, No. 7, 3) overlapping of fire zones, and also find the values of parameters q_1 (Table 7, No. 9, 3), q_2 (Table 7, No. 10, 3) and γ_1 (Table 7, No. 19, 5) and γ_2 (Table 7, No. 20, 5). We also note that the mathematical expectation of the enemy aircraft number that broke through to the target (N_{missed}) and the relative error (ΔP_{serv}) in the service probability forecast for the Erlang model are not difficult to find using formulas (33):

$$\left.\begin{aligned}&N_{missed}=N_{total.En}\cdot P_{refuse}\,;\\&\Delta P_{serv}=\left(P_{service.Erl}-P_{service}\right)/\,P_{service}\cdot 100\%\,.\end{aligned}\right\}\quad(33)$$

Next, we use the data in Table 1 and the sequence of formulas (5), (6), (27), (20), (26), (22), (29), (31)-(33) and step by step (Table 2 Nos 21–26), we find a significant discrepancy between the final probabilities of the Erlang model and the model of a non-fully accessible queuing system (Fig. 3).

At the same time, the use of the fully accessible Erlang model shows the possibility of destroying more than 80% of enemy aircraft (Table 2, No. 28, 13). In this case, it is considered that the SAM grouping reliably fulfills its task, letting no more than two aircraft passes to the object (Table 2, No. 30, 13). At the same time, taking

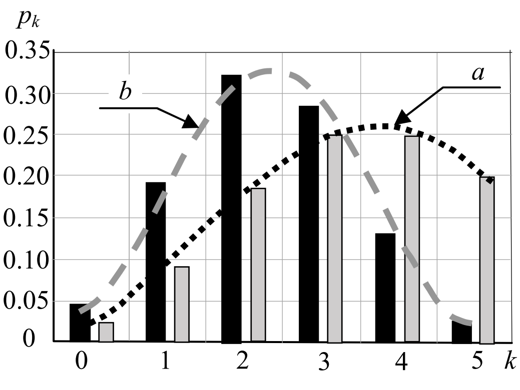

Figure 4 – Final probabilities p_k of the states in the same queuing systems with refusals and with the same intensity:
a) QS with refusals and with full accessibility of service devices (Erlang model M/M/5);
b) QS with refusals and with incomplete accessibility of service devices (incomplete accessible model M/M/5)

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Table 2 – Comparative assessment of the service probability and the efficiency of tasks performing by a SAMS group using the Erlang model and a more accurate incompletely accessible QS model with refusals (see Fig. 4)

#	Name	Value	#	Name	Value	#	k	$\rho^k/k!$	f_k	$\rho^k/k! \cdot f_k$	P_k	$k \cdot P_k$	$P_{k.Erl}$
1	2	3		4	5	6	7	8	9	10	11	12	13
1	$N_{SAM\,S}$	5	11	v_0	0.2	21	0	1	1	1	0.048	0	0.023
2	$R_{SAM\,S}$	20	12	v_1	0.21	22	1	4	1	4	0.191	0.191	0.093
3	T_{avr}	1	13	v_2	0.22	23	2	8	0.842	6.732	0.321	0.642	0.187
4	μ	1	14	v_3	0.23	24	3	10.667	0.557	5.945	0.284	0.851	0.249
5	$Sar.mx$	6283.185	15	v_4	0.24	25	4	10.667	0.258	2.749	0.131	0.525	0.249
6	$Sar.tot$	5202.477	16	N_{total}	10	26	5	8.533	0.062	0.531	0.025	0.127	0.199
7	$Sar.1$	4121.77	17	T_{work}	2.5	27	Σ	42.867	3.719	20.957	1	2.336	1
8	$Sar.2$	1080.708	18	ρ	4	28	$M_{busy.div}$	2.336	$P_{service}$	0.584	$P_{service.Erl}$		0.801
9	q_1	0.792271	19	γ_1	0.16	29	I	4	P_{refuse}	0.416	$P_{refuse.Erl}$		0.199
10	q_2	0.207729	20	γ_2	0.02	30	ΔP_{serv}.%	37.16%	N_{missed}	4.161	$N_{missed.Erl}$		1.991

into account the incomplete accessibility of the SAM grouping (devices for service) leads to a noticeable decrease (Table 2, No.30, 8) in the probability of service (Table 2, No. 28, 10) and to the appearance of the possibility of passing to the object more than four enemy aircraft, which jeopardizes the performance of the task by the SAM grouping and requires the adoption of additional measures to protect the object.

5 RESULTS

For the first time analytical expressions are obtained for the final probabilities of the queuing system states with refusals and with incomplete accessibility of service devices for the input flow of requirements, which makes it possible to evaluate as well as forecast values of all known system efficiency indicators. The resulting description, when an incompletely accessible queuing system degenerates into a fully accessible one, asymptotically transforms into the well-known Erlang formulas, which testifies in favor of its correctness. At the same time, ways of formalizing processes and transition to a model of incompletely accessible QS with refusals in real systems that perform similar service functions in the field of medicine and in the field of topical air defense tasks are given.

CONCLUSIONS

In the course of the research, the analytical expressions for the final probabilities of states in the *M*/*M*/*n* queuing system with refusals and with incomplete accessibility of service devices for the input flow of requirements were received. The results of the numerical experiment testify in favor of correctness the obtained analytical expressions for the final probabilities and in favor of possibility for their practical application in real queuing systems when solving problems of forecasting efficiency, as well as analyzing and synthesizing the parameters of real queuing systems.

The scientific novelty of the results obtained lies in the creation of possibilities for forecasting the effectiveness of known type of Markov queuing systems with refusals and with incomplete accessibility of service devices for the input flow of requirements. The obtained description (22), (25)-(27), (31) of a queuing system is a general for known Erlang model *M*/*M*/*n*.

The practical significance of the results obtained lies in creating conditions for the directed solution the problems of analysis, synthesis and control of Markov queuing systems in the general case of incomplete accessibility of service devices for the input flow of requirements. The formulas obtained for calculating the values of the incomplete accessibility functions are recurrent and convenient for practical calculations.

Prospects for further research may include the development of methods and formula schemes of algorithms for the transition from the parameters of real processes in the areas of management of microeconomic, financial and other systems with restrictions on servicing input requirements, to the parameters of an incompletely accessible QS model.

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АНАЛІТИЧНИЙ ОПИС ФІНАЛЬНИХ ЙМОВІРНОСТЕЙ СТАНІВ У НЕПОВНО ДОСТУПНІЙ СИСТЕМЕ МАСОВОГО ОБСЛУГОВУВАННЯ З ВІДМОВИМИ

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АНОТАЦІЯ

Актуальність. Існує проблема прогнозування працездатності реальних систем масового обслуговування із відмовами у разі неповної доступності пристроїв обслуговування для вхідного потоку вимог. Вирішення проблеми необхідно для створення можливості більш точного проектування та контролю роботи таких систем у режимі реального часу.

Метод. Можливості станів СМО з відмовами при неповній доступності приладів обслуговування для вхідного потоку вимог описуються диференціальними рівняннями Колмогорова. У стаціонарному стані ці рівняння перетворюються на лінійно залежну однорідну систему алгебраїчних рівнянь. Кількість рівнянь визначається безліччю-степенем і для сучасних систем масового обслуговування і зв'язку може обчислюватися тисячами, мільйонами і більше. Тому спроба прогнозувати ефективність системи стикається з необхідністю запису та чисельного вирішення лічильної множини систем алгебраїчних рівнянь, що досить складно.

Ключовою ідеєю запропонованого методу знаходження аналітичного опису фінальних ймовірностей для зазначеної системи масового обслуговування було прагнення перейти від опису окремих станів (у кількості 2^n) до опису груп станів системи (у кількості n+1) та локалізувати вплив неповної доступності приладів обслуговування для вхідного потоку вимог у мультиплікативні функції неповної доступності. Такі функції дозволяють отримати необхідний аналітичний опис та оцінити ступінь перетворення фінальних ймовірностей порівняно з відомими системами, а також оцінити прогнозні значення показників ефективності зазначеної системи масового обслуговування при побудові системи та виборі параметрів її управління.

Результати. Вперше отримано аналітичні вирази для фінальних ймовірностей станів СМО з відмовами та з неповною доступністю приладів обслуговування для вхідного потоку вимог, що дозволяє оцінювати, а також прогнозувати значення всіх відомих показників ефективності системи.

Висновки. Отриманий опис виявився загальним випадком для відомого типу Марківських систем масового обслуговування із відмовами. Результати чисельного експерименту свідчать на користь коректності отриманих аналітичних виразів для фінальних ймовірностей та на користь можливості їх практичного застосування в реальних системах масового обслуговування під час вирішення завдань прогнозування ефективності, а також аналізу та синтезу параметрів реальних систем масового обслуговування.

КЛЮЧОВІ СЛОВА: Марківські моделі, системи масового обслуговування, неповна доступність приладів обслуговування.

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АНАЛИТИЧЕСКОЕ ОПИСАНИЕ ФИНАЛЬНЫХ ВЕРОЯТНОСТЕЙ СОСТОЯНИЙ В НЕПОЛНО ДОСТУПНОЙ СИСТЕМЕ МАССОВОГО ОБСЛУЖИВАНИЯ С ОТКАЗАМИ

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АННОТАЦИЯ

Актуальность. Существует проблема прогнозирования работоспособности реальных систем массового обслуживания с отказами в случае неполной доступности устройств обслуживания для входного потока требований. Решение проблемы необходимо для создания возможности более точного проектирования и контроля работы таких систем в режиме реального времени.

Метод. Вероятности состояний СМО с отказами при неполной доступности приборов обслуживания для входного потока требований описываются дифференциальными уравнениями Колмогорова. В стационарном состоянии эти уравнения преобразуются в линейно зависимую однородную систему алгебраических уравнений. Количество уравнений определяется множеством-степенью и для современных систем массового обслуживания и связи может исчисляться тысячами,

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миллионами и более. Поэтому попытка прогнозировать эффективность системы сталкивается с необходимостью записи и численного решения счетного множества систем алгебраических уравнений, что достаточно сложно.

Ключевой идеей предлагаемого метода нахождения аналитического описания финальных вероятностей для отмеченной системы массового обслуживания было стремление перейти от описания отдельных состояний (в количестве 2^n) к описанию групп состояний системы (в количестве n+1) и локализовать влияние неполной доступности приборов обслуживания для входного потока требований в мультипликативных функциях неполной доступности. Такие функции позволяют получить требуемое аналитическое описание и оценить степень преобразования финальных вероятностей, по сравнению с известными системами, а также оценить прогнозные значения показателей эффективности отмеченной системы массового обслуживания при построении системы и выборе параметров ее управления.

Результаты. Впервые получены аналитические выражения для финальных вероятностей состояний СМО с отказами и с неполной доступностью приборов обслуживания для входного потока требований, что позволяет оценивать, а также прогнозировать значения всех известных показателей эффективности системы.

Выводы. Полученное описание оказалось общим случаем для известного типа марковских систем массового обслуживания с отказами. Результаты численного эксперимента свидетельствуют в пользу корректности полученных аналитических выражений для финальных вероятностей и в пользу возможности их практического применения в реальных системах массового обслуживания при решении задач прогнозирования эффективности, а также анализа и синтеза параметров реальных систем массового обслуживания.

КЛЮЧЕВЫЕ СЛОВА: Марковские модели, системы массового обслуживания, неполная доступность приборов обслуживания.

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