APPLICATION OF BLOW-UP THEORY TO DETERMINE THE SERVICE LIFE OF SMALL-SERIES AND SINGLE ITEMS

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ABSTRACT

Context. The actual task of developing a method for determining the service life of small-series and single items based on the blow-up modes theory has been solved.

Objective. Application of the blow-up theory in conditions where there are no statistical data on the dynamics of behaviour during the operation of small-series and single items.

Method. To determine the service life of a particular product manufactured in large series, information obtained for a set of similar products of the same type is used. This information is based on numerous experiments, mathematical statistics and probability theory. When operating small-series and single items, such information is not available. In this case, it is necessary to determine the individual resource of an individual product based on the results of an analysis of its behaviour in the past. The method presented in the article is based on the application for such an analysis of the method used when considering systems operating in blow-up mode. The essence of the technique is to extract the periodic component from the temporal realization of the control parameter. This component is modelled by a Fourier series consisting of log-periodic functions. The main coefficients of these functions are the time equal to the operating time of the product until the end of its service life.

Results. The method under consideration has been successfully tested in determining the service life of the transport-dumping bridge, related to products that are actually single items.

Conclusions. An analysis of the experimental data on the behaviour of the load-bearing elements of a transport-dump bridge confirms the assumption about the behaviour of the bridge structure as a system operating in a blow-up mode. This made it possible to determine in advance the service life of the power units of the bridge and obtain the result directly in units of time, without requiring information about the maximum permissible value of the controlled parameter to obtain this information.

For the first time, the possibility is shown to consider the behaviour of small-series and single items as dynamic systems operating in a blow-up mode.

Practical significance. A solution to the topical problem of determining the service life of small-series and single items is proposed.

KEYWORDS: life time, transport and dump bridge, small-series products, log-periodic component, direct and indirect control methods, blow-up theory.

NOMENCLATURE

- $T$ – service life of products;
- $t_f$ – moment of exacerbation;
- $a$ – actual exponent;
- $\alpha + \beta i$ – complex exponent;
- $a_i$ – polynomial coefficients;
- $F(t)$ – some periodic function;
- $\tau$ – size phase;
- $m, \alpha, \beta$ – exponent;
- $C, \eta, \gamma$ – coefficients;
- $f(t)$ – probability density function;
- $F(t)$ – distribution function;
- $R(t)$ – reliability function;
- $n$ – number of experiments;
- $B_{FR}$ – controlled signal trend;
- $A_{PER}$ – variable component of the controlled signal;
- $A_{SEM}$ – controlled signal value;
- $B_{MOD}$ – model of periodic composition;
- $A_0$ – amplitude;
- $A_{EXT}$ – variable component extremum $B_{VC}$;
- $A_{EXTM}$ – extreme array model of the variable component $B_{VC}$;
- $l_n, l_{n+1}, l_{n+2}$ – extreme time of the controlled signal variable component;
- $\rho$ – time parameter;
- $\omega$ – log-periodic frequency;
- $\phi$ – phase of log-periodic oscillations;
- $T_{RUL}$ – residual useful life;
- $a_0, a_2, b_2$ – Fourier series coefficients;
- $\kappa$ – serial number of the Fourier series term;
- $t_0$ – integration lower limit;
- $l_n$ – integration upper limit;
- $n$ – number of Fourier series terms.

INTRODUCTION

The service life of multi-series products is traditionally determined by the results of a mass, collective forecast based on numerous experimental data on the resource of products, mathematical statistics and probability theory. At the same time, the determination of the service life of a wide class of technical products is carried out, as a rule, indirectly upon the fact that the controlled parameter reaches the maximum permissible level according to the standards. This level is an average statistical value established by the results of a sufficiently long operation of a significant number of the same type of products. For small-series products, as well as products produced in single copies, this approach is not applicable. The solution to this problem is to determine for each product sample its...
individual service life. In this case, it is desirable to determine the duration of operation not indirectly, but directly in units of time and to obtain the information of interest long before the end of the service life of the product.

These requirements are met by information obtained, for example, from the results of approximating the trend of the controlled parameter by a predictive model, which is a smooth monotonically changing function. The coefficients of the model determined in this case include a coefficient that coincides in magnitude and dimension with the end time of the product’s operation. This coefficient can be determined long before the expiration of its service life each time when monitoring the condition of the product.

If the initial information does not meet these requirements, and fluctuations are superimposed on a smooth trend, the frequency of which increases with time, then the product can be considered as a dynamic system developing in a blow-up mode. A distinctive feature of the behaviour of these systems is that as the catastrophe approaches, which means the destruction of the system or a radical change in the law of its development, the frequency value reaches infinity.

The fact of frequency change is fixed long before the catastrophe, and the oscillation model allows you to directly determine the service life of the product, without requiring knowledge of the maximum permissible standard level of the controlled parameter.

The object of work is the process of determining the service life of small-scale and single items.

The subject of the work is a model that describes the change in the periodic component of the control parameter recorded during the operation of products.

The purpose of the work is to develop a method for determining the service life of small-series and single items.

1 PROBLEM STATEMENT

Periodic processes, apparently, are one of the foundations for constructing theories in various fields, including for determining the service life of small-scale and single products. Periodicity – the regular repetition of something in time – testifies to the cognoscibility of the world, in the causal conditionality of phenomena. Understanding the nature of periodicity makes it possible to predict events, and such predictions are the basis of a method for determining the life of a dynamic system under control.

In dynamic systems developing in blow-up mode, a periodic process is superimposed on the main trend of the controlled parameter. This process is described by a model, one of whose coefficients coincides in value and dimension with the moment of system destruction or a radical change in the law of its development [1].

Such modes are described by the following equation

\[ \frac{dx}{dt} = x^{1+1/\alpha}. \] (1)

The equation solution increases without limit as we approach the peaking moment \( t_f \):

\[ x(t) \sim (t_f - t)^{-\alpha}. \] (2)

To obtain a solution acceptable for practice (2), we pass from the real indicator \( a \) to the complex one \( \alpha + \beta i \), which allows us to obtain an equation of the following form:

\[ x(t) = \Re \sum_k a_k (t_f - t)^{\alpha + k\beta} = (t_f - t)^{\alpha} \cdot F(\ln(t_f - t)) \] (3)

The function \( F(\cdot) \) is described by several multiple harmonics, characterizing in the general case the significant nonlinearity of systems developing in the blow-up mode. However, in practice [2], the function \( F(\cdot) \) is limited to one first harmonic:

\[ x(t) = (t_f - t)^{\alpha} \left[ a_0 + a_1 \cos \left( \beta \ln \left( \frac{t_f - t}{\tau} \right) \right) \right]. \] (4)

This expression is a smooth trend, on which log-periodic fluctuations are superimposed, which serve as precursor of approaching the blow-up moment \( t_f \). Taking \( t \to t_f \), the oscillation frequency tends to infinity, which meets the dynamic law requirements followed by the blow-up mode. The continuous increase in the log-periodic oscillations frequency allows them to react sensitively to the course of catastrophically developing processes long before the blow-up moment.

If we consider the exhaustion moment of the tool life \( T \) as the blow-up moment \( t_f \), then the materials cutting can be attributed to the blow-up modes. At the same time, to improve the quality of predicting tool life, it is necessary to isolate the sensitive log-periodic part of the recorded signal. In practice, this means that the total signal periodic component must be separated from the smooth trend and its behavior should be analyzed separately throughout the entire cutting process.

The periodic component model should be subjected to direct analysis, which fully describes the complex polyharmonic in structure of the actually recorded signal.

2 REVIEW OF THE LITERATURE

The service life of similar products produced in significant quantities is determined using a mass, statistical model, when the behavior of a set of products over time is observed. Based on the observation made over a certain time, a prediction is made of the behavior of one specific product in the future time interval. The service life of products in this case is determined, as a rule, in three ways [2]. The first method is based on the use of statistics on the cumulative probability function
P(t) of the normal distribution, showing the probability of failure of a given type of product depending on its service life (Fig. 1).

Figure 1 – Probability of product failure depending on from its service life [2]

The second method establishes the degree of similarity of current information about the trajectory of the controlled parameter that characterizes the behaviours of the product of interest (degradation profile), with statistical data on similar trajectories compiled from the results of operating similar products (Fig. 2).

Figure 2 – Statistics of degradation profiles and the actual trajectory of the controlled parameter [2]

In Fig. 2, the statistical set of degradation profiles is highlighted in blue, the current trajectory of the controlled parameter is highlighted in red. In this case, based on the degree of closeness of the current curve and the set of blue curves, the residual life of the product is estimated by the authors at about 65 conditional cycles.

The third method predicts the moment of product failure by the value of the coordinate of the time axis (applicator) of the point of intersection of the skeletal curve of the trajectory of the controlled parameter with its threshold level (Fig. 3).

In practice, smooth monotonically varying functions are used as models of backbone curves [3]. In Fig. 4, for example, the behaviours of 4 types of backbone curves is shown, described by linear, parabolic, S-shaped (Gompertz curve) and exponential dependences.

Figure 3 – Intersection of the skeletal curve, described by the exponent, with a threshold level of the controlled parameter [2]

Figure 4 – Behaviours of 4 different analytical description of backbone curves

Forecasting the service life of a product using skeletal curves requires knowledge of the threshold level of the controlled parameter, which is not feasible for small-scale products, and, moreover, single products.

The controlled parameter of a given product sample changes along a single (individual) trajectory. Therefore, the threshold level, being in essence an average statistical value, refers to a specific sample of even the same type of products only with a certain degree of probability.

Thus, according to the most common normal distribution law for the uptime of a product, 50% of products fail before the threshold level is reached, and, accordingly, 50% work after it is crossed [4]. This serves as a serious, error-prone, problem of predicting the life of products in general, and not just small-scale or single ones.

In this case, it is legitimate to use an individual model, in which it is required to regularly monitor the technical condition of the product, comparing the recorded data with the skeletal curve. The analytical description of the skeletal curve model contains a coefficient that coincides in value and dimension with the operating time of the product until the end of its service life.

So, in [5], the model (5) is considered, which includes a similar coefficient $T$. The model was obtained on the basis of the expression for the fatigue curve.
The coefficients of the model $A(t)$, including the coordinate $T$, are determined numerically by minimizing the deviation from the backbone curve of the time series $A_{\text{con}}(t)$ compiled from the results of measuring the control parameters (6).

$$U(t) = \sum_{i=1}^{m} (A_{\text{con}}(t) - A(t))^2.$$  \hspace*{1cm} (6)

A similar approach to determining the service life is proposed in [6]. Here, as a model of the backbone curve, the analytical expression (7) is used to describe the Weibull distribution.

$$A(t) = A_0 + \eta \left( \frac{t-t_0}{T-t} \right)^{\alpha} \cdot \left( \frac{t-t_0}{T-t} \right)^{\beta}. \hspace*{1cm} (8)$$

When choosing models of backbone curves (5) and (7), following the phenomenological approach to modeling, a generalized scheme of the change in the controlled parameter during the life cycle of the product was reproduced (Fig. 5). Three characteristic sections are distinguished in this diagram: where an increased flow of product failures is noted, due to the running in of its components and parts; stationary site – the main time of operation of the product; site of catastrophic failures of the product, leading to the termination of its operation. The generalized scheme of the change in the controlled parameter (Fig. 5), consisting of three sections, characterizes the change in the process of the life cycle of the product of gradual failures, the negative impact of which on the quality of the product’s functioning gradually accumulates. For this reason, these failures in the literature are often referred to as “wear-out”, and “wear-out” is understood in an extended sense [4]. Accordingly, the generalized scheme is called the “wear curve” [7].

The use of this approach to determine the service life of small-scale and single products in various fields of human activity is clearly shown in the monograph [9].

The introduction of digital measurement systems made it possible to refine the method for monitoring the state of the product. In particular, what earlier, with the analogy method of monitoring the state of the product, was considered as a measurement error, for example, fluctuations (fluctuations) of the measured value relative to some of its average value (skeletal curve) in practice turned out to be an informative component of the measured data [10].

The wear curve (Fig. 5, 6) ends with a catastrophe section, which characterizes a sharp change in the value of the controlled parameter, which is the result of the cumulative impact on the product of its gradual failures. This gives grounds to believe that the behaviour of the product during its operation can be interpreted as the behaviour of a system developing in a blow-up mode [1].

In this case, the trajectory of the control parameter should be considered as the sum of a smooth skeletal curve and periodic oscillations about it. Periodic oscillations obey the log-periodic law, according to which the frequency of oscillations increases as it approaches the moment of product failure.

Monitoring the fact of changing the frequency of log-periodic oscillations makes it possible to predict the moment of product failure individually for each of its samples, regardless of the stage of its operation and does not require knowledge of statistical data on the threshold level of the controlled parameter.
The latter is very important, since it allows solving the urgent problem of determining the service life of products manufactured in small batches or, in general, in single copies, which were the goal of the research, the results of which are presented in this article.

3 MATERIALS AND METHODS

The controlled parameter \( A_{\text{CON}}(t) \) is considered as the sum of the smooth (trend) \( B_{\text{TR}} \) and the periodic component \( A_{\text{PER}} \).

\[
A_{\text{CON}}(t) = B_{\text{TR}} + A_{\text{PER}} \tag{9}
\]

According to (4), at \( T = t_0 \), \( B_{\text{TR}} \) is determined from the following expression

\[
B_{\text{TR}} = a_0 \cdot (T - t)^{\alpha}. \tag{10}
\]

The periodic component \( A_{\text{PER}} \) is extracted from the information (total) signal \( A_{\text{SUM}} \) by decomposing it into empirical modes [11].

\[
A_{\text{PER}} = -0.25A_{\text{SUM}1} + 0.5A_{\text{SUM}1} - 0.25A_{\text{SUM}1+1}. \tag{11}
\]

The periodic component \( A_{\text{PER}} \), according to (4), is determined from the following expression

\[
A_{\text{PER}} = a_1 \cdot \cos\left(\beta \cdot \ln\left(\frac{T - t}{\tau}\right) \right). \tag{12}
\]

For the convenience of further research, expression (12) should be reduced to the classical form of the log-periodic function (13), considering it as a \( B_{\text{MOD}} \) model of the periodic component \( A_{\text{PER}} \).

\[
B_{\text{MOD}} = A_0 \cos(\omega \cdot \ln(T - t) - \varphi),
\]

where \( A_0 = a_1 \cdot (T - t)^{\alpha} ; \omega = \beta \cdot \ln(\tau) \tag{13}\)

Expression (13) contains four unknown parameters: \( T, \omega, \varphi, A_0 \). The first three parameters are determined by solving the system of two nonlinear equations (14).

\[
\begin{align*}
\ln(T - t_n) - \ln(T - t_{n+1}) & = \frac{2\pi}{\omega}, \\
\ln(T - t_{n+1}) - \ln(T - t_{n+2}) & = \frac{2\pi}{\omega}.
\end{align*} \tag{14}
\]

Equations (14) are based on the knowledge of the time \( t_n \), which account for the extremes \( A_{\text{EXT}} \) of the periodic component \( A_{\text{PER}} \).

To search for these extremes, the following algorithm is used

– at least three local extreme stand out in the periodic component \( A_{\text{PER}} \). They are separated from each other in phase by an angle \( 2\pi \), and there are consecutive and identical in sign (maximum or minimum);

– the time \( t \) is marked when extremes occur (\( t_1, t_{n+1}, t_{n+2} \));

– the parameter \( \rho \) is calculated that characterizes the relationship between the extremes occurrence time.

\[
\rho = \frac{t_{n+1} - t_n}{t_{n+2} - t_{n+1}}, \quad \rho > 1. \tag{15}
\]

Parameter \( \rho \) must exceed one. This indicates a decrease in the period of its oscillations, characteristic of the log-periodic function, over time.

A decrease in the period leads to an increase in the oscillation frequency of the log-periodic function in the limit to infinity. This function feature was the basis for choosing it as a model \( B_{\text{MOD}} \) (13) for describing systems operating in the blowup mode [1].

The set of extremes forms an array composed of discrete values of \( A_{\text{EXT}} \) extremes of the periodic component \( A_{\text{PER}} \).

The solution of system (14) gives the following expressions for the first three unknowns of equation (13) [12]:

\[
T = \frac{t_{n+1}^2 - t_{n+2}t_n}{2t_{n+1} - t_{n+2} - t_n}, \quad \omega = 2\pi/(\ln(\rho)), \quad \varphi = \pi - \omega \cdot \ln(T - t_{n+2}). \tag{16}
\]

To check the correctness of the obtained unknowns values (16) and, if necessary, to refine them, the difference between the components of the array of extreme values \( A_{\text{EXT}} \) and their model \( B_{\text{MOD}} \) (13) is minimized. In this case, the parameter \( A_0 \) is also determined.

\[
\sum_{i=0}^{m} \left( A_{\text{EXT}i} - B_{\text{MOD}i} \right)^2 \Rightarrow \min. \tag{17}
\]

In practice, the array of extreme values \( A_{\text{EXT}} \) contains a number of components, indicating the polyharmonic nature of the oscillations of the periodic component \( A_{\text{PER}} \). Therefore, when refining the values of parameters (16), as a model \( A_{\text{EXT}M} \) (predictive model) describing an array of extreme values \( A_{\text{EXT}} \), one should use a trigonometric polynomial composed of log-periodic functions (Fourier series).

\[
A_{\text{EXT}M} = a_0 + \sum_{k=1}^{n} \left[ a_k \cos(k \cdot \omega \cdot \ln(T - t)) + b_k \sin(k \cdot \omega \cdot \ln(T - t)) \right]. \tag{18}
\]

The coefficients of the series \( a_0, a_k, b_k \) are determined from the following expressions...
The remaining useful life \( T_{\text{RUL}} \) is determined from the following expression.

\[
T_{\text{RUL}} = T - t.
\]  

(20)

4 EXPERIMENTS

The purpose of the experiment was to test the effectiveness of determining the service life of small-series and single products, considering their behavior during operation, as systems operating in the blow-up mode.

As an object of study, a hydro turbine, a representative of a small-series product [9] (Fig. 7), and a transport and dump bridge, related to single products [12] (Fig. 13), were considered.

5 RESULTS

The initial information for the analysis was the vibration of the turbine support, which was measured along the y-axis (Fig. 7).

![Diagram of the hydro turbine](image)

Figure 7 – Diagram of the hydro turbine indicating the point of its vibration monitoring: 1 — generator bearing; 2 — turbine bearing; 3 — turbine impeller; 4 — vibration sensor of the turbine bearing

The trajectory of the turbine vibration level during the observed period (120 days) is shown in Fig. 8.

![Changing the vibration level of the turbine support](image)

Figure 8 – Changing the vibration level of the turbine support

Fig. 9 shows the change over time in the frequency of log-periodic oscillations superimposed on the trajectory of turbine vibrations.

![Changing the frequency of logo-periodic oscillations](image)

Figure 9 – Changing the frequency of logo-periodic oscillations, superimposed on the trajectory turbine vibration level changes

Fig. 10 shows the approximation by the log-periodic oscillations model \( B_{\text{MOD}} \) (13) of the periodic component of turbine vibration.

![Approximation by the model B_MOD](image)

Figure 10 – Approximation by the model \( B_{\text{MOD}} \) (13) of the periodic component \( A_{\text{PER}} \) turbine vibration

The results of determining the service life of hydro turbine are shown in Fig. 11.
Fig. 12 shows the change during the operation of the residual useful life of the $T_{RUL}$ (20) of the hydro turbine.

Changes during the controlled period of the truss deflection trajectories in nodes No. 8 and No. 9 (marked in red in Fig. 13) are shown in Fig. 14.

Fig. 15 and 16 show the change over time in the frequency of logo-periodic oscillations superimposed on the trajectory of the deflection of nodes No. 8 and No. 9 and the model approximating them.
6 DISCUSSION

Monitoring of the state of the turbine and the bridge confirmed the assumption that the trajectory of change of the control parameter (Fig. 8, 14) contains a variable component (Fig. 10, 17, 18). This component is described by the model of logo-periodic fluctuations. The frequency of these oscillations increases as the products approach and the expiration of their service life (Fig. 9, 15, 16), which corresponds to the behaviour of systems operating in the blow-up mode. The calculation of the coefficients of the logo-periodic oscillations model made it possible to determine the operating time of the turbine before the accident (127 days, fig. 11) and the calendar date of the resource exhaustion of nodes No. 8 and No. 9 (fig. 19, 20). The remaining service life of the turbine and bridge at the time of the last control of their condition was 2 days for the turbine, and 2.38 years for the bridge (node No. 8) and 1.69 years (node No. 9).

CONCLUSIONS

The scientific novelty of obtained results is that the analysis of experimental data on the behaviour of the hydro turbine and of the transport-dump bridge confirms the assumption about the behaviour of the turbine and bridge structure as a system operating in the blow-up mode.

The practical significance of obtained results is made it possible to determine in advance the moment when the service life of the turbine and power units of the bridge is exhausted, and to obtain a forecast directly in units of time, and without requiring information about the maximum permissible value of the controlled parameter (turbine vibration level and deflection of the truss unit) to obtain this information.

Prospects for further research are to study the possibility of using the developed technique to predict the service life of products of various purposes and designs.

ACKNOWLEDGEMENTS

The work is supported by the state budget scientific research project of Sumy State University “Models and methods of information technologies for the analysis and synthesis of structural, information and functional models of automated objects and processes” (state registration number 0120U103071).

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УДК 616.1:67.05

ЗАСТОСУВАННЯТЕОРЕІ РЕЖИМІВ ІЗ ЗАГОСТРЕННЯМ ДЛЯ ВИЗНАЧЕННЯ ТЕРМІНУ ЕКСПЛУАТАЦІЇ МАЛОСЕРІЙНИХ ТА ОДИНІЧНИХ ВИРОБІВ

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АНОТАЦІЯ

Актуальність. Вирішено актуальні завдання розробки на основі теорії режимів із загостренням методики визначення терміну служби малосерійних виробів та виробів, виготовлених у одиничному екземплярі.

Ціль. Застосування теорії режимів із загостренням за умов, коли відсутні статистичні дані про динаміку поведінки в процесі експлуатації малосерійних виробів та виробів, виготовлених у одиничному екземплярі.

Метод. Для визначення терміну служби конкретного виробу, який виготовляють великими серіями, використовується інформація, отримана для сукупності подібних однотипних виробів. Ця інформація складена на основі числових експериментів, математичної статистики та теорії ймовірностей. При експлуатації малосерійних виробів та виробів, виготовлених у одиничному екземплярі, такої інформації немає. І тут необхідно визначити індивідуальний ресурс «виробу-індивіда» за результатами аналізу його поведінки у минулому. Піданий у статті метод заснований на застосуванні для такого аналізу методики, яка використовується при розгляді систем, що працюють у режимі із загостреннями. Суть методики полягає у визначенні з часової реалізації контрольної періодичної компоненти. Ця компонента моделюється рядом Фур’є, що складається з лого-періодичних функцій. Основним коефіцієнтом цих функцій є час, що діяє в напрямку накопичення виробу до закінчення терміну його служби.

Результати. Метод, що розглядається, успішно апробований при визначенні терміну служби транспортно-відałового мосту, що відноситься до виробів, які виготовляються фактично в одиничних екземплярах.

Висновки. Аналіз експериментальних даних про поведінку несучих елементів транспортно-відальному мосту підтверджує припущення щодо поведінки конструкції моста як системи, що працює в режимі з загостренням. Це дозволило заздалегідь визначити термін служби силових агрегатів моста і отримати результат безпосередньо в одиничних часу, не вимагаючи для отримання інформації про річній догорістре значення контролюваного параметра. Вперше показана можливість розглядати поведінку малосерійних виробів та виробів, що виготовляються в одиничному екземплярі, як динамічних систем, що працюють у режимі з загостреннями.

Практична значимість. Запропоновано вирішення актуальног завдання щодо визначення терміну служби малосерійних виробів та виробів, що виготовляються в одиничних екземплярах.

КЛЮЧОВІ СЛОВА: ресурс, транспортно-відальній міст, дрібнісерійна продукція, логістична складова, пряма та непряма методи керування, теорія режимів із загостренням.

ЛІТЕРАТУРА

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DOI 10.15588/1607-3274-2023-3-19
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