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# THE CURVE ARC AS A STRUCTURE ELEMENT OF AN OBJECT CONTOUR IN THE IMAGE TO BE RECOGNIZED

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## ABSTRACT

**Context**. The proposed article relates to the field of visual information processing in a computer environment, more precisely to the determination the parameters of the interest object in the image, in particular, the contour of the interest object In most cases, the contour of an object is a simply connected sequence of curve arcs.

**Objective.** The purpose and subject of the study is to find and to propose such a definition of the digital curve arc, as the most important element of the object contour in the recognizable image, which does not contradict modern neurophysiological conceptions about visual perception, and to recognize the object contour as a sequence of the digital curve arcs.

**Method.** The representation of the image in the form of a structural model is used, one of the structural elements of which is the contour of the object, consisting of digital curve arcs. Also, the image is considered as a cellular complex which corresponds to modern ideas about human visual perception.

**Results.** The new definition for arc of a digital curve as a sequence of digital straight segments is proposed, which does not contradict to modern concepts of neurophysiology. In contrast to the known definitions of a curve arc, the proposed definition of a digital curve arc makes it possible to determine the start and end points of the arc. According to the description of the contour of an object as a simply connected closed sequence of line segments, it is proposed to construct a description of the contour as a sequence of arcs of digital curves.

**Conclusions.** The use of the proposed definition of the digital curve arc in image processing makes it possible to recognize the contour of an object in an image and present it in a form close to visual perception. For best results, the use of variable resolution in image processing algorithms is recommended.

**KEYWORDS:** image, contour, curve arc, straight line segment, cellular complex, neurons, receptive field.

## NOMENCLATURE

$AB$ is the curve arc, corresponds to $x = \varphi(\tau)$, $y = \phi(\tau)$ at $l \le \tau \le u$;
$a$ is an example of 0-cell;
$b$ is an example of 0-cell;
$c$ is an example of 0-cell;
$C$ is an abstract cell complex;
$C_m$ is an $m$-dimensional Cartesian complex;
$d$ is the pixel size;
$dim$ is a dimension function;
$E$ is the set of abstract elements (cells);
$e$ is an example of 0-cell;
$e'$ is formal cell example;
$e''$ is formal cell example;
$f$ is an example of 0-cell;
$F$ is a binary relation;
$g$ is an example of 0-cell;
$I$ is the set of non-negative integers
$h$ is the height of the arc segment;
$h_s$ is the height of the $s$-th arc segment;
$l$ is lower bound of the parameter $\tau$ definition area;
$m$ is a dimension;
$n$ is the cell number in the cell sequence;
$N$ is the cell quantity in the cell sequence;
$p$ is an example of 0-cell;
$P$ is a set of points;
$r$ is a brightness function;
$s$ is the straight segment number in the sequence;
$S$ is the straight segment quantity in the sequence;
$t$ is the cell number in the cell sequence of the straight line segment;
$T_bT_e$ is a straight line segment;
$T_b$ is a begin point $x_b$,$y_b$ of the straight line segment $T_bT_e$;
$T_e$ is an end point $x_e$,$y_e$ of the straight line segment $T_bT_e$;
$u$ is upper bound of the parameter $\tau$ definition area;
$x_n$ is the cell abscissa with number $n$ in the sequence;
$y_n$ is the cell ordinate with number $n$ in the sequence;
$x_s$ is the abscissa of the boundary common point (0-cell) of adjacent line segments, number $s$ in the sequence;
$y_s$ is the ordinate of the boundary common point (0-cell) of adjacent line segments, number $s$ in the sequence;
$\underline{x}_s$ is the boundary point (0-cell) number $s$ abscissa of line segment belonging to curve arc;
$\underline{y}_s$ is the boundary point (0-cell) number $s$ ordinate of line segment belonging to curve arc, in the sequence;
$x_t$ is the cell abscissa with number $t$ in the sequence of the straight line segment;
$y_n$ is the cell ordinate with number $t$ in the sequence of the straight line segment;


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φ is continuous parametrically defined function;
ψ is continuous parametrically defined function;
τ is the parameter on the segment $[l,u]$.

## INTRODUCTION

Many tasks of image analysis and processing consist in detecting an object of interest and determining its parameters. Typically, visual information and, in particular, a grayscale image is represented as a set of pixels, densely, without gaps, filling the field of the image. The boundaries (contour) of the detected object are often not defined as the result of image processing, as, for example, for statistical [1, 2, 3, 4] or neural network [5, 6, 7] approaches to image processing.

At the same time, other approaches are known for solving image recognition tasks, in particular, structural recognition [8], which involve the representation of an image in the form of a hierarchical structure.

**The object of study** is the arc of the curve as an structural element of the image while its processing.

**The subject of study** is the development of the curve definition arc, suitable for the analysis and processing of an image in its discrete representation. The well-known mathematical definitions of the curve arc are not very suitable for use in discrete image processing, since they define abstract curves in a continuous space.

**The purpose of the work** is to develop a definition of the curve arc, which is suitable for finding the arc of the curve in the image in the process of recognizing the contour of an object.

## 1 PROBLEM STATEMENT

Let $\{x_n,y_n\}$, $1\le n\le N$, be a sequence of points (0-cells) describing the contour of an object. Let the contour of the object be approximated by line segments and $\{x_s,y_s\}$, $1\le s\le S$, be a sequence representing boundary points (0-cells) of adjacent line segments. That is, each adjacent two line segments are represented in the sequence by pairs of points (0-cells): $(x_{s-1},y_{s-1};\ x_s,y_s)$ and $(x_s,y_s;\ x_{s+1},y_{s+1})$, respectively.

The problem: whether there are in a sequence of line segments such that they form an arc of a curve can be reduced to checking the condition of belonging or not belonging to an curve arc of two adjacent line segments, followed by checking all pairs of line segments in the sequence.

Then it is necessary and sufficient to find the condition of belonging or not belonging to the arc of the curve of two adjacent line segments in the sequence and apply it to all pairs of adjacent segments in the sequence.

## 2 REVIEW OF THE LITERATURE

Consider the halftone image structure. The first hierarchical structure of an image is discussed in [8]. A more general structural image model, compared to [8], is presented here. In most cases, a grayscale image can be considered as a realization of an unknown brightness function $r = f(x,y)$ depending on two spatial variables $x$, $y$. This function defines a piecewise smooth surface. Objects in the field of view are regular pieces of a piecewise-smooth surface. The projections of the surface pieces contours onto the image plane coincide with the definition domains boundaries of the unknown functions that define the surface pieces, and are the objects contours in the field of view. The objects forming the background are the objects in the field of view, the contours of which partially or completely coincide with the boundaries of the image. Possible objects of interest may include objects in the field of view that do not have common boundaries with the boundaries of the image. Visual information should be presented taking into account the physiological characteristics of visual perception so that optimal results can be achieved with its automatic processing. In particular, one of the most important and natural features of human visual perception is its ability to segment the visual field into objects that differ from the background in brightness, color, texture. The main characteristic of any object is its shape, determined by the contour – the boundary between the object and the background. The contour, in turn, is perceived by a person as a sequence of straight line segments and curve arcs. The shape of halftone objects is also determined by the brightness function based on the color, texture within each of the objects.

These features of human visual perception are reflected in the structural model of the image. The structural model makes it possible to represent arbitrary images uniformly in form, invariant with respect to affine transformations – position in the field of view, scale, rotation. The problem of reduction to a structural model of arbitrary images given in a raster form, distorted by noise, in the general case, has not yet been solved. However, the transformation of images into a structural model can significantly increase the speed and quality of visual information processing in some rather numerous cases. The basis for the structural analysis of a halftone image is a model that determines its structural elements (Fig. 1). In particular, the objects of interest and the background of the image are such structural elements according to the known ideas about the mechanisms of visual perception. Objects, in turn, are defined by bounding contours and a three-dimensional brightness function within the object.

Contours are closed sequences formed by segments of straight lines and arcs of curved lines. Representation of grayscale images in the form of such or similar model is invariant to affine transformations of objects.

Consider the image as a cellular complex. There are many problems in image analysis that cannot be solved based on classical Euclidean geometry. The reason is that in classical geometry the assumption of space continuity is used. That is, each point in space contains in its neighborhood an infinite number of points, no matter how small this neighborhood is. According to the topological foundations of classical geometry, even the smallest neighborhood of each point contains an infinite number of other points. Thus, classical geometry has no means for


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processing discrete images, because discrete image is presented as the set of isolated points, sufficiently small neighborhoods of which do not contain points at all, except for the isolated point itself. But then the discrete image can be described in classical geometry very approximately, with an accuracy of several distinctly small spatial elements.

It was proposed to use the mathematical apparatus of abstract cellular complexes to describe the image [9, 10]. An abstract cellular complex $C = (E, F, \dim)$ is understood as a set $E$ of abstract elements (cells) that are in an antisymmetric, irreflexive and transitive binary relation $F \subset E x E$, which is called the limit relation, and with a dimension function dim: $E \rightarrow I$ with $E$ on the set $I$ of non-negative integers, such as $\dim(e') < \dim(e'')$ for all pairs $(e', e'') \in B$. If $(e', e'') \in B$ then one usually writes $e' < e''$ or says that cells $e'$ limit cells $e''$. Such cells are called incident to each other.

Fig. 2a shows an arbitrary halftone image, which can be described as an abstract two-dimensional cellular complex. First of all, let's note the 0-dimensional cells: *a*,*b*,*c*,*d*,*e*,*f*,*g*. 0-dimensional cells correspond to points in the two-dimensional Euclidean space. The attributes of each point are coordinates. Lines correspond to 1-dimensional cells in Euclidean space: *ab*, *bc*, *cd*, *de*, *ef*, *fg*, *ga*. These lines in Euclidean space correspond to an interval – an open set of points, the closure of which is the points corresponding to 0-dimensional cells. For example, a 1-dimensional cell *ab* is bounded by 0-dimensional cells (corresponding to points in the two-dimensional Euclidean space) *a*,*b*. One-dimensional cells have no thickness. The attributes of each line are the coordinates of its boundary points. The pieces of the plane correspond to 2-dimensional cells *abg*, *bcfg*, *cdf*, *def* in the Euclidean space. The sets of points that form pieces of the plane are open. 2-dimensional cells are bounded by the corresponding 1-dimensional and 0-dimensional cells. The sets of points corresponding to 1-dimensional and 0-dimensional cells, bounding the 2-dimensional cells, are the closures of the point sets of these 2-dimensional cells. For example, a 2-dimensional cell *abg* is bounded by 1-dimensional cells *ab*, *bg*, *ga* and 0-dimensional cells *a*, *b*, *g*. The attributes of each piece of plane are the brightness value of its points, the coordinates of the bounding points and lines.

The discretized image can be represented as a Cartesian two-dimensional cell complex [10]. Each coordinate axes can be considered as a sequence of 0-cells and alternating 1-cells in the space where the image is presented (Fig. 2b). 0-cells are assigned to the points of intersection of the grid lines with the axis, 1-cells are assigned to the

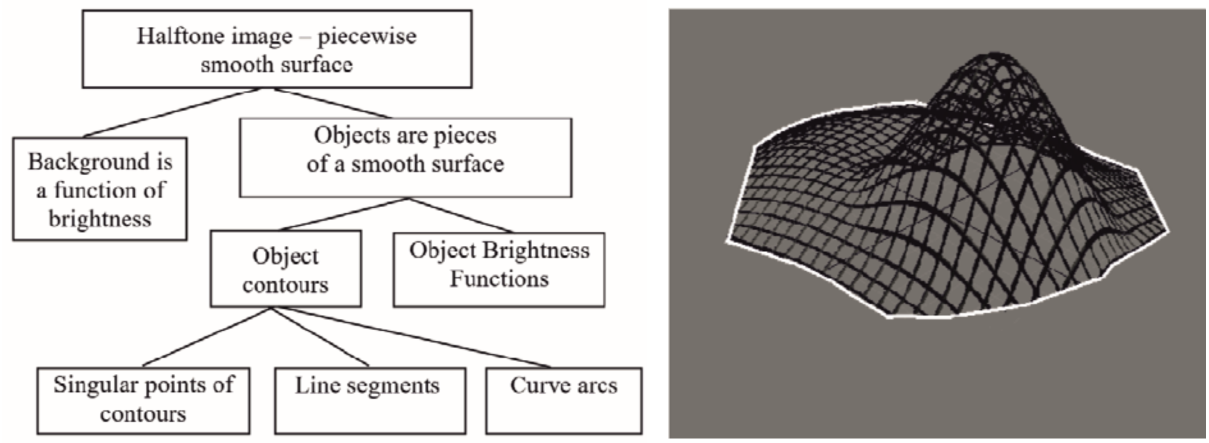


Figure 1 – The structural model of halftone image

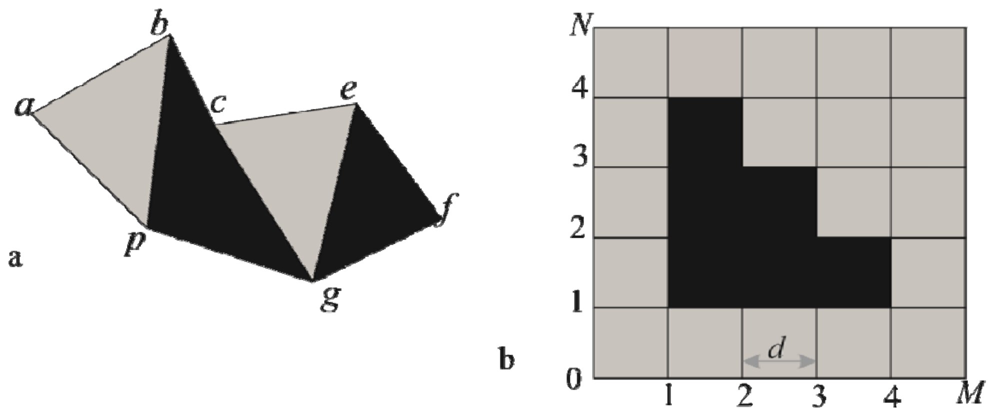


Figure 2 – Image as a: a – cell complex; b – Cartesian cell complex


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segments between adjacent points of intersection of the grid lines with the axis. The set of cells of a two-dimensional Cartesian complex is the Cartesian product of sets of axes cells. This means that a cell of the $m$-dimensional Cartesian complex $C_m$ is an $m$-tuple of axis cells. The bound ratio in $C_m$ is derived from the boundary ratios of the axes. The dimension of the cell $C_m$ is the sum of dimensions of its factors. 2-dimensional cells or grid cells correspond to pixels.

In the object in an image description the attributes of 0-cell are its coordinates, the attributes of 1-cell are its initial and final coordinates. The attributes of a 2-cell are the coordinates of its boundary 0-cell closest to the origin, as well as the brightness value of the corresponding image pixel. Each 2-cell corresponding to an image pixel is bounded by four 1-cells – cracks and four 0-cells – dots. Each 1-cell is bounded by two 0-cells. The most important feature of the object on an image is its boundary – contour. Contour of the object, when presented as a cellular complex, is a closed sequence of 0-cells and 1-cells. Fig. 2b shows the object of the halftone image and the contour of the object in the form of sequence of 0-cells and 1-cells. So, the contours are represented by lines without thickness. Thus, the description of a halftone image as a Cartesian cell complex contains sets of 0-cells, 1-cells, and 2-cells. These sets can be represented in computer memory as separate arrays.

Populations of neurons [11] have been found in the striate cortex, whose neuronal responses form the contours of figures in the visual field if their receptive fields correspond to the boundary segments of contrasting objects. The properties of these neurons' responses correspond to properties of 1-dimensional cells of cell complexes. These neurophysiological studies were carried out completely independently of mathematical work in the field of image description by methods of discrete topology.

The functioning of neurons comparable to 1-dimensional cells is described below. Most cortical cells (neurons) respond poorly to diffuse illumination and respond well to contrasting borders with a suitable orientation. For an arbitrary figure contrasting with the background, such a cell will respond if and only if a boundary segment with a certain orientation intersects its receptive field. "The same cells, whose receptive fields are located inside the boundaries of the figure, will not react in any way – they will continue to give a spontaneous impulse discharge regardless of the presence or absence of this figure. However, to excite a simple cell, it is not enough that the boundary section corresponds to the optimal orientation – the contour must also almost exactly hit the edge of the inhibitory and excitatory zones of the receptive field, because it is necessary for the answer that the light falls on the excitatory zone, but does not spread to the inhibitory one. If you the section of the contour was shifted even slightly without changing its orientation, the stimulation of this cell will turn out to be insufficient, and now another population of simple cells will begin to excite". It is known that each point of the visual field corresponds to a set of neurons (cells) with different orientational selectivity. The cells whose receptive fields and orientation coincide with the contour of the figure will answer to stimulus. The set of responses of such cells completely describes the contour of the figure. The following is also noteworthy: the output signals of cells whose receptive fields coincide with the contour of the figure and are oriented accordingly correspond to sections of the contour, one-dimensional segments without line width. That is, a material object – a neuron and its receptive field, corresponding to a part of the image, put in correspondence with an intangible object of a straight line segment – an element of the contour. Each of these contour elements can be considered as a 1-dimensional cell of the cellular complex. Here is another quote from [11]: "Another type of cells is found in the striate cortex. Typically, simple and complex cells are characterized by spatial summation – the longer the stimulus line, the better the response. However, the response only intensifies until the length of the line reaches the size of the receptive field: further lengthening of the line does not lead to a more vigorous response. In contrast, in cells that respond to line ends (end stopped cells), lengthening the line to a certain limit continues to improve the response, and if the line goes beyond this limit (in one or both directions), then the response weakens. Some cells, which we call "completely end stopped cells", do not respond at all to the presentation of a stimulus in the form of a long line. The zone from which a cell response can be elicited is called the activation zone (or excitatory zone), and the zones located at one or both ends are called inhibition zones (inhibitory zones). Thus, the entire receptive field of such a cell consists of an excitatory zone and an inhibitory zone (or zones) at the edges. A stimulus of optimal orientation, activating a cell from the excitatory zone, causes maximum inhibition outside this zone (on one or both sides)." From the standpoint of representing an image as a cellular complex, the above example can be considered as an experimental confirmation of the implementation of 0-dimensional cells in the visual system of a living organism.

Consider the mathematical definitions of the curve arc. Usually the objects contours are presented for further analysis as a closed sequence of curves arcs. The concept of a curve arc (meaning a continuous curve) is used in various fields of science and technology, in particular, recently in the processing of visual information. The concept of a continuous curve is one of the concepts that seems intuitively simple, but is actually very difficult to define. The greatest mathematicians defined the continuous curve in various ways at different periods in the development of this field of knowledge. Each new definition proceeded from the needs of human practical activity and the mathematical knowledge level of the corresponding era. The most modern definitions closely related to set theory are as follows [12].

Definition (according to Jordan). A plane curve is a set of points in a plane whose coordinates are determined by two equations:


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$$x = \varphi(\tau),$$
$$y = \psi(\tau),$$

where $\tau$ is defined on some segment $[l,u]$. The choice of the segment values does not violate the generality of the definition.

Definition (according to Cantor) [13]. A curve in a plane is any connected, compact set P of points in the plane that does not contain any interior point.

Definition (according to Urysohn) [14]. A curve is a one-dimensional connected and at the same time compact set.

The following difficulties occur when using the above definitions while processing the visual information.

1. It is assumed in the given definitions of continuous curves that the exact boundaries of the segment or data set to be examined for suitability or non-suitability with these definitions are known. But this part of the total data set that corresponds to one or another curve arc is not known in advance, when processing signals and visual information. This part of the data set can only be formed during processing. For example, when processing an image, it is not known which part of pixels belongs to the object of interest, or to its boundaries. Often it is obtained as the result of image processing.

2. It is essential to consider the difference between curve arcs and line segments when processing visual information. This difference is not provided in the above definitions.

3. The above definitions are found and fulfilled for continuous curves in a continuous Euclidean space. But in the modern view, visual information, signals, as well as visual perception, are inherently discrete. That is, we are talking about a discrete space, where each point is isolated.

4. Traditionally, the points of a discrete curve arc in a real image are represented by pixels that have real dimensions, while the points of mathematical curves refer to infinitesimal values.

It should be recognized that another definition is required for the visual information and signals processing, taking into account the above considerations, while fully recognizing the great scientific and practical significance of the known definitions of a continuous curve.

## 3 MATERIALS AND METHODS

In general, the task of object contour recognition as finding simply connected closed sequence of 0-dimensional and 1-dimensional cells includes processing of grayscale image using variable resolution. The presentation of this problem and its solution are beyond the scope of this work. At the same time, the solution of this problem for a binary image is given in [15]. In the same way, the construction of an object contour for a large number of halftone images is reduced to the mentioned problem if it is possible to binarize a halftone image using a specially selected threshold. We will assume that for the object of interest in a grayscale image, a contour is constructed as a simply connected closed sequence of 0-dimensional and 1-dimensional cells. But 0-dimensional and 1-dimensional cells are elements of the contour, commensurate with the pixel, which in turn is not structural semantic, meaningful element of the contour.

As already mentioned, in accordance with the structural model of a halftone image, the contour is a simply connected closed sequence of structural elements – straight line segments and arcs of curve, which are formed from parts of a sequence of 0-dimensional and 1-dimensional cells. The task is to represent the object contour as a closed simply connected sequence of line segments and arcs of curve using the mentioned closed simply connected sequence of 0-dimensional and 1-dimensional cells as input data.

First of all, the simply connected closed sequence of line segments must be calculated for the sequence of 0-dimensional and 1-dimensional cells. Each segment corresponds to a certain simply connected part of the original sequence. The definition of line segments must be performed by 0-dimensional cells sequentially, starting from the first 0-dimensional cell of the sequence. Let $\{x_n,y_n\}$, $n=\overline{1,N}$ be a sequence of coordinates of 0-dimensional cells.

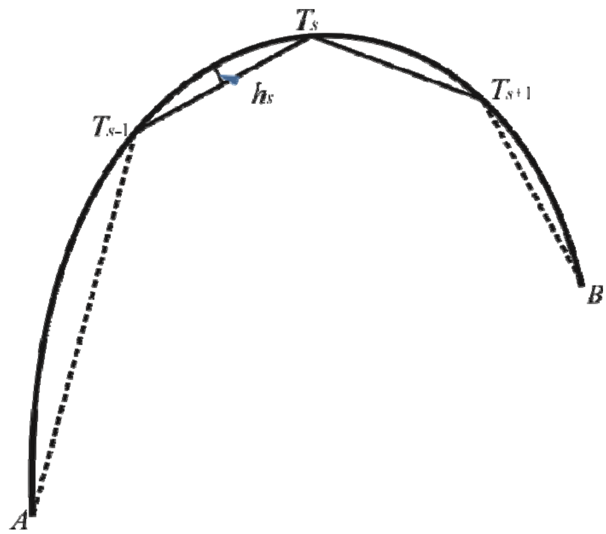


Figure 3 – The curve arc with an inscribe polyline

The possibility of representing the next part of a sequence of 0-dimensional cells $\{(x_b,y_b),\dots (x_t,y_t)\dots,(x_e,y_e)\}$, between the begin point $x_b,y_b$ and end point $x_e,y_e$, as a segment of a straight line with the begin point $T_b=(x_b,y_b)$ and end point $T_e=(x_e,y_e)$ , is determined by the condition [16] that

$$\max_{\forall(b\le t\le e)} dist(T_bT_e,(x_t,y_t)) \le d/2 ,$$

here $dist(T_bT_e,(x_t,y_t))$ – distance of the cell $(x_t,y_t)$ to a line segment $T_bT_e$. That is, the distance from any 0-dimensional cell belonging to segment $T_bT_e$ should not


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exceed half the length of a 1-dimensional cell. Consistent application of the above condition allows us to represent the entire sequence as a simply connected sequence of line segments or $\{(x_1,y_1),\ldots,(x_s,y_s),\ldots,(x_S,y_S)\}$ – a sequence of boundary common 0-cells (points) of adjacent line segments.Thus, the contour of the object is presented as a simply connected, closed sequence of digital straight segments.

The definition of the arc of a digital curve proposed below makes it possible to establish or reject the fact that a sequence of digital straight segments is received due to that some arc of the curve has been discretized. We will assume, that the arcs of the curve used in graphic images represent segments of smooth functions and correspond to Jordan curves. The arcs of an arbitrary curve [9], are given by the equations $x = \varphi(\tau)$, $y = \phi(\tau)$, without multiple points or simple arcs, that is, such that for any two different values τ' and τ′′ the corresponding points on the plane $[\varphi(\tau'), \phi(\tau')]$ and $[\varphi(\tau''), \phi(\tau'')]$ are different. Let $x = \varphi(\tau)$, $y = \phi(\tau)$, where the parameter τ defined on the segment $[l,u]$. As τ increases from $l$ to $u$, the point with coordinates $x$, $y$ describes the arc $AB$ (Fig. 4). Consider a partition of the segment $[l, u]$ by division points

$$l=t_0<\ldots\ t_{s-1} < t_s < t_{s+1}\ldots < t_S = u,$$

and let these points of division correspond to the points of the curve $A,\ldots, T_{s-1}, T_s, T_{s+1}, \ldots, B$.

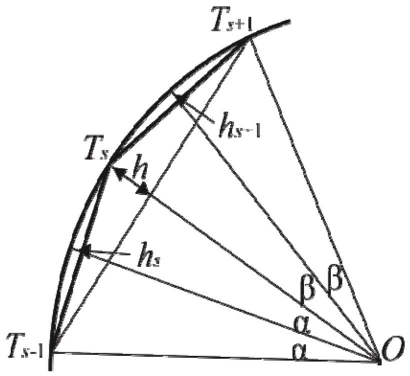


Figure 4 – The correspondence checking of straight line segments pairs to the definition of an curve arc

A polyline, inscribed in the arc $AB$, will be constructed if we connect successively point $A$ with point $T_1$, point $T_1$ with… point $T_{s-1}$, point $T_{s-1}$ with point $T_s$, point $T_s$ with point $T_{s+1}$, point $T_{s+1}$ with … point $B$ by segments of straight lines. The figure bounded by the segment of the polyline $T_s,T_{s+1}$ and the corresponding arc link $\cap T_s,T_{s+1}$ will be called the segment of the arc $T_s,T_{s+1}$, and the maximum length of the line between the segment $T_s,T_{s+1}$ and $\cap T_s,T_{s+1}$, perpendicular to the segment $T_s,T_{s+1}$ is the height of the arc segment $h_s$. Let be

$$\beta = \max_{s=0,1,\ldots,S-1} l(T_s,T_{s+1}).$$

If β tends to zero with a corresponding increase in $s$, then the length of any of the links of the inscribed polyline will tend to zero, as well as the height of each segment of the arc, due to the continuity of the functions $\varphi(t)$, $\phi(t)$.

While an arc and an inscribed polyline represent in a discrete space of discreteness $d$, segments of the inscribed polyline are displayed as line segments. Since the coordinate values take integer multiples of $d$ in a discrete space, then objects smaller than half the discreteness the heights of the segments, in particular, will not be displayed in this space, their lengths will become equal to zero, starting from the moment when $h_s < d/2$. So, the discrete mappings of the parts of the arc will coincide with the corresponding links of the inscribed polyline – segments of digital lines for $h_s < d/2$. Thus, the contour, which consists of straight segments and arcs of arbitrary curves, after discretization is defined as a sequence of digital straight segments. Sequences of digital straight segments that correspond to arcs of curves can be considered as polylines inscribed in these arcs of curves. Such inscribed polylines will be called arcs of digital curves. The contour can include both individual segments of straight lines, and sequences of such segments – polylines that are not arcs of digital curves.

Consider pairs of adjacent segments of digital lines in sequence. In general, many curves can be drawn through three points defined by a pair of line segments. Nevertheless, as already noted, the lengths of the arcs segments heights that correspond to the segments of the inscribed polyline should not exceed the value of the space discreteness $d/2$. Thus, in order to consider pairs of segments of digital lines $T_{s-1}T_s$, $T_sT_{s+1}$ as part of an arc of a digital curve, it is necessary to establish the existence of a curve that passes through the points $T_{s-1}$, $T_s$, $T_{s+1}$, such that the condition is satisfied: $(h_s<d/2)\&(h_{s+1}<d/2)$ (Fig. 4).

The curvature of a plane curve is usually identified with the curvature of a contacting circle [9]. The contacting circle of a plane curve at the point $T_s$ is the limiting position of the circle passing through two neighboring points $T_{s-1}$ and $T_{s+1}$ as $T_{s-1}$ and $T_{s+1}$ tend to $T_s$. We can formulate the following definition, based on the above considerations.

***Under the arc of a digital curve in a two-dimensional discrete space of discreteness d we mean such a sequence of straight line segments that through the three end points of each pair of adjacent segments it is possible to draw such a circle that the heights of the circle segments do not exceed d/2.***

This definition is valid to the extent that it is legitimate to identify a segment of an arc of an arbitrary curve that corresponds to a pair of neighboring segments with an arc of a contacting circle.

Having constructed a circle in accordance with the definition of the digital curve arc for the points $T_{s-1}, T_s, T_{s+1}$, let us estimate the distance of the common point $T_s$ of the pair of segments $(T_{s-1}, T_s),(T_s,T_{s+1})$ to the segment $T_{s-1}$, $T_{s+1}$, that is, the height $h$ (Fig. 4). The


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lengths of each segment in this pair cannot differ significantly, since this would contradict the smoothness condition – that is, $l(T_{s-1}, T_s) \sim l(T_s, T_{s+1})$. As already noted, the maximum distance between the points of the arc lines and the corresponding segment of the digital straight line is $h_s = h_{s+1} = d/2$. At the same time

$$h_s = OT_{s-1} - OT_{s-1} \times \cos\alpha = r - r\cos\alpha = r(1-\cos\alpha).$$
$$h = OT_{s-1} - OT_{s-1} \times \cos 2\alpha = r - r\cos 2\alpha = r(1-\cos 2\alpha) = 2r(1-\cos^2\alpha).$$

$h/h_{s-1} = 2(1+\cos\alpha)$; or $h = 2(1+\cos\alpha) \times h_{s-1}$. If $h_{s-1} \approx d/2$ and $\alpha \le 10°$, that is $\cos\alpha \approx 1$, then the height of the triangle $(T_{s-1}T_s\ T_{s+1})$ $h \approx 2d$. Using the value $h_s$ instead of $h_{s-1}$ will not affect the result, since both $\beta \le 10°$ and $\cos\beta \approx 1$.

This means that in order to be related to the digital arc of curve for the considered pair of segments, it is necessary that the value of the maximum deviation of $h$ does not exceed $2d$. The minimum deviation is $h > d/2$, since at a smaller deviation the directions of the segments $T_{s-1}T_s$ and $T_sT_{s+1}$ are indistinguishable, and a pair of segments of different directions turns into one straight segment. If $h > 2d$, then the segments under consideration are segments of a polyline. Thus, taking into account the above considerations, the sequence of common points of adjacent segments takes the form: $\{(x_1,y_1),\ldots,(\underline{x_{s-1},y_{s-1}}),(\underline{x_s,y_s}),(\underline{x_{s+1},y_{s+1}}),\ldots,(x_S,y_S)\}$, where the points belonging to the digital curve arc are underlined.

## 4 EXPERIMENTS

Experimental verification of the proposed method consists in representing the contour of the interest object as a sequence of digital curve arcs and segments of digital straight lines. Moreover, the contour elements sequence of the object must be the same for various affine transformations – the rotation of the interest object, changing the position in the image field. For comparison, the representation the contour of the interest object as a sequence of digital curve arcs and segments of digital straight lines was performed using a well-known tool – the graphical editor Corel Draw.

For the experiment, binary images of object contours that were not distorted by noise were used, since noise filtering, as well as recognition of the object contours in a grayscale image, are separate tasks that must be solved by appropriate means. Separate works will be devoted to solving these problems.

An example of the image used in the experiment is shown in Fig. 5. Objects in the image are the identical sectors of the ellipse, differing in space position and angle of rotation. The result of processing each object is its contour, represented by a closed sequence of line segments in the form of boundary points (0-cells) of adjacent line segments. Some parts of the line segments sequence are defined as arcs of curves with indication of their boundary points (0-cells).

Figure 5 – The example of the image for the experiment to determine the contour as a sequence of curve arcs and line segments

The experimental program was executed in the Visual C environment. The required RAM is no more than 512 MB.

The main blocks of the program:

1. Represent the image as a cell complex and determine the sequences of boundary 0-cells and 1-cells for each object.
2. Define the contour of each object as a sequence of line segments in the form of a sequence of boundary points of neighboring segments.
3. Determine for the contour of each object the parts of the line segments sequence that form the arcs of the curves, indicating the boundary points of the arcs of the curves. The use of curve arcs as the structural elements of image contours description would approach its description to intuitional, natural representation of images by a man, substantially would shorten the expenses of memory for storage of image and image processing time. As an example we will consider description of contours of binary images which are got with the use of tools of wide-spread graphics editor of Corel Draw. On Fig. 5a the contours of three identical objects are represented, not to be distorted by noises. Each of objects contains the arc of ellipse and differs from the other objects by spatial position and rotation angle. The boundary points which divide contours into the curve arcs and the straight segments are marked by the squares. Identical with each other arcs, to belong to different objects, are represented by sequences containing the different amount of different arcs of curves. Each of identical objects in the image is represented with the different elements. Such description of objects can not be directly used in the intelligence systems for interpretation of images, as supposes enough hard processing. The represented example shows existence of the problem even at the images not distorted by noises and actuality to solve the problem.A special program that implements the proposed method and algorithms has been developed.

The example of contour recognition as the sequences of digital straight segments and of digital curve arcs by the program is demonstrated on a Fig. 6b. The image is used from the Fig. 6a, but the offered algorithms are implemented in the program. Unlike the contours of Fig. 6a, got by means of the program of Corel Trace, the arcs of contours are represented without laying out by intermediate points on a few arcs regardless of different spatial positions and rotation angles for the each of objects.


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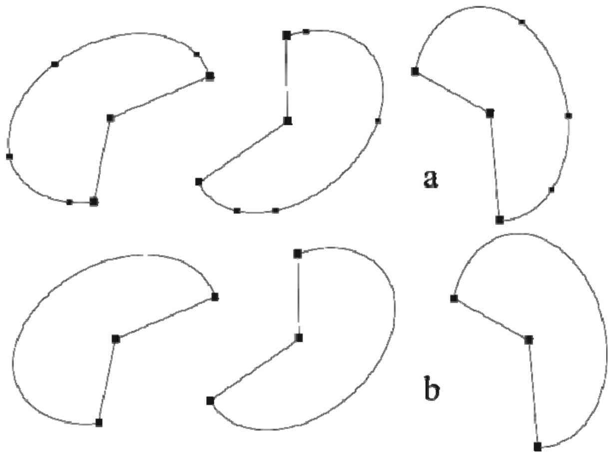


Figure 6 – a – Contour recognition in three identical, rotated in relation to each other objects by facilities of Corel Draw;
b – Contour recognition by the program, using the proposed method and algorithms

## 5 RESULTS

The main result, apparently, should be considered the ability to determine the arcs of curves in sequences of 0-cells and 1-cells that form the contour of an object. The same contour configurations must correspond to the same sequences of curve arcs, regardless of affine transformations – shift, rotation angle, scale.

The significance of the result obtained is clear only when comparing the processing of the same object by known and proposed methods. The result of the experiment is shown in Fig. 6. The contours of objects from Fig. 5, recognized by means of the Corel Draw graphic editor are shown in Fig. 6a. Each of objects contains the arc of ellipse and differs from the other objects by spatial position and rotation angle. The boundary points which divide contours into the curve arcs and the straight segments are marked by the squares. Identical with each other arcs, to belong to different objects, are represented by sequences containing the different amount of different arcs of curves. Each of identical objects in the image is represented with the different elements. The arc of each ellipse is represented by several unequal arcs. Such representation of processing results cannot be used in artificial intelligence tasks, in particular, in recognition tasks.

The proposed method is free from these defects. Recognition of the same objects by the developed experimental program is shown in Fig. 6b

## 6 DISCUSSION

The representation of image within the framework of the structural model, on the one hand, is natural for visual perception, on the other hand, it is fully consistent with the theory of cell complexes. The image object is a 2-dimensional cell. The contour of the object, its boundary, is, most often, a simply connected, closed sequence of 1-dimensional cells that form segments of straight lines and arcs of curve. The boundaries of line segments and arcs of curve, i.e. singular points correspond to 0-dimensional cells. Compared to recent imaging concepts, an important advantage to using cell complexes in image processing is the following [16].

One of the simplest tasks of image processing is to encode the object contour of the binary image as a single-connected closed sequence of the object boundary elements in the image. The contour of the object is a closed curve that divides the image into two parts: the object itself and the other part of the image. Traditionally, for a binary discretized image, contour pixels bounding with background pixels or, conversely, background pixels bounding with object pixels are used as contour elements. In order to construct a single-connected closed sequence from boundary pixels that corresponds to the contour of the image object, the concept of connectivity – the pixels neighborhood – must be defined. The following ideas about the connectivity of pixels in two-dimensional discrete space are generally accepted:

1. Pixels are considered adjacent if they have a common side. In this case, each pixel has four adjacent pixels.
2. Pixels are considered adjacent if they have a common side or a common point. In this case, each pixel has eight adjacent pixels. Examples of closed lines and contours of objects formed by boundary pixels are shown in Fig. 6. As follows from the above examples, this representation of contour has significant disadvantages [13].

Fig. 7a shows a closed line drawn according to the rules of 4-neighborhoods. As a result of the use of 4-neighborhoods, the contour line divides the image field not into two areas, as it should be, but into three.

A closed simply connected line drawn according to the rules of 8-neighborhoods is shown on Fig. 7b. But, due to 8 neighborhoods, the space inside the closed line and outside of it is not divided: there are connections between pixels inside and outside the line.

Fig. 7c shows an attempt to construct a closed contour of the object using the boundary pixels adjacent to the pixels of the object according to the rules of 4-neighborhoods: the contour line is not closed.

Fig. 7d illustrates the case of constructing a closed contour of an object using boundary pixels adjacent to the pixels of the object according to the 8-neighborhood rules: the contour line is not simply connected.

A curved line in continuous space, as follows from its definition, has no thickness. That is, each of the infinitely large set of points that form a line is an infinitesimal value. This also applies to the closed curve of the object contour line (boundary). At the same time, traditionally used representations of a curved line in a discrete space assume that the curve consists of minimal elements of this space – points. But the point in this case corresponds to a pixel – the minimum element of the image that has finite dimensions. It is this difference that is the reason for the above paradoxes of representing lines as a sequence of pixels. That is, the correct representation of initially continuous images in discrete space is possible using the theory of cell complexes.


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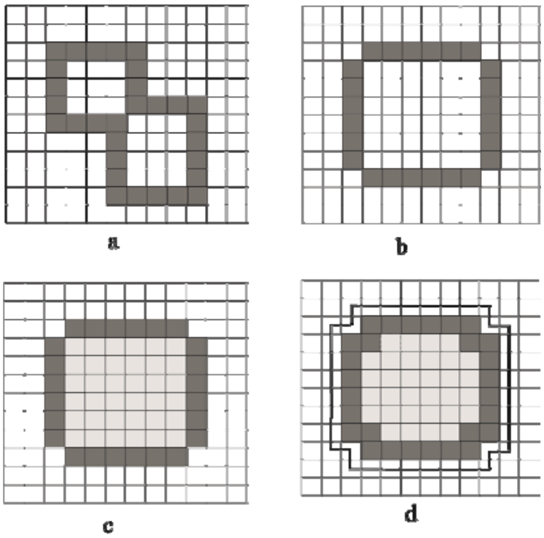


Figure 7 – Closed lines formed by pixels: problems and paradoxes

In the primary visual (striate) cortex, neurons were found that generate signals – responses to extended pieces at the border of contrast areas of the visual field, and those and only those neurons are excited whose receptive fields match the border and their orientation match the orientation of the corresponding sections of the contrast area border. That is, the excited neurons respond to the boundary segments of the straight line in certain orientations, which can be considered as 1-cells if the contrast area is considered as a cellular complex. That is, the representation of an image as a cellular complex can be considered as an approximation to the implementation of the mechanisms of visual perception. It can also be assumed that the signals of a curved line are formed by pairs of segments of the corresponding directions, as proposed in this paper.

## 6 CONCLUSIONS

The paper considers the arc of the curve as a structural element of the image, more precisely as a structural element of the interest object contour in the image, and the image is presented as a cellular complex.

**The scientific novelty** is that the arc of a digital curve is defined in the discrete space of a digital image, in contrast to the known definitions of continuous curves, which are oriented to use in a continuous space.

**The practical significance** is that the interest objects contours are presented as sequences of line segments and arcs of digital curves. This representation of the object contour does not depend on affine transformations, such as position in the field of view and rotation, which greatly simplifies image processing.

**Prospects for further research** are as follows. The successful result of object contour recognition in the form of a sequence of straight line segments and arcs of a digital curve depends on the choice of *d* – the resolution value when sampling the image. The task of determining the most appropriate resolution for a particular image has not been solved. It is all the more possible that different parts of the same image must be processed at different resolutions. Therefore, the recognition of line segments and arcs of curves using variable resolution will be considered in subsequent publications.

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## ДУГА КРИВОЇ ЯК СТРУКТУРНИЙ ЕЛЕМЕНТ ЗОБРАЖЕННЯ, ЩО МАЄ БУТИ РОЗПІЗНАНЕ

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### АНОТАЦІЯ

**Актуальність.** Пропонована стаття стосується галузі обробки візуальної інформації в комп'ютерному середовищі, а саме визначення параметрів об'єкта інтересу на зображенні, зокрема контуру об'єкта інтересу. У більшості випадків контур об'єкта інтересу є однозв'язна послідовність дуг кривих.

**Мета.** Мета і предмет дослідження – знайти і запропонувати таке визначення дуги цифрової кривої, як найважливішого елемента контуру об'єкта в розпізнаваному образі, яке не суперечить сучасним нейрофізіологічним уявленням про зорове сприйняття, і розпізнати контур об'єкта як послідовність дуг цифрових кривих.

В якості **методу** використовується подання зображення у вигляді структурної моделі, одним із структурних елементів якої є контур об'єкта, що складається з цифрових дуг кривих. Також зображення розглядається як клітинний комплекс, що відповідає сучасним уявленням про зорове сприйняття людини.

**Результати.** Запропоновано нове визначення дуги цифрової кривої як послідовності відрізків цифрових прямих, що не суперечить сучасним уявленням нейрофізіології. На відміну від відомих визначень дуги кривої, запропоноване визначення дуги цифрової кривої дає можливість визначити початкову та кінцеву точки дуги. За описом контуру об'єкта як однозв'язної замкнутої послідовності відрізків пропонується побудувати опис контуру як послідовності дуг цифрових кривих.

**Висновки.** Використання запропонованого визначення дуги цифрової кривої при обробці зображень дає змогу розпізнати контур об'єкта на зображенні та представити його у формі, наближеній до зорового сприйняття. Для досягнення найкращих результатів рекомендується використовувати змінну роздільну здатність в алгоритмах обробки зображень.

**КЛЮЧОВІ СЛОВА:** зображення, контур, дуга кривої, відрізок прямої, клітинний комплекс, нейрони, рецептивне поле.

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