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SYNTHESIS OF THE SYMBOLOGIES OF MULTICOLOR INTERFERENCE-RESISTANT BAR CODES ON THE BASE OF MULTI-VALUED BCH CODES

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ABSTRACT

Context. The problem of constructing a set of barcode patterns for multicolor barcodes that are resistant to distortions of one or two elements within each pattern is considered.

Objective. The goal of the work is ensuring the reliability of the reading of multi-color barcode images.

Method. A multicolor barcode pattern has the property of interference immunity if its digital equivalent (vector) is a codeword of a multi-valued (non-binary) correcting code capable to correct errors (distortions of the pattern elements). It is shown that the construction of barcode patterns should be performed on the basis of a multi-valued correcting BCH code capable to correct two errors. A method is proposed for constructing a set of interference-resistant barcode patterns of a given capacity, which ensure reliable reproduction of data when they are read from a carrier. A procedure for encoding data with a multi-valued BCH code based on the generator matrix of the code using operations by the modulo of a prime number has been developed. A new method of constructing the check matrix of the multivalued BCH code based on the vector representation of the elements of the finite field is proposed. A generalized algorithm for generating symbologies of a multi-color barcode with the possibility of correcting double errors in barcode patterns has been developed. The method also makes it possible to build symbology of a given capacity based on shortened BCH codes. A method of reducing the generator and check matrices of a multi-valued full BCH code to obtain a shortened code of a given length is proposed. It is shown that, in addition to correction double errors, multi-valued BCH codes also make it possible to detect errors of higher multiplicity – this property is enhanced when using shortened BCH codes. The method provides for the construction of a family of multicolor noise-immune barcodes.

Results. On the basis of the developed software tools, statistical data were obtained that characterize the ability of multi-valued BCH codes to detect and correct errors, and on their basis to design multi-color interference-resistant bar codes.

Conclusions. The conducted experiments have confirmed the operability of the proposed algorithmic tools and allow to recommend it for use in practice for developing interference-resistant multi-color barcodes in automatic identification systems.

KEYWORDS: barcoding, multicolor barcodes, interference immunity of barcodes, BCH codes.

ABBREVIATIONS

BC – barcode;
BCH code – Bose-Choudhuri-Hocquenghem code;
BC-pattern – barcode pattern;
BC-symbol – barcode symbol;
HCC2D barcode – high capacity colored two dimensional barcode;
HCCB – high capacity color barcode;
JAB code – just another barcode;
LCM – least common multiple;
QR code – quick response barcode;
URL – uniform resource locator.

NOMENCLATURE

$B(\ )$ is an information word;
barcode-pattern (B) is a program procedure, provides the printing of the barcode pattern for the given information word $B(\ )$;
$B(x)$ is a polynomial which corresponds to information word $B(\ )$;
$\det M$ is a determinant of the matrix M;
$d_{\min}$ is a minimal Hamming distance of the correcting code;
do – execute;
for – start of cycle;
$G_{(s,\,u)}$ is a generator matrix of $(s,\ u)$-BCH code;
$GF(\ )$ is a Galois field;
$g(x)$ is a generator polynomial of the correcting code;
$H_{(s,\,u)}$ is a check matrix of the $(s,\ u)$-BCH code;
$M^{(\ )}(x)$ is a minimal polynomial of the field’s element;
m is a degree of minimal polynomials;
$p(x)$ is an irreducabel polynomial;
q is an amount of colors for barcode-patterns painting;
S is an error syndrome;
s is a the total length of the codewords;
u is an amount of informational positions in codewords;
V is a capacity of the barcode symbology;
v is a degree of generator polynomial;
x is a root of the error locator polynomial;
X is an error locator;
Y is an error value;
$Z(\ )$ is a codeword (vector of the barcode pattern);
$Z'(\ )$ is a the resived vector;
$Z(x)$ is a polynomial which corresponds to codeword $Z(\ )$;
α is a primitive element of Galois field;
σ is a coefficient of the error locator polynomial;

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σ(x) is an error locator polynomial;
Ω is a barcode symbology.

INTRODUCTION

Data barcoding is one type of automatic identification. The advantages of barcoding are the speed of entering data into the computer system, the low cost of making barcode symbols and ease of use; BC is read from the accounting objects optically, including at a distance.

During its more than 60-year history of development, BCs have undergone a certain evolution: linear BCs, stacks, matrix, and eventually multi-color BCs. During the last decade, there has been a steady tendency to expand the scope of application of multi-color BCs, because multi-color allows to significantly (several times) increase the information density – to provide a greater of information quantity per unit area of the carrier without changing the geometric dimensions of the image elements, than is allowed by black-and-white BCs.

However, the processes of recognition and decoding of barcode images become more complicated when using multi-color barcodes. The researchers note that the reasons for this may be: distortion of the geometric dimensions of image elements during reading due to certain optical effects; mixing of colors on the border of neighboring elements with different colors; aging of dyes; folding and blurring of paint, etc. [1]. Therefore, for reliable reading of barcode data from the accounting object, it is necessary to ensure the interference immunity of barcode images [2].

Structurally, the BC-symbol consists of an array of BC-patterns arranged in the form of a rectangle or a square (Fig. 1). To ensure an adequate level of reliability of data reading, a multi-color BC should be constructed so that the property of immunity to interference is provided at two levels – at the level of the entire BC-symbol, as well as at the level of BC-patterns – the minimal structural units of the image.

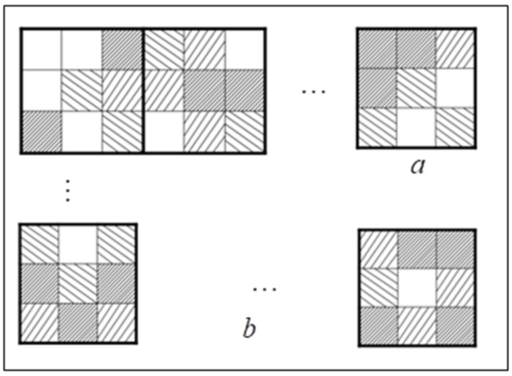

Figure 1 – The structure of a multi-color barcode symbol:
a – BC-pattern; b – BC-symbol

The object of study is the process of developing multi-color barcodes with improved reliability indicators.

Single-level system for ensuring interference resistance is used in the existing BCs – the BC-symbol, which is an array of BC-patterns, is coded with a correcting code capable to correct multiple errors, usually the Reed-Solomon code. To increase the interference resistance of a multi-color barcode image, it is suggested to additionally apply interference-resistant coding within each barcode pattern.

The subject of study is a method of ensuring interference-resistance of BC-patterns, which is based on the use of multi-valued BCH codes, which are capable to correct double errors in BC-patterns.

The purpose of the work is to develop the technology of designing multi-color interference-resistant BC-patterns, which would ensure reliable reproduction of data when read from the carrier.

1 PROBLEM STATEMENT

Every BC has its own set of BC-patterns, each of which denotes a symbol (or a combination of symbols) of computer alphabet. This set of barcode patterns is called barcode symbology.

Let it be necessary to create the symbology of the capacity V of an interference-resistant multi-color matrix BC with parameters q, s; q is the number of colors used to color the elements of the BC-patterns when q >2; s is the number of elements (cells) in the BC-pattern (see Fig. 1).

The colors with which each element of the BC-pattern can be painted will be denoted by numbers: 0, 1, 2, ..., q –1. Let's match the digital equivalent of the BC-pattern consisting of s elements – the vector of the BC-pattern $Z =(z_0 z_1 \dots z_{s-1})$, where $z_i \in \{0,1, 2, \dots, q-1\}$.

In order for the BC-pattern to be interference-resistant, its vector Z must be a codeword of a multi-valued (q >2) correcting code capable to correct distortions (errors). We will assume that during the reading from the carrier of BC-patterns, the most probable cases are damages (distortions) of one or two elements in BC-patterns. In this case, the vector Z of BC-pattern must be a codeword of a correcting code with a minimal code distance of $d_{\min}$ =5. Such a correcting code can be a multi-valued (q-valued) BCH code, q >2.

Thus, it is necessary to solve the problem of construction the symbologies of multi-colored barcodes with the possibility of correction single or double errors in readable barcode patterns, as well as detecting errors (distortions) of greater multiplicity.

2 REVIEW OF THE LITERATURE

In 2007, Microsoft developed a 4-color High Capacity Color Barcode (HCCB), which uses black, red, green and yellow colors. Some researchers rightly believe that it is from this code (which is also called Microsoft Tag) multi-color barcoding has started. The first studies of HCCB were performed by Devi Parikh and Gavin Jancke, who proposed a method for decoding this code, the main procedures of which are the localization and segmentation of the color image of the BC-symbol [1]. The color image is divided into clusters, each center of the cluster is matched with one of the reference colors of the palette, which is used when printing the BC-symbol. In HCCB, informa-

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tion is provided by colored triangles, which are placed on the carrier in the form of a matrix – the number of rows in the BC-symbol can be from 10 to 60, and in a row – from 20 to 120 triangles. The triangle gives a quadruple value of 0, 1, 2 or 3. Thus, the capacity of the BC-symbol can be from 200 to 7200 quadruple digits.

In subsequent scientific works on this topic, researchers began to use a square (instead of a triangle) as a minimal structural element of a multi-color barcode image, and also improved color recognition procedures during optical reading. For example, in [2] a series of algorithms were developed for recognizing color elements of an image using a small number of reference colors under illuminant of different intensities. The illuminant was considered as parametric color converter. This transformation was used to visualize (recognize) an unknown color element under a reference illuminant that can be identified using training data.

The authors of the study [3] proposed a method of recognizing a color barcode image without using reference colors, and also improved the decoding algorithm, which resulted in an increase in the speed of data reading and a decrease in the intensity of errors. In addition, it was possible to reduce the computational complexity of data reading and data processing.

The subject of the research in [4] was the spectral difference in the color channels of the devices for printing BC-symbols, where the color palette C, M, Y is used (C – cyan, M – magenta, Y – yellow), and color channels devices for reading BC-symbols, in which the R, G, B palette is used (R – red , G – green , B – blue). In order to mitigate the effect of inter-channel interference in color channels when printing BC-symbols and color channels of reading devices, the authors developed an algorithm for eliminating interference during the implementation of printing and reading (scanning) processes. As a result, it was possible to significantly reduce the probability of errors and increase the decoding speed.

A team of researchers in [5] introduced High Capacity Colored Two Dimensional (HCC2D) QR-based code with improved information density indicators due to the use of 4, 8 or 16 colors; at the same time, high reliability of data reproduction during scanning is ensured. The authors experimentally proved that HCC2D has the same information density as HCCB (Microsoft), and is not inferior in stability (reliability) QR code.

In [6] continued the study of HCC2D code. In particular, the authors investigated the frequency of decoding errors using different classifiers: the Minimum Distance, Decision Trees, K-means, Navie Bayes, and Support Vector Machines. It is shown that the K-means algorithm is the most effective classifier in experiments. An efficient algorithm for decoding multi-color BCs with reasonable computational costs is proposed.

Some researchers, in particular in [7, 8], proposed ways to increase the information density of QR-codes by using multicolor barcode elements in the QR-code structure. The palette C, M, Y when printing BC-symbols and the palette R, G, B in code scanning devices were studied, and on their basis the possibility of obtaining 8-color components in QR-codes was analyzed.

The authors of the publication [9] performed a comparative analysis of two-dimensional barcodes and outlined some directions for improving their characteristics – information density and interference resistance. The issue of compression of alphanumeric data before their presentation in barcode form is considered in [10]. A method is proposed that allows you to compress data 1.4 – 1.7 times.

In addition to compression, some researchers, in particular the authors of [8], also suggest applying multiplexing and multilayered technique of color barcode images. For example, in [11] a three-layer 8-color BC is proposed for the presentation of three independent alphanumeric messages in a single barcode symbol, in particular, the presentation of URL as one of them.

Eight colors for data presentation were also used by the developers of the JAB Code (Just Another Barcode), which managed to triple the information density of the barcode labels without changing the geometric dimensions of the barcode image elements [12].

In multi-color barcoding the problem of ensuring interference resistance of barcode images is extremely important. For reliable reading of the BC-symbols, the data before being applied to the carrier are coded using a correcting code with a high corrective ability, usually the Reed-Solomon code [4, 7–9, 12]. Besides, structural methods of increasing reliability can also be additionally applied [13].

However, it is guaranteed to achieve reliable reproduction of data when scanning a large-capacity multicolor barcode image only under the conditions of two-level reliability assurance – at the level of the entire barcode symbol (upper level), as well as at the level of the smallest structural units (lower level). The study of this approach was started in [14], where it was proposed to use a multi-valued Hamming code at the level of the BC-patterns. In this investigation, a more powerful multi-valued BCH code is proposed for this purpose.

3 MATERIALS AND METHODS

The BCH code is a cyclic correcting code in which information encoding is reduced to the multiplication of a polynomial of $B(x)=\sum_{i=0}^{u-1} b_i x^i$ of degree u–1, which corresponds to information word $B=(b_0 b_1 \ldots b_{u-1})$, to the generator polynomial

$$g(x)=\sum_{i=0}^{v} g_i x^i = g_0 + g_1 x + \ldots + g_v x^v \text{ of degree } v:$$

$$Z(x)=B(x)g(x),$$

where $Z(x)=\sum_{i=0}^{s-1} z_i x^i$ is a polynomial of degree s–1 corresponding to s-bit codeword $Z=(z_0 z_1 \cdots z_{s-1})$, which is

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the digital equivalent of the BC-pattern; $s = u + v$; $b_i, g_i, z_i \in GF(q)$ [15].

If the generator polynomial $g(x)$ is known, then the generator matrix $G_{(s,\,u)}$ of the (s, u)-BCH code can be written as follows:

$$G_{(s,u)} = \begin{Vmatrix} g(x) \\ xg(x) \\ \vdots \\ x^{n-1}g(x) \end{Vmatrix} = \begin{Vmatrix} g_0 g_1 \dots & g_v & 0 & \dots 0 \\ 0 \; g_0 \dots & g_{v-1} & g_v 0 \cdots 0 & \\ \vdots & & & \\ 0 \;\; 0 \dots 0 \, g_0 & g_1 & \dots g_v & \end{Vmatrix},$$

it has dimension $u \times s$.

The generator polynomial $g(x)$ of the (s, u)-BCH code with the minimal code distance $d_{\min} = 5$, capable to correct two errors, is defined as the least common multiple of the minimal polynomials $M^{(1)}(x)$, $M^{(2)}(x)$, $M^{(3)}(x)$, $M^{(4)}(x)$ for elements α^1, α^2, α^3, α^4 of the field $GF(q^m)$, where α^i is the root of the irreducable polynomial $M^{(i)}(x)$, that is, $M^{(i)}(\alpha^i)=0$:

$g(x)=\mathrm{LCM}\,(M^{(1)}(x), M^{(2)}(x), M^{(3)}(x), M^{(4)}(x))$,

$\alpha \in GF(q^m)$, m is the degree of minimal polynomials [15]. $GF(q)$ is called the field of characters, and $GF(q^m)$ is the field of locators.

Next, we will consider the case when q – prime number; the case when q is an exponent of a prime number is the subject of a separate study.

The BCH code is called full if $s = q^m - 1$.

Some non-binary ($q > 2$) full BCH codes with $d_{\min} = 5$, which are suitable for the problem to be solved, are listed in Table 1.

Table 1 – Some non-binary full (s, u)-BCH code

Number of colors	$q = 3$	$q = 5$	$q = 7$
Field of characters	$GF(3)$	$GF(5)$	$GF(7)$
(s, u)-BCH code	(8, 3)– (26, 17)–	(24, 16)–	(48, 40)–

Let $q = 3$ (the case of a three-color barcode).

Consider, for example, the ternary (8, 3)-BCH code (see Table 1).

To build such a code, two fields are used: $GF(3)$ – the field of characters (Fig. 2), and $GF(3^2)$ – the field of locators (Table 2), which is an extension over $GF(3)$. The construction of $GF(3^2)$ is based on the irreducable polynomial $p(x)= x^2 + x + 2$.

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

–	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

·	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

:	0	1	2
0	–	0	0
1	–	1	2
2	–	2	1

Figure 2 – Performing operations in the $GF(3)$

Each element α^i of the field $GF(3^2)$, $i \geq 1$, corresponds to the minimal polynomial $M^{(i)}(x)$, the degree of which does not exceed two.

In the field $GF(3^2)$ $\alpha^i = x^{i-8}$, $\alpha^{-i} = \alpha^{8-i}$, $\alpha^8 = \alpha^{-8} = \alpha^0 = 1$.

Let's find the generator polynomial $g(x)$ of the ternary (8, 3)-BCH code:
$g(x)= \mathrm{LCM}(M^{(1)}(x), M^{(2)}(x), M^{(3)}(x), M^{(4)}(x)) =$
$\mathrm{LCM}(x^2+x+2,\ x^2+1,\ x^2+x+2,\ x+1)=(x^2+x+2)(x^2+1)(x+1)=$
$=x^5+2x^4+x^3+x^2+2 \rightarrow g_0 g_1 \dots g_5 = 2\ 0\ 1\ 1\ 2\ 1$.

The generator polynomial $g(x)$ corresponds to the generator matrix $G_{(8,\,3)}$:

$$\begin{matrix} & z_0 & z_1 & z_2 & z_3 & z_4 & z_5 & z_6 & z_7, \end{matrix}$$
$$G_{(8,3)} = \begin{Vmatrix} 2 & 0 & 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 1 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 & 1 & 1 & 2 & 1 \end{Vmatrix}.$$

Table 2 – Elements of the field $GF(3^2)$ and their minimal polynomials

Exponential notation		Polyno-mial notation	Vector nota-tion	Deci mal nota tion	Minimal polynomial
With a non-negative degree of the primi-tive element α of the field	With a negative degree of the primi-tive ele-ment α of the field				
–	–	0	(0, 0)	0	–
α^0	α^{-8}	1	(0, 1)	1	–
α^1	α^{-7}	α	(1, 0)	3	x^2+x+2
α^2	α^{-6}	2α+1	(2, 1)	7	x^2+1
α^3	α^{-5}	2α+2	(2, 2)	8	x^2+x+2
α^4	α^{-4}	2	(0, 2)	2	$x+1$
α^5	α^{-3}	2α	(2, 0)	6	x^2+2x+2
α^6	α^{-2}	α+2	(1, 2)	5	x^2+1
α^7	α^{-1}	α+1	(1, 1)	4	x^2+2x+2

On the basis of the generator matrix $G_{(s,\,u)}$ of the BCH code, it is possible to construct the symbology of an interference-resistant barcode with the possibility of correcting single or double distortions of elements (errors) in BC-pattern. For this, the u-bit informational word $B = (b_0\, b_1 \dots b_{u-1})$ must be converted into the s-bit word $Z = (z_0\, z_1 \dots z_{s-1})$, which is a vector (the digital equivalent) of the BC-pattern, i.e., encode the word B with the (s, u)-BCH code, and then match the BC-pattern to the vector Z. The coding operation is given by the equation $Z = B \cdot G_{(s,\,u)}$ and performed according to the rules of the field $GF(q)$.

For example, if $B=(b_0\, b_1\, b_2)$, where $b_i \in \{0, 1, 2\}$, then $Z = B \cdot G_{(8,\,3)}$, i.e.

$$\begin{Vmatrix} b_0 b_1 b_2 \end{Vmatrix} \cdot \begin{Vmatrix} 2\ 0\ 1\ 1\ 2\ 1\ 0\ 0 \\ 0\ 2\ 0\ 1\ 1\ 2\ 1\ 0 \\ 0\ 0\ 2\ 0\ 1\ 1\ 2\ 1 \end{Vmatrix} = \begin{Vmatrix} z_0 z_1 \cdots z_7 \end{Vmatrix}, \qquad (1)$$

whence it follows that

$z_0 = 2b_0$, $z_1 = 2b_1$, $z_2 = b_0 + 2b_2$, $z_3 = b_0 + b_1$,
$z_4 = 2b_0 + b_1 + b_2$, $z_5 = b_0 + 2b_1 + b_2$, $z_6 = b_1 + 2b_2$, $z_7 = b_2$
(operations should be performed according to modulo 3).

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Let B = (1 0 2), then Z = (2 0 2 1 1 0 1 2), and the corresponding BC-pattern is shown in Fig. 3

Taking all possible values of the vector B – from (0 0 0) to (2 2 2) and applying the coding procedure (1) to each word, we get 3^3 = 27 different BC-patters that form the symbology Ω of an interference-resistant three-color barcode, in which correction of single or double errors is possible within each BC-pattern (Table 3).

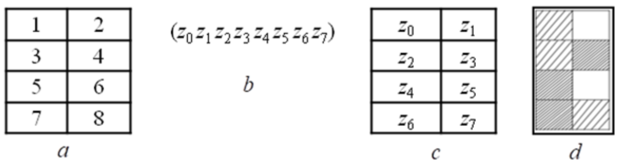


Figure 3 – Creation of a three-color BC-pattern with the possibility of correction double distortion of the elements of the BC-pattern; *a* – the structure of the BC-pattern; *b* – vector of the BC-pattern; *c* – filling out the BC-pattern; *d* – coloring the BC-pattern

The capacity of the Ω symbology is 27 BC-patterns (N=27); it can be matched with the numerical set Ω={0, 1, 2, …, 26}.

The process of synthesizing the symbology of a three-color interference-resistant barcode with the possibility of correcting double errors in BC-patterns can be described by the following generalized algorithm:

for B =000 to 222 do
$\|Z\|=\|B\|\cdot\|G_{(8,3)}\|$
$Z[1...8]:=(z_0 z_1 \ldots z_7)$
$Z[1…8]\rightarrow$ barcode_pattern (B).

The alphanumeric sequence, which is to be presented in the form of a barcode, is converted into a numerical form, the elements of which are numbers from the range 0 – 26 (from the set Ω), and then each number is matched with a BC-pattern (Table 3). Next, BC-patterns are applied to the carrier, forming a BC-symbol.

While reading BC-symbol successively allocate BC-patterns, each of which is matched with a digital equivalent – s-bit vector $Z' = (z'_0 z'_1 \cdots z'_{s-1})$, which is decoded according to the rules of the (s, u)-BCH code with $d_{\min}$ =5.

Decoding is carried out on the basis of the check matrix of the of the BCH code, which is presented in the form:

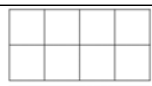
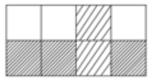
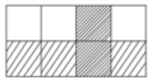
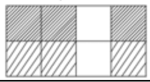
$$H_{(s,u)} = \begin{Vmatrix} \alpha_i \\ \alpha_i^2 \\ \alpha_i^3 \\ \alpha_i^4 \end{Vmatrix}, \; i = 0, 1, 2, ..., s-1,$$

where α_i are elements of $GF(q^m)$.

For the ternary (8, 3)-BCH code, it looks like this:

$$H_{(8,3)} = \begin{Vmatrix} \alpha^0 & \alpha^1 & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 & \alpha^7 \\ \alpha^0 & \alpha^2 & \alpha^4 & \alpha^6 & \alpha^8 & \alpha^{10} & \alpha^{12} & \alpha^{14} \\ \alpha^0 & \alpha^3 & \alpha^6 & \alpha^9 & \alpha^{12} & \alpha^{15} & \alpha^{18} & \alpha^{21} \\ \alpha^0 & \alpha^4 & \alpha^8 & \alpha^{12} & \alpha^{16} & \alpha^{20} & \alpha^{24} & \alpha^{28} \end{Vmatrix}.$$

Table 3 – Symbology of the three-color interference-resistant barcode with the possibility of correction single or double errors in the BC-patterns based on the ternary (8, 3)-BCH code

The serial number of the BC-pattern	BC-pattern	Vector of BC-pattern
0		00000000
1		00201121
2		00102212
⋮		
26		11012202

Taking into account that $\alpha^0=\alpha^8=\alpha^{16}=\alpha^{24}$ and substituting the corresponding two-digit vector columns instead of α^i (see the vector representation of the field elements in Table 2), we obtain

$$\begin{matrix} z'_0 & z'_1 & z'_2 & z'_3 & z'_4 & z'_5 & z'_6 & z'_7, \end{matrix}$$

$$H_{(8,3)} = \begin{Vmatrix} 0 & 1 & 2 & 2 & 0 & 2 & 1 & 1 \\ 1 & 0 & 1 & 2 & 2 & 0 & 2 & 1 \\ 0 & 2 & 0 & 1 & 0 & 2 & 0 & 1 \\ 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \\ 0 & 2 & 1 & 1 & 0 & 1 & 2 & 2 \\ 1 & 2 & 2 & 0 & 2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \end{Vmatrix}.$$

The product of the matrix $G_{(8,3)}$ to the transposed matrix $H_{(8,3)}$ is zero ($G_{(8,3)} H^T_{(8,3)}$=0).

The received vector Z' is decoded according to the algorithm in Fig. 4.

The error syndrome $S = S_1S_2S_3S_4$ is calculated as $S=Z'\cdot H^T_{(8,3)}$, operations are performed according to modulo 3:

$$S_1 = z'_0\begin{pmatrix}0\\1\end{pmatrix} + z'_1\begin{pmatrix}1\\0\end{pmatrix} + z'_2\begin{pmatrix}2\\1\end{pmatrix} + z'_3\begin{pmatrix}2\\2\end{pmatrix} + z'_4\begin{pmatrix}0\\2\end{pmatrix} + z'_5\begin{pmatrix}2\\0\end{pmatrix} + z'_6\begin{pmatrix}1\\2\end{pmatrix} + z'_7\begin{pmatrix}1\\1\end{pmatrix},$$

$$S_2 = z'_0\begin{pmatrix}0\\1\end{pmatrix} + z'_1\begin{pmatrix}2\\1\end{pmatrix} + z'_2\begin{pmatrix}0\\2\end{pmatrix} + z'_3\begin{pmatrix}1\\2\end{pmatrix} + z'_4\begin{pmatrix}0\\1\end{pmatrix} + z'_5\begin{pmatrix}2\\1\end{pmatrix} + z'_6\begin{pmatrix}0\\2\end{pmatrix} + z'_7\begin{pmatrix}1\\2\end{pmatrix},$$

$$S_3 = z'_0\begin{pmatrix}0\\1\end{pmatrix} + z'_1\begin{pmatrix}2\\2\end{pmatrix} + z'_2\begin{pmatrix}1\\2\end{pmatrix} + z'_3\begin{pmatrix}1\\0\end{pmatrix} + z'_4\begin{pmatrix}0\\2\end{pmatrix} + z'_5\begin{pmatrix}1\\1\end{pmatrix} + z'_6\begin{pmatrix}2\\1\end{pmatrix} + z'_7\begin{pmatrix}2\\0\end{pmatrix},$$

$$S_4 = z'_0\begin{pmatrix}0\\1\end{pmatrix} + z'_1\begin{pmatrix}0\\2\end{pmatrix} + z'_2\begin{pmatrix}0\\1\end{pmatrix} + z'_3\begin{pmatrix}0\\2\end{pmatrix} + z'_4\begin{pmatrix}0\\1\end{pmatrix} + z'_5\begin{pmatrix}0\\2\end{pmatrix} + z'_6\begin{pmatrix}0\\1\end{pmatrix} + z'_7\begin{pmatrix}0\\2\end{pmatrix}.$$

If S =0, then there are no errors in the word Z', otherwise ($S \neq 0$) – the word Z' contains one or two errors (Fig. 4).

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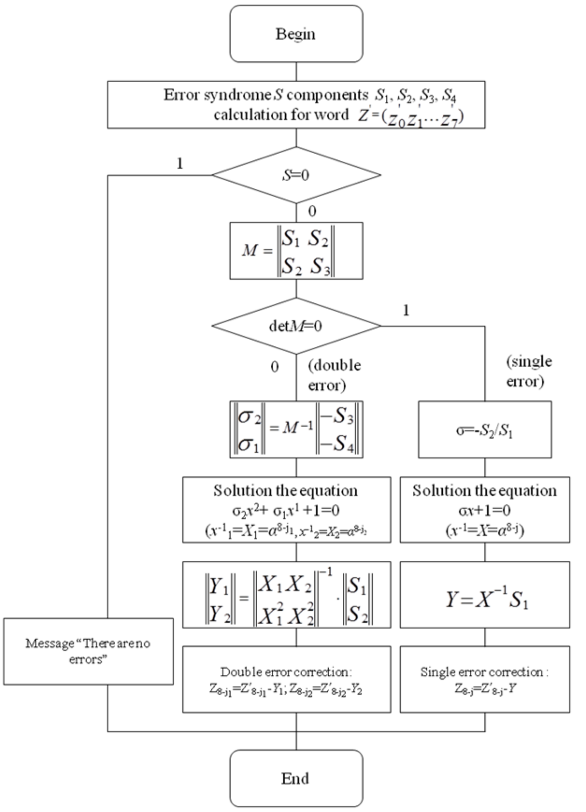

Figure 4 – Block-diagram of the single and double error correction algorithm for the word $Z'=(z'_0 z'_1 \ldots z'_7)$

Further we form the matrix $M = \begin{Vmatrix} S_1 S_2 \\ S_2 S_3 \end{Vmatrix}$ and calculate determinant det$M$. If det$M$ =0, then there is one error in the word Z', otherwise (det$M \neq 0$) – two errors.

Consider the case when det$M \neq 0$.

Find the roots x_1, x_2 of the error locator polynomial $\sigma(x)=\sigma_2 x^2+\sigma_1 x+1$, where the coefficients σ_2, σ_1 determine like

$$\begin{Vmatrix} \sigma_2 \\ \sigma_1 \end{Vmatrix} = M^{-1} \begin{Vmatrix} -S_3 \\ -S_4 \end{Vmatrix}.$$

The error locators X_1, X_2 are values: $X_1 = x_1^{-1}$, $X_2 = x_2^{-1}$.

The equation $\sigma_2 x^2+\sigma_1 x+1=0$ is solved in the field $GF(3^2)$ according to Chien search algorithm [15], which consists in the successive calculation of $\sigma(\alpha^{j})$ for j=0, 1, …, 7, and checking the received value for zero. There is no other way of solving equations in finite fields.

Positions (digits) of the codeword Z correspond to the degrees of the primitive element α of the field:

$$\alpha^{-8}\ \alpha^{-7}\ \alpha^{-6}\ \alpha^{-5}\ \alpha^{-4}\ \alpha^{-3}\ \alpha^{-2}\ \alpha^{-1},$$
$$\alpha^{0}\ \alpha^{1}\ \alpha^{2}\ \alpha^{3}\ \alpha^{4}\ \alpha^{5}\ \alpha^{6}\ \alpha^{7},$$
$$Z=(\ z_0\ z_1\ z_2\ z_3\ z_4\ z_5\ z_6\ z_7).$$

Therefore, if $\sigma(\alpha^{j})=0$, then the error locator X is equal to $\alpha^{-j}=\alpha^{8-j}$, and the location of the error is the digit numbered $8-j$.

The equation $\sigma_2 x^2+\sigma_1 x+1=0$ has two roots: x_1 , x_2 such that $x_1^{-1} = X_1 = a^{-j_1} = \alpha^{8-j_1}$, $x_2^{-1} = X_2 = a^{-j_2} = a^{8-j_2}$.

If $X_1 = a^{-j_1}$, $X_2 = a^{-j_2}$, then the errors are in the digits of z'_{8-j_1}, z'_{8-j_2} the received word Z', respectively.

Next, the values Y_1 , Y_2 of the errors are calculated

$$\begin{Vmatrix} Y_1 \\ Y_2 \end{Vmatrix} = \begin{Vmatrix} X_1 & X_2 \\ X_1^2 & X_2^2 \end{Vmatrix}^{-1} \cdot \begin{Vmatrix} S_1 \\ S_2 \end{Vmatrix} \qquad (2)$$

and perform error correction in the word Z'

$$: z_{8-j_1} = (z'_{8-j_1} - Y_1)\bmod 3, z_{8-j_2} = (z'_{8-j_2} - Y_2)\bmod 3.$$

If det$M$=0 (one error in the accepted word Z'), then the root x of the polynomial $\sigma x+1$ is found, where $\sigma = - S_2/S_1$. The equation $\sigma x+1=0$ is also solved by Chien algorithm. The solution is $x=\alpha^{j}$ such that x^{-1} is the error locator ($X=x^{-1}=\alpha^{-j}=\alpha^{8-j}$) and the location of the error is the digit numbered $8-j$ (that is z'_{8-j}).

Let's consider an example of correcting two errors in a read BC-pattern.

Let's assume that BC-pattern was printed on carrier, the vector of which was equal to Z =(2 0 2 1 1 0 1 2).

Let a vector be obtained during the reading of this pattern is

$$\begin{matrix} & z'_0 & z'_1 & z'_2 & z'_3 & z'_4 & z'_5 & z'_6 & z'_7 \\ Z'=(& \underline{1} & 0 & 2 & \underline{2} & 1 & 0 & 1 & 2), \end{matrix}$$

which contains two errors (underlined units).

Let's calculate syndrome components S_1, S_2, S_3, S_4:

$$S_1 = 1\begin{pmatrix}0\\1\end{pmatrix} + 0\begin{pmatrix}1\\0\end{pmatrix} + 2\begin{pmatrix}2\\1\end{pmatrix} + 2\begin{pmatrix}2\\2\end{pmatrix} + 1\begin{pmatrix}0\\2\end{pmatrix} + 0\begin{pmatrix}2\\0\end{pmatrix} + 1\begin{pmatrix}1\\2\end{pmatrix} + 2\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix} = \alpha^2,$$

$$S_2 = 1\begin{pmatrix}0\\1\end{pmatrix} + 0\begin{pmatrix}2\\1\end{pmatrix} + 2\begin{pmatrix}0\\2\end{pmatrix} + 2\begin{pmatrix}1\\2\end{pmatrix} + 1\begin{pmatrix}0\\1\end{pmatrix} + 0\begin{pmatrix}2\\1\end{pmatrix} + 1\begin{pmatrix}0\\2\end{pmatrix} + 2\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix} = \alpha^7,$$

$$S_3 = 1\begin{pmatrix}0\\1\end{pmatrix} + 0\begin{pmatrix}2\\2\end{pmatrix} + 2\begin{pmatrix}1\\2\end{pmatrix} + 2\begin{pmatrix}1\\0\end{pmatrix} + 1\begin{pmatrix}0\\2\end{pmatrix} + 0\begin{pmatrix}1\\1\end{pmatrix} + 1\begin{pmatrix}2\\1\end{pmatrix} + 2\begin{pmatrix}2\\0\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix} = \alpha^6,$$

$$S_4 = 1\begin{pmatrix}0\\1\end{pmatrix} + 0\begin{pmatrix}0\\2\end{pmatrix} + 2\begin{pmatrix}0\\1\end{pmatrix} + 2\begin{pmatrix}0\\2\end{pmatrix} + 1\begin{pmatrix}0\\1\end{pmatrix} + 0\begin{pmatrix}0\\2\end{pmatrix} + 1\begin{pmatrix}0\\1\end{pmatrix} + 2\begin{pmatrix}0\\2\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} = \alpha^0.$$

Let's put it together the matrix

$$M = \begin{Vmatrix} S_1 & S_2 \\ S_2 & S_3 \end{Vmatrix} = \begin{Vmatrix} \alpha^2 & \alpha^7 \\ \alpha^7 & \alpha^6 \end{Vmatrix}.$$

Let's calculate the determinant of the matrix M:

det$M=\alpha^2\alpha^6 - \alpha^7\alpha^7=\alpha^8 - \alpha^{14}=\alpha^0 - \alpha^6$=(0, 1) – (1, 2)= =(0, 1)+(2, 1)=(2, 2)=$\alpha^3 \neq 0$ (see Table 2).

Since det$M \neq 0$, there are two errors in the word Z'. Let's find the coefficients σ_2 , σ_1 of the error locator polynomial $\sigma(x)=\sigma_2 x^2+\sigma_1 x+1$:

$$\begin{Vmatrix} \sigma_2 \\ \sigma_1 \end{Vmatrix} = M^{-1} \begin{Vmatrix} -S_3 \\ -S_4 \end{Vmatrix}.$$

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To do this, first calculate M^{-1}:

$$M^{-1} = (1/\det M)\cdot\begin{Vmatrix} S_3 & -S_2 \\ -S_2 & S_1 \end{Vmatrix} = (1/\alpha^3)\cdot\begin{Vmatrix} \alpha^6 & -\alpha^7 \\ -\alpha^7 & \alpha^2 \end{Vmatrix} = \begin{Vmatrix} \alpha^3 & -\alpha^4 \\ -\alpha^4 & \alpha^{-1} \end{Vmatrix}.$$

Since $-\alpha^4=-(0, 2)=(0, 1)=\alpha^0$, and $\alpha^{-1}=\alpha^7$, then

$$M^{-1} = \begin{Vmatrix} \alpha^3 & \alpha^0 \\ \alpha^0 & \alpha^7 \end{Vmatrix}.$$

Then, $\begin{Vmatrix} \sigma_2 \\ \sigma_1 \end{Vmatrix} = \begin{Vmatrix} \alpha^3 & \alpha^0 \\ \alpha^0 & \alpha^7 \end{Vmatrix}\cdot\begin{Vmatrix} -\alpha^6 \\ -\alpha^0 \end{Vmatrix} = \begin{Vmatrix} \alpha^3 & \alpha^0 \\ \alpha^0 & \alpha^7 \end{Vmatrix}\cdot\begin{Vmatrix} \alpha^2 \\ \alpha^4 \end{Vmatrix} = \begin{Vmatrix} \alpha^3 \\ \alpha^1 \end{Vmatrix}$

(since $-\alpha^6=-(1, 2)=(2,1)=\alpha^2$, and $-\alpha^0=-(0,1)=(0,2)=\alpha^4$). Next, we will solve the equation $\alpha^3x^2+\alpha^1x+1=0$ in the field $GF(3^2)$.

For this, we will apply Chien algorithm:

$x=\alpha^0\to\alpha^3(\alpha^0)^2+\alpha^1(\alpha^0)+1=\alpha^3+\alpha^1+1=(2,2)+(1,0)+1=(0,0)=0$,
$x=\alpha^1\to\alpha^3(\alpha^1)^2+\alpha^1(\alpha^1)+1=\alpha^5+\alpha^2+1=(2,0)+(2,1)+1=(1,2)\neq0$,
$x=\alpha^2\to\alpha^3(\alpha^2)^2+\alpha^1(\alpha^2)+1=\alpha^7+\alpha^3+1=(1,1)+(2,2)+1=(0,1)\neq0$,
$x=\alpha^3\to\alpha^3(\alpha^3)^2+\alpha^1(\alpha^3)+1=\alpha^1+\alpha^4+1=(1,0)+(0,2)+1=(1,0)\neq0$,
$x=\alpha^4\to\alpha^3(\alpha^4)^2+\alpha^1(\alpha^4)+1=\alpha^3+\alpha^5+1=(2,2)+(2,0)+1=(1,0)\neq0$,
$x=\alpha^5\to\alpha^3(\alpha^5)^2+\alpha^1(\alpha^5)+1=\alpha^5+\alpha^6+1=(2,0)+(1,2)+1=(0,0)=0$,
$x=\alpha^6\to\alpha^3(\alpha^6)^2+\alpha^1(\alpha^6)+1=\alpha^7+\alpha^7+1=(1,1)+(1,1)+1=(2,0)\neq0$,
$x=\alpha^7\to\alpha^3(\alpha^7)^2+\alpha^1(\alpha^7)+1=\alpha^1+\alpha^0+1=(1,0)+(0,1)+1=(1,2)\neq0$.

As we can see, the roots of the equation are $x_1=\alpha^0$ and $x_2=\alpha^5$.

So, $X_1 = x_1^{-1} = \alpha^0 = \alpha^{8-0} = \alpha^8 = \alpha^0$, and $X_1 = x_2^{-1} = \alpha^{-5} = \alpha^{8-5} = \alpha^3$. This means that errors locate in digits z'_0 and z'_3 of accepted word.

Let's calculate the error values based on (2):

$$\begin{Vmatrix} Y_1 \\ Y_2 \end{Vmatrix} = \begin{Vmatrix} \alpha^0 & \alpha^3 \\ \alpha^0 & \alpha^6 \end{Vmatrix}^{-1}\cdot\begin{Vmatrix} \alpha^2 \\ \alpha^7 \end{Vmatrix}.$$

First, we find the inverse matrix:

$$\begin{Vmatrix} \alpha^0 & \alpha^3 \\ \alpha^0 & \alpha^6 \end{Vmatrix}^{-1} = (1/\alpha^5)\cdot\begin{Vmatrix} \alpha^6 & -\alpha^3 \\ -\alpha^0 & \alpha^0 \end{Vmatrix} = (1/\alpha^5)\cdot\begin{Vmatrix} \alpha^6 & \alpha^7 \\ \alpha^4 & \alpha^0 \end{Vmatrix} =$$

$$= \begin{Vmatrix} \alpha^1 & \alpha^2 \\ \alpha^{-1} & \alpha^{-5} \end{Vmatrix} = \begin{Vmatrix} \alpha^1 & \alpha^2 \\ \alpha^7 & \alpha^3 \end{Vmatrix}.$$

Then $\begin{Vmatrix} Y_1 \\ Y_2 \end{Vmatrix} = \begin{Vmatrix} \alpha^1 & \alpha^2 \\ \alpha^7 & \alpha^3 \end{Vmatrix}\cdot\begin{Vmatrix} \alpha^2 \\ \alpha^7 \end{Vmatrix} = \begin{Vmatrix} \alpha^4 \\ \alpha^0 \end{Vmatrix}$.

But $\alpha^4=(0, 2)=2$, $\alpha^0=(0, 1)=1$.

Therefore, $Y_1=2$, $Y_2=1$.

Let's correct the errors:

$$z_0=(z'_0-Y_1)\text{mod}3=(1-2)\text{mod}3=2$$
$$z_3=(z'_3-Y_2)\text{mod}3=(2-1)\text{mod}3=1.$$

Thus, the right vector of the read BC-pattern is $Z=(2\ 0\ 2\ 1\ 1\ 0\ 1\ 2)$; double error is corrected.

To construct the symbologies of multi-color BCs of different capacities, shortened BCH code should be used. To obtain shortened BCH code it is needed to remove the required number of columns and rows from the original generator matrix and appropriate number of columns in check matrix.

We will consider the construction of shortened codes on the example of a full triple (26, 17)-BCH code (see Table 1), which uses two fields: $GF(3)$ – the field of characters (Fig. 2) and $GF(3^3)$ – the field of locators (Table 4). The field of locators is built on the basis of an irreducable polynomial of third degree $p(x)=x^3+2x+1$.

In $GF(3^3)$ $\alpha^i=\alpha^{i-26}$, $\alpha^{-i}=\alpha^{26-i}$, $\alpha^{26}=\alpha^{-26}=\alpha^0=1$.

Table 4 – Elements of the field $GF(3^3)$ and their minimal polynomials (fragment)

Exponential notation		Polynomial notation	Vector notation	Decimal notation	The minimal polynomial
With a non-negative degree of the primitive element α of the field	With a negative degree of primitive element α of the field				
–	–	0	(0,0,0)	0	–
α^0	α^{-26}	1	(0,0,1)	1	–
α^1	α^{-25}	α	(0,1,0)	3	x^3+2x+1
α^2	α^{-24}	α^2	(1,0,0)	9	x^3+x^2+x+2
α^3	α^{-23}	$\alpha+2$	(0,1,2)	5	x^3+2x+1
α^4	α^{-22}	$\alpha^2+2\alpha$	(1,2,0)	15	x^3+x^2+2
α^5	α^{-21}	$2\alpha^2+\alpha+2$	(2,1,2)	23	x^3+x^2+x+1
⋮	⋮	⋮	⋮	⋮	⋮
α^{25}	α^{-1}	$2\alpha^2+1$	(2,0,1)	19	x^3+2x^2+1

The generator polynomial $g(x)$ of the ternary (26, 17)-BCH code is defined as follows: $g(x)$=LCM(x^3+2x+1, x^3+x^2+x+2, x^3+2x+1, x^3+x^2+2)=$(x^3+2x+1)(x^3+x^2+x+2)(x^3+x^2+2)=x^9+2x^8+x^7+x^6+x^5+2x^4+2x^3+2x^2+x+1\to$
$\to g_0\, g_1 \ldots g_9$=1 1 2 2 2 1 1 1 2 1.

It corresponds to the generator matrix $G_{(26,\ 17)}$ of the full code.

If, for example, we remove 10 columns to the right (z_{16} – z_{25}) and 10 botton rows from $G_{(26,\ 17)}$; and 10 columns to the right ($z'_{16}-z'_{25}$) from $H_{(26,\ 17)}$ then we get generator matrix $G_{(16,\ 7)}$ and, accordingly, check matrix $H_{(16,\ 7)}$, of shortened (16, 7)-BCH code (Fig. 5).

Moving the right columns and bottom rows from $G_{(26,17)}$, and the corresponding right columns from $H_{(26,\ 17)}$, we will obtain different shortened triple (q=3) BCH codes: (17, 8)–; (16, 7)–; (15, 6)–; (14, 5)–; (13, 4)–, on the basis of which it is possible to synthesize the symbologies of interference-resistant three-colored BCs of different capacities (Table 5).

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$$G_{(16,7)}=\left\|\begin{matrix} z_0 & z_1 & z_2 & z_3 & z_4 & z_5 & z_6 & z_7 & z_8 & z_9 & z_{10} & z_{11} & z_{12} & z_{13} & z_{14} & z_{15} \\ 1 & 1 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 1 \end{matrix}\right\|$$

$$H_{(16,7)}=\left\|\begin{matrix} z'_0 & z'_1 & z'_2 & z'_3 & z'_4 & z'_5 & z'_6 & z'_7 & z'_8 & z'_9 & z'_{10} & z'_{11} & z'_{12} & z'_{13} & z'_{14} & z'_{15} \\ 0 & 0 & 1 & 0 & 1 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 & 0 & 2 & 1 & 2 & 2 & 1 & 0 & 2 & 2 & 2 & 0 & 0 \\[8pt] 0 & 1 & 1 & 1 & 2 & 1 & 1 & 0 & 0 & 1 & 2 & 0 & 2 & 0 & 1 & 1 \\ 0 & 0 & 2 & 1 & 0 & 1 & 0 & 2 & 2 & 2 & 1 & 2 & 2 & 0 & 0 & 2 \\ 1 & 0 & 0 & 1 & 2 & 0 & 2 & 0 & 1 & 1 & 1 & 2 & 1 & 1 & 0 & 0 \\[8pt] 0 & 0 & 1 & 0 & 1 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 & 0 & 0 & 2 & 0 & 2 & 1 & 2 & 2 & 1 & 0 & 2 & 2 \\ 1 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 2 & 0 & 2 & 1 & 2 \\[8pt] 0 & 1 & 2 & 1 & 0 & 2 & 2 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 2 & 0 & 0 & 2 & 1 & 2 & 0 & 1 & 1 & 2 & 2 & 2 & 0 & 2 & 0 \\ 1 & 0 & 2 & 2 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 2 & 1 & 0 & 2 \end{matrix}\right\|$$

Figure 5 – Generator (*G*) and check (*H*) matrices of the shortened ternary (16, 7)-BCH code based on the full (26, 17)-BCH code

Table 5 – Capacity (*V*) of the symbologies of multi-color (*q*) interference-resistant BCs based on shortened (*s*, *u*)-BCH codes

$q = 3$		$q = 5$		$q = 7$	
(*s*, *u*)–	*V*	(*s*, *u*)–	*V*	(*s*, *u*)–	*V*
(13, 4)–	81	(10, 2)–	25	(10, 2)–	49
(14, 5)–	243	(11, 3)–	125	(11, 3)–	343
(15, 6)–	729	(12, 4)–	625	(12, 4)–	2401
(16, 7)–	2187	(13, 5)–	3125	(13, 5)–	16807
(17, 8)–	6561	(14, 6)–	15625		

Similarly, on the basis of shortened quinary (q=5) and septenary (q=7) BCH codes, it is possible to synthesize the symbologies of interference-resistant five-color and seven-color BCs with the possibility of correction double errors in BC-patterns. So, assigned in the Table 5 shortened quinary codes, formed on the basis of the full quinary (24, 16)-BCH code (see Table 1), which uses the field of characters *GF*(5) and the field of locators *GF*(5^2), and the shortened septenary codes, formed on the basis of full septenary (48, 40)-BCH code, which uses the character field *GF(7)* and the locator field *GF*(7^2).

Such a series of BCH codes makes it possible to build the family of multi-color interference-resistant BCs with symbologies of different capacity.

4 EXPERIMENTS

The considered BCH codes with a minimal code distance of $d_{\min}$=5 ensure the correction of a single or double error inside each BC-pattern when read from the carrier. In order to explore the correction capabilities of shortened BCH codes, in particular, the ability to detect multiple errors in BC-patterns, a software product was developed in Java in the environment Intelij idea.

Experimental studies were carried out on a computer with the macOS operating system BigSur, 32 GB RAM, 2.4 GHz 8-Core processor Intel Core i9.

This software product makes it possible to carry out statistical studies of the corrective ability of BCH codes in conditions of multiple damages to the elements of BC-patterns. All possible cases of occurrence of one to seven errors in BC-patterns were studied. For each case, one of three possible events were recorded: an error is detected (for single and double error detection is equivalent to correction – according to the algorithm in Fig. 4); error syndrome is equal to zero; the combination of errors is undetected.

Not only all possible locations in words of probable errors, but also all possible values of errors were generated.

5 RESULTS

Statistical data characterizing the ability of multi-valued BCH codes to detect and correct multiple errors was obtained. It has been proven that all single and double errors in data words (vectors of BC-patters) are corrected. The ability to detect (3–7)-tuple errors for 18 BCH codes – 8 ternary codes, 6 quinary codes, and 4 septenary codes – was studied. For each code, corresponding indicators were obtained – for example, for the full ternary (8, 3)-BCH code, they are presented in the Table 6.

Generalized indicators for ternary BCH-codes are reflected in Fig. 6 (the abscissa axis indicates the investigated ternary (*s*, *u*)-BCH codes, the ordinate axis indicates

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the percentage of detected errors), where the upper curve is the percentage of 3-tuple errors that are detected, and the lower curve is the percentage of detection (4–7)-tuple errors.

Table 6 – Corrective ability of the full ternary (8, 3)-BCH code in the case of multiple errors

Multiplicity of error	The percentage of errors which		
	are detected	are undetectable	give zero syndrome
3	46.4%	53.6%	0
4	39.3%	60.7%	0
5	29.5%	69.7%	0.8%
6	32.6%	66.9%	0.5%
7	34.4%	64.9%	0.7%

Quinary BCH codes allow to detect of 70.7 – 96.0% of 3-tuple errors and 67.9–93.8% (4–7)-tuple errors; septenary BCH codes, respectively, 97.9–99.0% 3-tuple errors and 97.1–98.3% (4–6)-tuple in data words.

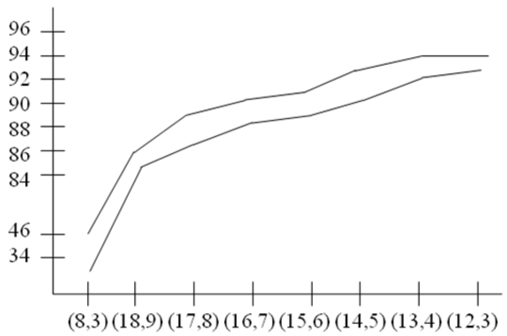

Figure 6 – Ability to detect multiple errors with ternary codes

6 DISCUSSION

Research shows that full BCH codes, such as a ternary (8, 3)-code or a quinary (24, 16)-code, provide fewer multiple-error detections compared to shortened codes. This is because shortened codes have more redundancy. It was also found out that combinations of multiple errors, which give a zero syndrome during decoding, are extremely rare (0.004 – 0.009%) precisely in shortened BCH codes, and they are not detected by the decoder.

The obtained results are highly reliable, since the verification of the proposed algorithm for decoding BCH codes was carried out on data words of different bit sizes, with an overview of all possible combinations of errors that may occur during data processing, as well as for a sufficiently large number of codes.

The obtained results give reason to conclude about the expediency of the two-level security of multi-color barcode images, when the lower level should be based on the use of a multi-valued BCH code (the level of the BC-pattern; the digital vector of the BC-pattern is the codeword of the BCH code), and the upper level (the BC-symbol in general) – on the use of the Reed-Solomon code, which is capable to correct two types of distortion – errors and erasures, and for which the minimal structural units (word bits) are the BC-patterns of the barcode image. For the Reed-Solomon code, an “error” is considered to be a situation when neither the location of the distortion in the read word (BC-symbol) nor the value of distortion is unknown; and “erasure” is a situation when the location of the distortion is known, and only the value of the distortion is not known.

When reading a multi-color barcode, the program sequentially separates BC-patterns from the barcode image, thus forming a Reed-Solomon codeword, in which each BC-pattern is a separate digit of the word. A software decoder of the BCH-code operates inside every BC-pattern, the result of which can be three conclusions: “BC-pattern is not damaged”, “BC-pattern is corrected”, “BC-pattern is erased”.

The solution “BC-pattern is not damaged” is formed if the error syndrome is equal to zero. It should be noted that those rare cases (the number of which is less than one-hundredth of a percent) when combinations of element distortions in the BC-patterns give zero syndrome, and may occur with an error multiplicity of more than four, will be detected by the software decoder of the Reed-Solomon code.

The solution “BC-pattern is corrected” is formed when the BCH code decoder corrects a one- or two errors in the BC-pattern vector. If the BC-pattern contains three or more damages, and the decoder of the BCH code perceives them as a single or double error, and, accordingly, will correct it incorrectly (and in such cases, for example, for a ternary (15, 6)-code or a quinary (13, 5)-code, – about 8–10%), then such a BC-pattern will be perceived by the decoder of the Reed-Solomon code as an “error”.

The solution “BC-pattern is erased” is formed by the BCH code decoder, if it detects distortion of three or more elements in the BC-pattern. For the (15, 6)– and (13, 5)–BCH codes mentioned above, this will happen in 90 – 92% of cases. Such a situation would be qualified by a Reed-Solomon code decoder as “erasure”.

It is known that to correct each error in the structure of the codeword of the Reed-Solomon code, two check digits must be provided, and to detect each erasure – one check digit [15]. Therefore, the use of a multi-valued, for example, three-color (15, 6)-BCH code at the lower level of ensuring interference resistance of a multi-color BC, which, in addition to correcting single and double damage, also allows to detect about 90% of multiple (three or more) damages of elements in BC-pattern, strengthens the corrective capabilities of the Reed-Solomon code by an average of 45% by transferring “erasure” situations instead of “error” situations to the upper level of immunity protection.

CONCLUSIONS

The work solves the actual scientific problem of improving the interference resistance of multi-color barcodes.

The scientific novelty of the work lies in the fact that the method of constructing the symbology of a given capacity of a multi-color barcode is firstly proposed, the barcode patterns of which have the properties of interfer-

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ence resistance, which is consist in the fact that when reading a multi-color barcode image the data will be reliably reproduced from the barcode patterns even in case of damage one or two graphic elements of the pattern. This is achieved due to the fact that the vector (digital twin) of each barcode pattern is a codeword of the correcting multi-valued BCH code with a minimal code distance of five.

The construction (synthesis) of BC symbologies is considered in detail for the case of three-, five- and seven-color two-dimensional barcodes.

It is shown that in the BC-patterns synthesized on the basis of the BCH code, during their reading from the carrier, in addition to the correction of single and double errors, a significant part (from 33.9% to 97.1%) of (3 – 7)-tuple errors is also detected. It has been proven that the use of shortened multi-valued BCH codes for the synthesis of symbologies of multi-color barcodes significantly increases the ability to detect multiple errors in BC-patterns compared to full codes, in particular by 1.91 – 2.75 times – for three-color ones and by 1.30–1.38 times – for five-color BC-patterns, and also allows to receive symbologies of different capacity, which makes it possible to create family of multi-color interference-resistant barcodes.

The practical significance of the obtained results lies in the fact that the developed method of constructing interference-resistant BC-patterns based on BCH codes can be used at the lower level in the system of two-level interference resistance of multi-color barcode images, when the Reed-Solomon code is used at the upper level. At the same time, the corrective capabilities of the Reed-Solomon code are significantly strengthened (up to 45%) with an unchanged number of control digits, as a result of which the immunity of barcodes patterns of multi-color barcodes is significantly improved.

Prospects for further research should be focused on improving the mechanism of complementary application of correcting codes for two-level interference immunity of multi-color barcode images.

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СИНТЕЗ СИМВОЛІК БАГАТОКОЛІРНИХ ЗАВАДОСТІЙКИХ ШТРИХОВИХ КОДІВ НА ОСНОВІ МНОГОЗНАЧНИХ КОДІВ БЧХ

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АНОТАЦІЯ

Актуальність. Розглянуто задачу побудови набору (символіки) штрихкодових знаків для багатоколірних штрихових кодів, стійких до ушкодження одного або двох елементів у межах кожного знака.

Мета. Забезпечення надійності зчитування багатоколірних штрихкодових зображень.

Метод. Багатоколірний штрихкодовий знак має властивість завадостійкості, якщо його цифровий еквівалент (вектор) є кодовим словом многозначного (недвійкового) коректувального коду, здатного виправляти помилки (спотворення елементів знака). Показано, що побудову штрихкодових знаків слід виконувати на основі многозначного коректувального коду БЧХ, здатного виправляти дві помилки. Запропоновано метод побудови множини завадостійких штрихкодових знаків заданої потужності, які забезпечують достовірне відтворення даних при їх зчитуванні з носія. Розроблено процедуру кодування даних многозначним кодом БЧХ на основі твірної матриці коду з використанням операцій за модулем простого числа. Запропоновано новий спосіб побудови перевірної матриці многозначного коду БЧХ на основі векторного подання елементів скінченного поля. Розроблено узагальнений алгоритм генерування символіки багатоколірного штрихового коду з можливістю корекції двократних помилок у штрихкодових знаках. Метод також дозволяє будувати символіки заданої потужності на основі скорочених кодів БЧХ. Запропоновано спосіб скорочення твірної та перевірної матриць многозначного повного коду БЧХ для отримання скороченого коду заданої довжини. Показано, що крім виправлення двократних помилок, многозначні коди БЧХ дозволяють також виявляти помилки більшої кратності; ця властивість посилюється при використанні скорочених кодів БЧХ. Метод забезпечує побудову сімейства багатоколірних завадостійких штрихових кодів.

Результати. На основі розробленого програмного забезпечення отримані статистичні дані, що характеризують здатність многозначних кодів БХЧ виявляти та виправляти помилки, і на їх основі проєктувати багатоколірні завадостійкі штрихові коди.

Висновки. Проведені експерименти підтвердили працездатність розробленого алгоритмічного забезпечення і дозволяють рекомендувати його для використання на практиці при проєктуванні завадостійких багатоколірних штрихових кодів у системах автоматичної ідентифікації.

КЛЮЧОВІ СЛОВА: штрихове кодування, багатоколірні штрихові коди, завадостійкість штрихових кодів, коди БЧХ.

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СИНТЕЗ СИМВОЛИК МНОГОЦВЕТНЫХ ПОМЕХОУСТОЙЧИВЫХ ШТРИХОВЫХ КОДОВ НА ОСНОВЕ МНОГОЗНАЧНЫХ КОДОВ БЧХ

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АННОТАЦИЯ

Актуальность. Рассмотрена задача построения набора штрихкодовых знаков для многоцветных штриховых кодов, устойчивых к искажениям одного или двух элементов в пределах каждого знака.

Цель. Обеспечение надежности считывания многоцветных штрихкодовых изображений.

Метод. Многоцветный штрихкодовый знак имеет свойство помехоустойчивости, если его цифровой эквивалент (вектор) является кодовым словом многозначного (недвоичного) корректирующего кода, способного исправлять ошибки (искажения элементов знака). Показано, что построение штрихкодовых знаков следует выполнять на основе многозначного корректирующего кода БЧХ, способного исправлять две ошибки. Предложен метод построения множества помехоустойчивых штрихкодовых знаков заданной мощности, обеспечивающих достоверное воспроизведение данных при их считывании с носителя. Разработана процедура кодирования данных многозначным кодом БЧХ на основе образующей матрицы кода с использованием операций по модулю простого числа. Предложен новый способ построения проверочной матрицы многозначного кода БЧХ на основе векторного представления элементов конечного поля. Разработан обобщенный алгоритм генерирования символики многоцветного штрихового кода с возможностью коррекции двукратных ошибок в штрихкодовых знаках. Метод позволяет строить символики заданной можности на основе сокращенных кодов БЧХ. Предложен способ сокращения образующей и проверочной матрицы многозначного полного кода БЧХ для получения сокращенного кода за-

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данной длины. Показано, что кроме исправления двукратных ошибок, многозначные коды БЧХ позволяют также обнаруживать ошибки большей кратности – это свойство усиливается при использовании укороченных кодов БЧХ. Метод обеспечивает построение семейства многоцветных помехоустойчивых штриховых кодов.

Результаты. На основе разработанного программного обеспечения получены статистические данные, характеризующие способность многозначных кодов БЧХ обнаруживать и исправлять ошибки, и на их основе проектировать многоцветные помехоустойчивые штриховые коды.

Выводы. Проведенные эксперименты подтвердили работоспособность разработанного алгоритмического обеспечения и позволяют рекомендовать его для использования на практике при проектировании помехоустойчивых многоцветных штриховых кодов в системах автоматической идентификации.

КЛЮЧЕВЫЕ СЛОВА: штриховое кодирование, многоцветные штриховые коды, помехоустойчивость штриховых кодов, коды БЧХ.

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