

ON THE RECURSIVE ALGORITHM FOR SOLVING THE TRAVELING SALESMAN PROBLEM ON THE BASIS OF THE DATA FLOW OPTIMIZATION METHOD

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ABSTRACT

Context. The article considers a technique for the sequential application of flow schemes for distributing a homogeneous resource for solving the traveling salesman problem, which is formulated as the problem of finding a route to visit a given number of cities without repetitions with a minimum duration of movement. The task of formalizing the algorithm for solving the traveling salesman problem by the method of streaming resource distribution using the backtracking scheme is posed. The use of Orlin's method to optimize the flow distribution on the graph is proposed.

Objective. The goal of the work is to develop an algorithm for solving the traveling salesman problem based on the implementation of the method of streaming resource distribution and the backtracking scheme with the minimum duration of movement along the route.

Method. This paper proposes a method for solving the traveling salesman problem by the method of streaming resource distribution with the backtracking scheme. A scheme for formalizing the procedure for solving the traveling salesman problem with the minimum duration of movement along the route is described. A variant of accelerating the speed of the developed algorithm is proposed, which consists in using a greedy technique in the procedure for selecting route sections: planning each subsequent stage of movement is determined based on the choice of the fastest direction of movement. The results of the proposed algorithm for calculating solutions to the traveling salesman problem with minimization of the duration of movement are presented, the obtained solutions are compared with the solutions found by other exact and heuristic methods.

Results. The method for solving the traveling salesman problem using the method of streaming resource allocation and using the backtracking scheme is developed. A variant of accelerating the speed of the developed algorithm is proposed, which consists in using a greedy technique in the procedure for selecting route sections: planning each subsequent stage of movement is determined based on the choice of the fastest direction of movement. The application of the greedy approach makes it possible to obtain a constructive scheme for solving the traveling salesman problem. The results of the proposed algorithm for calculating solutions to the traveling salesman problem with minimization of the duration of movement are presented, the obtained solutions are compared with the solutions found by other exact and heuristic methods.

Conclusions. The paper considers a method for formalizing the algorithm for solving the traveling salesman problem using the method of streaming resource allocation and the backtracking scheme. The use of Orlin's method to optimize the flow distribution on the graph is proposed. The scheme of formalization of the procedure for using the method with the implementation of the backtracking scheme for solving the traveling salesman problem with the minimum duration of movement along the route is briefly described. A variant of accelerating the speed of the developed algorithm is proposed.

KEYWORDS: traveling salesman problem, resource allocation method, recursive backtracking scheme, greedy approach.

ABBREVIATIONS

TSP is a traveling salesman problem.

NOMENCLATURE

t is cyclic permutation of numbers;

j_1, \dots, j_n are different city numbers;

n is a number of cities;

c_{ij} are the travel time between all pairs of vertices;

C is a matrix of c_{ij} , $i, j = \overline{1, n}$;

i, j, k are the indexes;

I is the set of vertex indices;

X is a binary matrix of transitions between vertices

x_{ij} , $i, j = \overline{1, n}$;

x_{ij} are the elements of matrix X , which equal to 0 or 1;

$G(V, E)$ is a graph;

V is a non-empty set of vertices;

E is a set of edges;

v_i is a vertex of graph, $i = \overline{1, N}$;

N is a number of vertices of the graph;

e_i is a edge of graph, $i = \overline{1, M}$;

M is a number edges of the graph;

V_s is a subset of initial nodes of the graph;

V_p is a subset of intermediate nodes of the graph;

V_e is a subset of final nodes;

$F(i)$ is a subset of edges of the graph coming out of the i -th vertex;

e'_i time distribution coefficients, $i = \overline{1, K}$;

K is a the power of subset vertices V_d ;

V_d is a set that unites sets V_s and V_p ;

$f(\cdot)$ is a criterion for optimization;

J_k is a power of $F(k)$;

N_1 is a power of V_s ;

N_2 is a power of V_p ;

N_3 is a power of V_e ;

$t_k(\cdot)$ is a element of vector-function that determines the amount of time spent on moving to the k -th vertex of the graph, $k = \overline{1, N_3}$;

H^{in} is a matrix for the forward flows of dimension $N \times M$;

H^{out} is a matrix for backward flows of dimension $M \times N$;

S^r is a incidence matrix $N \times N$ for a path of multiplicity r ;

r is a number of edges through which there is a path from vertex v_i to vertex v_j ;

$w'_j(\cdot)$ is a element of vector-function of dimension N that determines the amount of the time resource formed by the initial vertices;

$\gamma_R^j(\cdot)$ is a element of the vector-function of dimension N ;

$\gamma_1^m(\cdot)$ is a element of the vector-function of dimension N ;

$\beta(\cdot)$ is a matrix-function;

Q is a vector of dimension N_3 whose elements determine the numbers vertices of the final points of the movement.

INTRODUCTION

Recently, most global companies have experienced disruptions in logistics caused by the pandemic and the war in Ukraine. Due to the sanctions and events related to the pandemic, managers of logistics companies have experienced serious disruptions in determining the routes and volumes of transportation, as the mentioned processes have exposed the weaknesses of the traditional existing supply chains in logistics.

A lack of vertical vision of manufacturing processes and connections, outdated demand management processes, insufficient resilience to changes in demand, and unexpected disruptions due to reliance on manual efforts in logistics operations have disrupted the supply chain.

Logistics companies are forced to analyze their logistics processes. It is clear that changes in customer behavior and expectations are unlikely to address these unexpected logistics challenges, as shoppers expected faster delivery and easier product tracking.

It is becoming clear that companies need to quickly optimize their logistics management. Depending on the task at hand, there are many different mathematical approaches to various logistical problems, such as linear programming, network optimization, decision analysis, genetic algorithms, and so on.

Logistics problems experience their own difficulties, some of which are solved thanks to the work of the management department, while others involve the analysis and optimization of logistics operations, including planning, coordination and control of the movement and storage of goods, services and information, optimization of network flows [1–3]. Simulation modeling methods and models allow you to create computer models of a logistics system and use them to test various scenarios and optimize system performance.

Attracting mathematical approaches to solving logistics problems is becoming widespread, the specific content of which depends on the nature of the problem and the available data. Sometimes it is possible to find atypical methods for solving known problems, one of which is the traveling salesman problem.

The object of study is the process of optimal route search for the traveling salesman problem with a minimum duration of movement.

The subject of study is the development of the efficient algorithm for solving for solving the traveling salesman problem by the method of streaming resource distribution using the backtracking scheme.

The purpose of the work is to develop an algorithm for solving the traveling salesman problem based on the implementation of the method of streaming resource distribution and the backtracking scheme with the minimum duration of movement along the route

1 PROBLEM STATEMENT

According to the content of the traveling salesman problem (TSP, Traveling Salesman Problem), it is necessary to create a route of movement within a given set of interconnected points (bridges) that form the transport network of a particular region [4]. A feature of the problem is that the route must contain all the points specified in the task, and each of the points must be visited no more than once. It is clear that such trips take a lot of time, so it is logical that it is necessary to plan the route in such a way that the distance to be covered, or the time to overcome it, is minimal (finding the path with the least cost can also be considered as a criterion).

The traveling salesman problem is a combinatorial problem that can be solved using mathematical programming methods. To reduce the problem to a general form, we number the cities by numbers $(1, 2, 3, \dots, n)$, and describe the traveling salesman's route by a cyclic permutation of numbers $t = (j_1, j_2, \dots, j_n, j_1)$, where all j_1, \dots, j_n are different numbers. The number j_1 , repeated from the very beginning and at the end, shows that the permutation is cyclic [5].

The set of cities can be considered as the vertices of some graph with given distances (or travel time) between

all pairs of vertices c_{ij} that form the matrix $C=(c_{ij})$, $i,j=\overline{1,n}$. We assume that the matrix is symmetric. The formal problem then is to find the shortest route (in time or length) t that goes through each city and ends at the starting point. In this formulation, the problem is called the closed traveling salesman problem, which is a well-known mathematical integer programming problem.

Let us formulate a mathematical model of the TSP problem. Let $I=\{1,\dots,n\}$ be the set of vertex indices of the problem graph. The objective function is the total distance or time of the route, including all the vertices of the task graph. The parameters of the problem are the elements of the matrix $C=(c_{ij})$, $i,j \in I$.

Shift tasks are elements of the binary matrix of transitions between vertices $X=\{x_{ij}\}$, $i,j \in I$, which are equal to 1 if there is an edge (v_i, v_j) in the constructed route for the task, 0 otherwise [6]. The shortest route in terms of distance or time is optimal:

$$\sum_{i \in I} \sum_{j \in I, j \neq i} c_{ij} x_{ij} \rightarrow \min \quad (1)$$

with constraints

$$\begin{aligned} \sum_{j \in I, j \neq i} x_{ij} &= 1, \quad i \in I, \\ \sum_{i \in I, i \neq j} x_{ij} &= 1, \quad j \in I, \end{aligned} \quad (2)$$

$$v_i - v_j + nx_{ij} \leq n - 1, \quad 1 \leq i \neq j \leq n.$$

The last inequality ensures the connectivity of the vertex traversal route; it cannot consist of two or more unconnected parts.

2 REVIEW OF LITERATURE

Algorithms that allow solving the problem of finding the optimal route are divided into exact and heuristic. Exact methods guarantee finding the optimal solution to the problem in a certain time or taking into account certain resource constraints. In this case, the search for solutions is based on optimization methods such as linear programming, dynamic programming, or the branch and bound method [7]. However, it is expedient to use exact methods only for small-scale problems (for example, for the purpose of primary design of a small-sized transport network), since their implementation requires large computing power.

On the other hand, heuristic methods are algorithms that do not guarantee finding an optimal solution, but are aimed at quickly finding a locally optimal solution. Traditionally, "trial and error" approaches, such as random search or greedy algorithm, are used to quickly explore the solution space and find a promising solution [8]. Heuristics are more flexible and can be applied to larger problems, but the solution they offer may not be optimal. Among such heuristic methods, attention should also be

paid to methods that imitate biological (ant colony algorithm and genetic algorithm [9, 10]) or physical processes (imitation annealing [11]).

3 MATERIALS AND METHODS

When forming a route, it is necessary to pay attention to the fact that each subsequent stage of movement can be chosen based on the consistent use of methods for optimizing the distribution of a homogeneous resource, one of the most effective among which is the Orlin method [12]. Then the problem of this study can be formulated by formalizing the algorithm for solving the traveling salesman problem using the method of streaming resource allocation and using a backtracking scheme.

Consider the application of the method for our problem. This method allows solving the problem of distribution of a homogeneous resource with intermediate points in the form of a directed graph without loops and parallel edges, given by a set of a non-empty set of vertices and a set of edges

$$E \subset \{v_i, v_j\} = \langle V, E \rangle, \quad G(V, E) = \langle V, E \rangle, \quad V \neq \emptyset, \quad v_i, v_j \in V, i \neq j \quad (3)$$

where $V = \{v_1, v_2, \dots, v_N\}$, $E = \{e_1, e_2, \dots, e_M\}$, N and M are the total number of vertices and edges of the graph, respectively. It is assumed that the set V of graph vertices $G(V, E)$ is represented by a set of non-intersecting subsets:

1. V_s – a subset of initial nodes (vertices) of the graph;
2. V_p – a subset of intermediate nodes (vertices) of the graph;
3. V_e – a subset of final nodes (vertices),

that is $V = V_s \cup V_p \cup V_e$, provided that $(V_s \cup V_p) \cap V_e = \emptyset$ and $|V_s| = N_1$, $|V_p| = N_2$, $|V_e| = N_3$, $N = N_1 + N_2 + N_3$, and the weight of the ribs is understood as the time to overcome the corresponding stage of the route.

Let's denote $V_d = V_s \cup V_p$. Then the weight of the edges from the set E emanating from the vertices of the subset V_d is determined by the value $E' = \{e'_1, e'_2, \dots, e'_K\}$ of the corresponding time distribution coefficients for the route, where $K - \emptyset$ the number of vertices of the graph $G(V, E)$ belonging to the subset V_d , i.e. $|V_d| = K = |V_s| + |V_p| = N_1 + N_2$. Let be $F(i) \subset E$ – a subset of edges of the graph $G(V, E)$ coming out of the i -th vertex, with $E = \bigcup_{i=1}^N F(i)$ and $\bigcap_{i=1}^N F(i) = \emptyset$.

Then the problem of optimal distribution of a homogeneous resource is the problem of determining the weight of edges emanating from the vertices of the subset V_d , taking into account the criterion

$$f(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K) = (-1) \sum_{k=1}^{N_3} t_k(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K) \rightarrow \min_{\substack{e'_1, \dots, e'_K \\ T_1, \dots, T_{N_1}}} \quad (4)$$

and the restriction on the distribution coefficients, which is given by the relation:

$$\sum_{j=1}^{J_k} e'^k_j = 1, \quad (5)$$

where $e'^k_j \in F(k)$, $0 \leq e'^k_j \leq 1$, $j = \overline{1, J_k}$, $J_k = |F(k)|$, $F(k) \subset E$, $k = \overline{1, K}$.

To find a solution of the optimization problem (4) in the form of a vector function $t(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K) = (t_1(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K), \dots, t_{N_3}(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K))$, the k -th element of which characterizes the time spent to move to the k -th vertex ($k = \overline{1, N_3}$), we introduce the notation. The indicative graph $G(V, E)$ will be specified in the form of incidence matrices for the forward H^{in} and backward H^{out} flows of dimensions $N \times M$ and $M \times N$, respectively, whose elements are defined as:

$$H^{in}_{i,m} = \begin{cases} 1, \text{ node } v_i \text{ is incident to edge } e_m \text{ and is its end} \\ 0, \text{ in opposite case} \end{cases} \quad (6)$$

$$H^{out}_{m,i} = \begin{cases} 1, \text{ node } v_i \text{ is incident to edge } e_m \text{ and is its beginning} \\ 0, \text{ in opposite case} \end{cases} \quad (7)$$

$$i = \overline{1, N}, m = \overline{1, M}.$$

For an indicative graph $G(V, E)$, we define a matrix S^r of dimension $N \times N$, which is the incidence matrix for a path of multiplicity r (r specifies the number of edges through which there is a path from vertex v_i to vertex v_j). The matrix S^r is defined by the equality:

$$S^r = \left(H^{in} (-H^{out})^T \right)^r \quad (8)$$

Consider a vector function $w(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K) = (w_1(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K), \dots, w_N(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K))$, the j -th element of which determines the amount of time spent on moving to the j -th vertex of the graph $G(V, E)$, $j = \overline{1, N}$:

$$w_j(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K) = w'_j(T_1, \dots, T_{N_1}) + \left[\sum_{p=1}^N \left(H^{in} * \text{diag}(\gamma_R(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K)) * (-H^{out})^T \right) \right]_j \quad (9)$$

where the sum in the second term is taken over all N elements of the j -th row of the matrix

$$H^{in} * \text{diag}(\gamma_R(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K)) * (-H^{out})^T,$$

the vector function $w'(T_1, \dots, T_{N_1}) = (w'_1(T_1, \dots, T_{N_1}), \dots, w'_N(T_1, \dots, T_{N_1}))$ of dimension N determines the amount of the time resource formed by the initial vertices of the graph $G(V, E)$, the vector function

$\gamma_R(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K) = (\gamma^1_R(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K), \dots, \gamma^N_R(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K))$ of dimension N , the elements of which are calculated recursively by the formula

$$\gamma^m_R(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K) = \gamma^m_1(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K) + \left[\sum_{q=1}^M (\beta(e'_1, \dots, e'_K) H^{in} \text{diag}(\gamma_{r-1}(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K))) \right]_m \quad (10)$$

$r = \overline{2, R}$, $m = \overline{1, N}$, and the sum in the second term is taken over all M elements of the m -th row of the matrix $\beta(e'_1, \dots, e'_K) H^{in} \text{diag}(\gamma_{r-1}(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K))$.

In the recursive expression (10), the initial values of the elements of the vector function $\gamma_1(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K)$ and the elements of the matrix function $\beta(e'_1, \dots, e'_K)$ are determined by relations:

$$\begin{aligned} \gamma^m_1(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K) &= \\ &= \left[\sum_{p=1}^N (\beta(e'_1, \dots, e'_K) * \text{diag}(v'(T_1, \dots, T_{N_1}))) \right]_m, \\ & \quad m = \overline{1, N}; \\ \beta(e'_1, \dots, e'_K) &= [(-H^{out}) * \text{diag}(E'(e'_1, \dots, e'_K))]^T. \end{aligned} \quad (11)$$

Then the elements of the vector function $w(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K)$ determine the elements of the original vector function $t(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K)$, which is the solution of the optimization problem (4):

$$t_k(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K) = w_{Q_k}(T_1, \dots, T_{N_1}, e'_1, \dots, e'_K), \quad (12)$$

where Q is a vector of dimension N_3 whose elements determine the numbers of the graph $G(V, E)$ vertices that make up the subset V_e of the final points of the movement, $k = \overline{1, K}$.

It is clear that such a search for a route involves the use of a solution technique with return (backtracking) [13]. The solution of the problem based on the use of backtracking is reduced to a consistent expansion of a particular solution. If the expansion fails at the next step, then a return to a shorter particular solution occurs and the search continues further in a new direction. This algorithm allows you to find all solutions to the problem, if any. It is known that the use of algorithms based on the backtracking scheme in solving practical problems is significantly limited by the low speed of operation and puts forward high requirements for computing resources. To speed up the work of the method, they try to organize calculations in such a way as to identify non-optimal options as early as possible, or use a selection scheme based on a greedy approach when constructing each step. This can significantly reduce the time to find a solution.

The greedy approach is formulated in accordance with the principle of choosing the optimal solution at every step, despite previous steps or those taken ahead. In other words, the greedy technique is based on a locally optimal choice with the hope that this choice will lead to a globally optimal solution.

It should be noted that there is no way to check the quality of the application of greedy algorithms in solving a specific applied problem, however, for problems in which the sequence of local optima goes to the global optimal solution, this approach is very promising.

The greedy method proposed by the authors assumes consideration at each stage of the formation of the route of the fastest in time section of the route of movement. A combined approach based on the method of resource allocation and greedy choice of the direction of movement made it possible to implement a constructive scheme for solving the traveling salesman problem, that can be formulated as the following *recursive algorithm* for a network of N nodes and a given travel time for each pair of vertices. :

Step 0. We form the initial information for the flow distribution method. The starting vertex of the traveling salesman route defines a subset of the initial nodes of the method, the set of directions from it defines a subset of intermediate nodes, and the graph vertices accessible from this subset defines the set of end nodes.

Based on the Orlin method of flow distribution, we determine the time to reach each of the end vertices on a subnet of initial, intermediate, and final vertices.

We select the shortest travel time and the corresponding stage of the route, mark the selected vertices and proceed to the formation of data for a new flow distribution problem. We pass to the next step of the algorithm.

Step s , $s=1,2,\dots$ We construct new subsets of initial, intermediate, and final vertices, excluding from further consideration the previously noted vertices.

If at the current step it is impossible to determine new subsets (all vertices are marked), we return to the previous step, unmark the route stage, marking the dead end direction, and move on to the next possible one by choosing the fastest direction of movement.

We repeat this process until we reach the end point of the route, which coincides with the starting point.

If the route is built, but does not include all the vertices of the graph, we return to the previous levels and rebuild all the working subsets, choosing new directions of movement, taking into account the speed of movement.

Final step. As a result of the work, we finally obtain a cyclic permutation of the numbers of the vertices of the graph, which determines the sequence of stages of the traveling salesman's route.

4 EXPERIMENTS

To analyze the efficiency of the algorithm, computational experiments were carried out, in which various methods (complete search, greedy, annealing and the one proposed above) were used to solve the traveling salesman problem on a network of 11 points [14]. The graph of the network of movements with the given time costs is shown in Figure 1.

5 RESULTS

The results of the numerical experiments performed are shown in Table 1.

The optimal route in the considered problem was found by full search and is determined by the sequence of numbers 1, 2, 5, 9, 7, 4, 3, 6, 8, 11, 10, 1 or 1, 2, 5, 9, 7, 4, 3, 6, 10, 11, 8, 1. The proposed algorithm made it possible to quickly find a route for visiting all graph vertices (1, 4, 3, 6, 2, 5, 9, 7, 10, 11, 8, 1), but the time it took more to move along this route.

As a result of the computational experiments the efficiency of using the developed algorithm was established, the obtained solutions are compared with the solutions found by other exact and heuristic methods.

6 DISCUSSION

Several remarks should be noted. The search procedure is based on the use of the backtracking technique, according to which the solution of the problem is reduced to a sequential expansion of a particular solution. If at the next step the expansion fails, then a return to a shorter particular solution occurs and the search continues further. This algorithm allows you to find all solutions to the problem, if any. It is known that the use of algorithms based on the backtracking scheme in solving practical problems is significantly limited by the low speed and significant requirements for computing resources. To speed up the work of the method, calculations are organized in such a way as to identify non-optimal options as early as possible, or use selection schemes based on a greedy campaign. This can significantly reduce the time to find a solution.

The greedy technique is based on a locally optimal choice under the assumption that this choice will lead to a globally optimal solution. Unfortunately, there is no way to check the quality of using greedy algorithms in solving a specific applied problem, however, for problems in which a sequence of local optima goes to a global optimal solution, this approach is very promising.

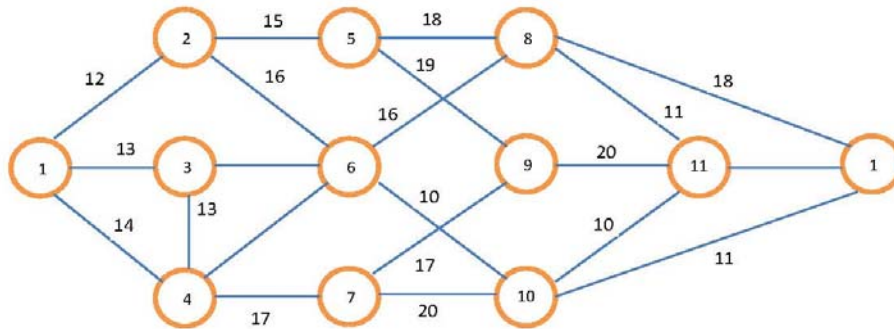


Figure 1 – The network of $n=11$ nodes and a given travel time for each pair of vertices for the traveling salesman problem

Table 1 – The comparison of search time and solutions of the traveling salesman problem for $n=11$

Calculation method	Operation time	Optimal solution	Solution characteristic
Complete search	30 sec	157 h	Exact
Greedy algorithm	21 sec	169 h	Approximate
Annealing method	23 sec	174 h	Approximate
The proposed algorithm	25 sec	169 h	Approximate

The greedy method proposed by the authors assumes consideration at each stage of the route formation of the fastest direction of movement in terms of time. A combined approach based on the method of resource allocation and greedy choice of the direction of movement made it possible to implement a constructive scheme for solving the traveling salesman problem.

CONCLUSIONS

The paper considers a method for formalizing the algorithm for solving the traveling salesman problem using the method of streaming resource allocation and using the backtracking scheme. The use of Orlin’s method to optimize the flow distribution on the graph is proposed. The scheme of formalization of the procedure for using the method with the implementation of the backtracking scheme for solving the traveling salesman problem with the minimum duration of movement along the route is briefly described. A variant of accelerating the speed of the developed algorithm is proposed, which consists in using a greedy technique in the procedure for selecting route sections: the planning of each next stage of movement is determined based on the choice of the fastest direction of movement, which makes it possible to obtain a constructive scheme for solving the traveling salesman problem. The results of the proposed algorithm for calculating solutions to the traveling salesman problem with minimization of the duration of movement are presented, the obtained solutions are compared with the solutions found by known exact and heuristic methods. The influence of the greedy approach on the speed of the developed algorithm was analyzed. Conclusions are drawn, further development of the proposed methodology for solving traveling salesman problems based on the use of other principles of greedy choice of direction of movement and for solving fuzzy and dynamic traveling salesman problems is proposed.

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ЕФЕКТИВНИЙ МЕТОД РОЗВ'ЯЗАННЯ ЗАДАЧІ РОЗПОДІЛУ ПОТУЖНОСТЕЙ КАНАЛІВ З УРАХУВАННЯМ НЕЧІТКИХ ОБМЕЖЕНЬ НА ОБСЯГИ СПОЖИВАННЯ

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АНОТАЦІЯ

Актуальність. Важливою сучасною проблемою є швидке відновлення та оптимізація управління логістикою. В залежності від поставленої задачі існує багато різних математичних методів та підходів до вирішення різних логістичних задач, розв'язування яких набуває широкого практичного впровадження. Його конкретний зміст залежить від характеру проблеми та повноти наявних даних. Іноді для розв'язання відомих задач, однією з яких є задача комівояжера, вдається знайти нетипові методи на основі поєднання декількох обчислювальних схем та методів.

Ціль. Мета роботи – розробити алгоритм розв'язання задачі комівояжера на основі реалізації методу потокового розподілу ресурсів і схеми backtracking з мінімальною тривалістю руху за маршрутом.

Метод. У статті розглядається методика послідовного застосування поточкових схем розподілу однорідного ресурсу для розв'язання задачі комівояжера, що формулюється як задача знаходження маршруту відвідування заданої кількості міст без повторень з мінімальною тривалістю руху. Поставлено та вирішено задачу формалізації алгоритму розв'язання проблеми комівояжера на основі методу розподілу ресурсів з використанням схеми backtracking. Запропоновано використання методу Орліна для оптимізації розподілу потоку на графі. Розроблено конструктивний алгоритм розв'язання задачі. Проведено обчислювальні експерименти.

Результати. Розроблено метод розв'язання задачі комівояжера з використанням методу потокового розподілу ресурсів і схеми пошуку з поверненням. Запропоновано варіант прискорення швидкості розробленого алгоритму, яке полягає в залученні жадібного способу в процедурі вибору ділянок маршруту: планування кожного наступного етапу переміщення визначається виходячи з відбору найбільш швидкого напрямку руху. Застосування жадібного підходу дозволило отримати конструктивну схему розв'язання задачі комівояжера. Представлено результати розрахунків за допомогою запропонованого алгоритму в задачах комівояжера з мінімізацією тривалості руху, проведено порівняння отриманих розв'язків з розв'язками, знайденими іншими точними та евристичними методами.

Висновки. У статті розглянуто метод формалізації алгоритму розв'язання задачі комівояжера з використанням алгоритму потокового розподілу однорідного ресурсу та схеми backtracking. Запропоновано використання методу Орліна для оптимізації розподілу потоку на графі. Описано схему формалізації процедури використання методу з реалізацією схеми з поверненням для розв'язання задачі комівояжера з мінімізацією тривалості руху за маршрутом. Запропонований варіант прискорення роботи розробленого алгоритму.

КЛЮЧОВІ СЛОВА: задача комівояжера, метод розподілу ресурсів, рекурсивна схема пошуку з поверненням, жадібний підхід.

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