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CONTROL IN TECHNICAL SYSTEMS

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THE FREQUENCY METHOD FOR OPTIMAL IDENTIFICATION OF CLOSE-LOOP SYSTEM ELEMENTS

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ABSTRACT

Context. The article is devoted to overcoming the contradictions between the assumptions adopted in known methods of closed-loop control system identification and the design and conditions of its operation. The article presents a new method of identifying the transfer functions matrix of a two-level closed-loop control system element, which functions under the conditions of multidimensional stationary centered random influences.

Objective. The purpose of the study, the results of which are presented in this paper, is to extend the indirect identification method to the case of estimating one of the two-level closed-loop control system elements' dynamics model based on passive experiment data.

Method. To solve the optimal identification problem, a variational method for minimizing the quality functional on the class of fractional-rational transfer function matrices was used.

Results. As a result of the research, the identification problem formulation was formalized, the rules for obtaining experimental information about the input and output signals were determined, the rules for identifying the transfer functions matrix of a two-level closed-loop control system element, which minimizes the sum of the variances of identification errors in the frequency domain, and the verification of these rules was carried out.

Conclusions. Justified rules allow to correctly determine transfer functions matrices of the closed-loop systems selected element when fulfilling the defined list of conditions. The closed-loop systems control paths signals analysis proves the possibility of the effect of changing these signals statistical means, even under conditions of only centered stationary input influences actions on the system. Based on this, the further development of research can be aimed at overcoming such effects.

KEYWORDS: Identification, transfer function matrix, spectral density, error variance, quality functional.

NOMENCLATURE

M_1 is a matrix of dimension $m\times n$, the elements of which are polynomials from the differentiation operator ρ;

m is a number of signals at the output of the local control system;

N_0 is a matrix of results of dividing the polynomials of the numerators by the polynomials of the denominator of the product on the right side of the expression;

N_+ is a matrix of fractional rational functions whose poles are located in the left half-plane of the complex plane;

N_- is a matrix of fractional rational functions with poles in the right half-plane;

n is a number of inputs of the local system;

$O_{m\times n}$ is a zero matrix of size $m\times n$;

P_1 is a matrix of dimension $m\times m$;

R is an additionally defined weight matrix;

r is a vector of programme signals;

S'_{rr} is a transposed spectral density matrix of the vector r;

$S'_{x_n x_n}$ is a transposed spectral density matrix of the vector x_n;

$S'_{\zeta x}$ is a transposed matrix of mutual spectral densities between the generalised input vector ζ and the vector x_n;

$S'_{x\zeta}$ is a transposed matrix of mutual spectral densities between the vectors x_n and ζ;

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$S'_{\delta\delta}$ is a transposed matrix of spectral densities of uncorrelated white noise of single intensity;

$S'_{\varphi\varphi}$ is a transposed spectral density matrix of measurement noise;

$S'_{\varphi_0\varphi_0}$ is a transposed spectral density matrix of the dummy noise vector φ_0;

$S'_{\psi\psi}$ is a transposed matrices of spectral densities of disturbances;

$S'_{\zeta\zeta}$ is a transposed spectral density matrix of the generalised input vector;

u_1 is a vector of input signals of the local system;

u_n is a mismatch vector;

W_n is a matrix of transfer functions that determines the relationship between the operator's reactions to changes in the components of the misalignment vector u_n;

W_p is a matrix of transfer functions that determines the relationship between the operator's actions to prevent and probe pulses δ;

x_1 is a vector of signals at the output of the local control system;

x_n is a vector of control signals;

y is a vector of master feedback signals;

Φ is a block matrix of transfer functions of size $n\times(n+m)$;

δ is a vector of sensing pulses;

ε_x is the vector of identification errors;

φ is a vector of measurement noise;

φ_0 is a dummy measurement noise vector;

ρ is a vector of additional signals;

ψ is a vector of disturbances with m components;

ζ is a generalised vector of input influences.

INTRODUCTION

According to the definition given in the well-known article [1], one of the central problems in systems theory is the problem of identification. According to L. Zadeh, this problem is "determination, on the basis of observation of input and output, of a system within a specified class of systems to which the system under test is equivalent; determination of the initial or terminal state of the system under test". If we limit ourselves to considering the works [2, 3] devoted to the determining automatic control systems elements dynamics models, it is obvious that the whole set of such studies is divided into two parts. The first part is, for example, works [3–5], which are devoted to determining open-loop systems and their elements dynamics models. The second part, for example works [2, 6–8], combines studies aimed at solving the closed-loop control system elements identification problem. Despite the large number of papers devoted to solving the latter problem, the search for new methods and means of determining the dynamics models of closed-loop control system elements is still relevant.

This relevance is due to the existence of contradictions between the assumptions made when formulating the identification method and the design and operating conditions of a closed-loop control system. In the context of the fourth industrial revolution, there is a rapid increase in the diversity of control objects and, accordingly, systems. Therefore, the requirements of practice require bringing the identification procedures into line with the conditions of the closed-loop control system design tasks.

The object of study in this paper is a two-level closed-loop control system.

The subject of study is identification of a transfer functions matrix of a two-level closed-loop control sytem element.

The purpose of the work is to substantiate the rules for estimating two-level closed-loop control system's one of the elements dynamics model based on passive experiment data.

1 PROBLEM STATEMENT

As a rule, modern control systems have physical subsystems with many inputs and outputs as objects. These subsystems operate under the influence of vector stochastic useful signals, measurement noise, and interference. For example, the flight control system of an unmanned aerial vehicle or aircraft. Thus, of all considered. identification methods [1–8], only the indirect and joint methods remain. They allow identifying a multidimensional closed-loop control system if its structure can be represented as shown in Fig. 1, and the sensors have low inertia and low intensity of measurement noise.

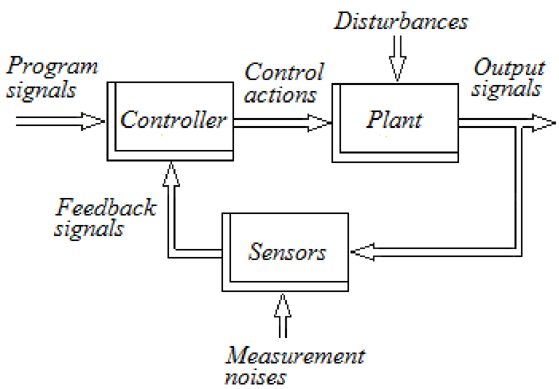

Figure 1 – Architecture of a closed-loop (local) control system

However, the development of the principles of controlling such objects has led to the emergence of closed-loop systems that have two control loops (Fig. 2) [4, 16].

The external loop is used to generate a vector of control signals that are transmitted to the local control system. This vector is generated by the master controller as a result of comparing the vectors of program signals and master feedback signals. The master feedback signals differ slightly from the controlled variables due to the influence of measurement noise and the inertial properties of the master sensors. The relationship between the controlled variables and the signals at the output of the control object is characterised by a kinematic link. If the control object is a moving vehicle, this link solves the inverse kinematics problem [17].

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Thus, the use of the known indirect and joint methods of identification to solve the problem of determining the system (Fig. 2) elements dynamics models requires modification of these methods.

We will assume that, due to preliminary experiments and the use of known identification methods, a linearized model of the dynamics of the local control system has been determined and presented as a system of ordinary differential equations with constant coefficients of the form

$$P_1 x_1 = M_1 u_1 + \psi \,, \quad (1)$$

where P_1 is a dimension matrix $m \times m$, the elements of which are polynomials from the differentiation operator

$$p = \frac{d}{dt} \,,$$

m is the number of signals at the output of the local control system; x_1 is a vector of signals at the output of the local control system (Output signals); M_1 is a matrix of dimension $m \times n$, the elements of which are polynomials from the differentiation operator p; n is the number of inputs of the local system; u_1 is a vector of input signals of the local system; ψ is a vector of disturbances with m components. In this case, the architecture of a two-level closed-loop control system is transformed into a block diagram (Fig. 3). As you can see, this diagram has two parts.

The first part combines the main controller and the communication system with the main sensors (Fig. 3). Three vectors act on the inputs of the main controller (Fig. 3): program signals r, main feedback signals y, and additional signals ρ. The origin and effect of additional signals depend on the purpose and design of the control system. At the outputs of the master controller, a vector of control signals x_n is formed. This vector is simultaneously the local control system input signals vector u_1. Relationship between these vectors is characterised by two transfer function matrices W_n and W_p. For example,

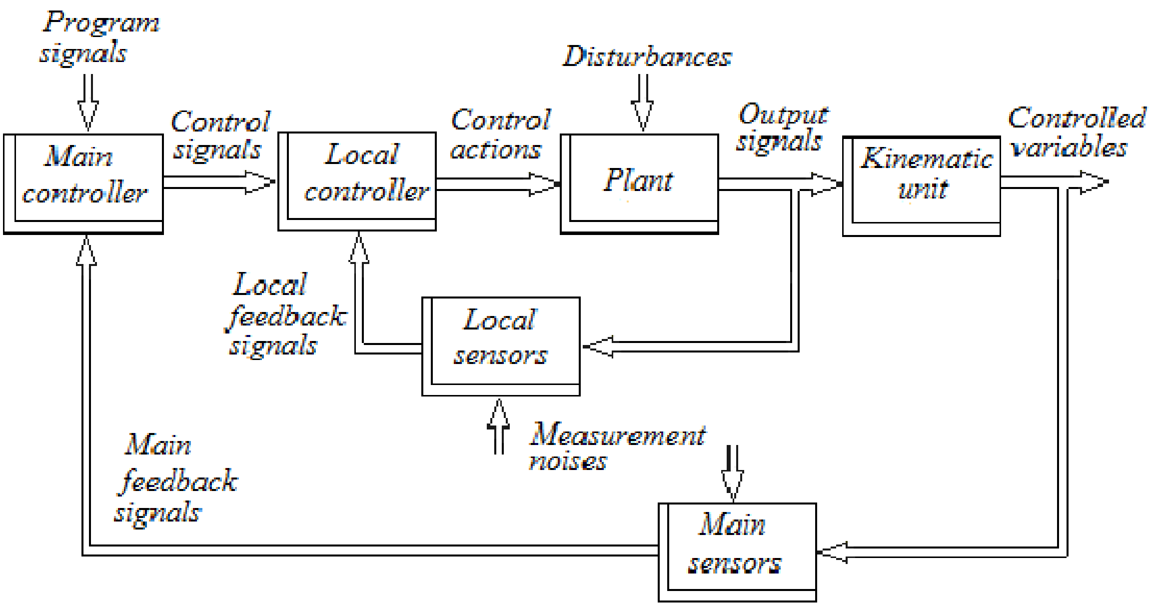

Figure 2 – Architecture of a two-level closed-loop control system

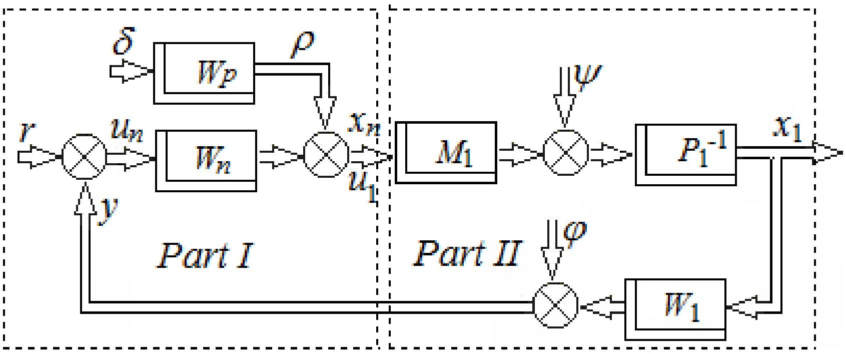

Figure 3 – Block diagram of a two-level closed-loop control system

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if the control system is designed to ensure the movement of an unmanned aerial vehicle along a given trajectory, then the central part of the master controller is the pilot-operator. Its dynamic properties are characterised by two transfer functions matrices [9, 10]. The first W_n determines the relationship of the operator's reactions to changes in the components of the misalignment vector u_n. The second matrix W_p describes the relationship between the operator's actions to prevent (remnant) and the sensing impulses of his nervous system δ.

The block diagram (Fig. 3) second part combines the local control system, the kinematic link, and the main sensors (Fig. 2). Its inputs are also three vectors: control signals u_1, disturbances ψ, and measurement noise φ. The output of the second part is a vector of main feedback signals y. The effect of changing the components of the signal vector at the output of the local system x_1 on the components of the vector y is characterised by the transfer function matrix W_1.

The structure and parameters of the transfer function matrix W_1 and the model of measurement noise dynamics can be determined separately from the system (Fig. 3) as a result of dynamic sensor certification according to the methodology given, for example, in monograph [4].

At the same time, it is possible to determine the transfer function matrices of the master controller, which it has in real operating conditions, only as a result of solving the problem of identifying a closed-loop control system in an appropriate manner.

Taking into account the statement of Peter Eykhoff, substantiated in article [11], about the possibility of unambiguous identification of only the transfer function of an element of a closed-loop control system, the following identification problem is formulated.

Let the polynomial matrices P_1, M_1, the transfer function matrix W_1, the transposed spectral density matrices of disturbances $S'_{\psi\psi}$ and measurement noise $S'_{\varphi\varphi}$ be given, and it is known that all signals in the control loops of the system (Fig. 4) are centred stationary random signals. It is also assumed that the vectors r, u_n, x_n are measured with sufficient accuracy. The optimal identification task is that, as a result of processing experimental data (records of vectors r, u_n, x_n) and a priori information about those elements of the system in the Fig. 4, the dynamics of which is known, to find algorithm of searching for such matrices W_n and W_p, at which the identification error vector ε_x components weighted variances sum would be minimal.

2 REVIEW OF THE LITERATURE

The analysis of methods for identifying closed-loop control systems based on the study of such literature sources as [2, 6–8] and [12], allows dividing them into four parts (Fig. 4).

Direct Methods [6, 12] are used to determine the model of the dynamics of the control object and sometimes the disturbances acting during the experiment, while ignoring the presence of feedback. The main restrictions on the use of this set of methods are:

– the requirement of a low intensity of disturbances with a high intensity of the useful signal;

– requirement of knowledge of the disturbance dynamics models;

– the requirement of low sensor inertia and high signal-to-noise ratio;

– the need to pre-determine the order of the control object;

– limitation of the control object dynamics to stable dynamic links only.

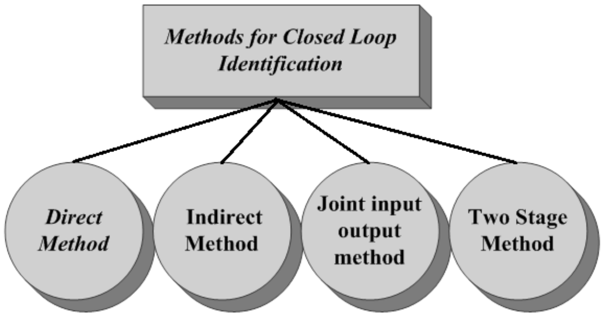

Figure 4 – Types of methods for identifying elements of closed-loop systems

An attempt to overcome the shortcomings of direct methods led to the development of a variational method for identifying the dynamics of multidimensional control objects, which is described in [13]. This method does not require a priori information about the model of disturbance dynamics and the order of the control object.

Indirect Method [6, 12] involves conducting an experiment, obtaining records of signals acting on the inputs and outputs of the system, identifying the dynamics model of a closed-loop system, calculating the dynamics model of an equivalent open system, and searching for the dynamics model of the control object. Algorithms that implement this method have an advantage over the direct method due to the absence of the requirement to have a model of the dynamics of disturbances. At the same time, the main disadvantages of these methods include:

– the need for prior knowledge of the controller's control law (transfer function);

– limitation of the class of systems that can be identified to one-dimensional ones;

– requirement of low sensor inertia and high signal-to-noise ratio;

– the need to pre-determine the order of the control object.

An attempt to overcome the disadvantages of indirect methods led to the development of a variational method for identifying the dynamics of multidimensional control objects, which is described in [14, 15]. This method overcomes all the disadvantages of the indirect method, but limits the class of useful signals, disturbances, and interferences that act during the experiment. All these signals must belong to centred stationary random processes or to an additive mixture of a stationary random process and a deterministic time function.

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Joint input-output method [6] involves combining the signals acting at the input and output of the system into a single signal vector. This vector is considered to be the output of some imaginary dynamic system, at the input of which there is a virtual test signal with the known dynamics, for example, "white noise". It is believed [12] that the main advantage of this method is associated with the absence of the need for a priori information about the system and disturbances. However, it also has certain limitations (disadvantages):

– the need to measure all useful signals, disturbances and interferences that are present during the experiment;

– the experiment must reproduce real conditions of the system;

– the requirement of low sensor inertia and high signal-to-noise ratio;

– the need to pre-determine the order of the control object.

Two Stage Method [6, 8] consists of reducing a closed-loop system to an equivalent open system and then parametrically identifying this open system. The main difficulty in applying this method is the need to fulfil several requirements:

– having a priori knowledge of the system's block diagram;

– the block diagram should have one input and one output;

– sensors must have low inertia and measurement noise;

– the controller must follow a linear control law.

The purpose of the study, the results of which are presented in this article, is to extend the indirect identification method to the case of estimating the dynamics model of one of the elements of a two-level closed-loop control system based on passive experiment data.

To achieve this goal, we solved the problem of determining the set of necessary a posteriori information about the signal vectors in the control paths of the system (Fig. 4), as well as the substantiation of the algorithm for identifying the dynamics of one of the elements of a closed-loop system, provided that the identified dynamics model delivers an extreme of the selected quality indicator.

3 MATERIALS AND METHODS

According to the problem statement and the block diagram (Fig. 3), the identification error vector must satisfy the equation

$$\varepsilon_x = x_n - \Phi\zeta, \tag{2}$$

where Φ is a block matrix of transfer functions of size $\times(n+m)$ type

$$\Phi=[\Phi_{11}\ \Phi_{12}], \tag{3}$$

that relate the output of the identified master controller model (the reconstructed vector x_n) to the input vectors δ, r, ψ, φ, ζ is a generalised input vector of the form

$$\zeta = \begin{bmatrix} \delta \\ r + \varphi_0 \end{bmatrix}, \tag{4}$$

where φ_0 is a dummy measurement noise vector equal to

$$\varphi_0 = \begin{bmatrix} -W_1 P_1^{-1} & -E_m \end{bmatrix} \begin{bmatrix} \psi \\ \varphi \end{bmatrix}. \tag{5}$$

The sum of the weighted variances of identification errors can be determined in the frequency domain [18] by applying the Wiener-Khinchin theorem in vector form [19] to expression (2) as

$$J = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\left\{\left(S'_{x_n x_n} - S'_{\zeta x_n}\Phi_* - \Phi S'_{x\zeta} + \Phi S'_{\zeta\zeta}\Phi_*\right)R\right\}\cdot ds, \tag{6}$$

where $S'_{x_n x_n}$ is a transposed matrix of spectral densities of the vector x_n determined as a result of statistical processing of records of the components of this vector; $S'_{\zeta x}$ is a transposed matrix of mutual spectral densities between the generalised input vector ζ and the vector x_n

$$S'_{\zeta x} = \begin{bmatrix} S'_{\delta x_n} & S'_{r x_n} + S'_{\varphi_0 x_n} \end{bmatrix}, \tag{7}$$

The index * denotes the Hermitian conjugation of the matrix; $S'_{x\zeta}$ is a transposed matrix of cross spectral densities between the vectors x_n and ζ, which is equal to

$$S'_{x\zeta}\left(S'_{\zeta x}\right)_*;$$

$S'_{\zeta\zeta}$ is a transposed spectral density matrix of the generalised input vector (4), which has the following form in the case of vectors' δ, r, ψ, φ different and independent origin sources

$$S'_{\zeta\zeta} = \begin{bmatrix} S'_{\delta\delta} & O_{n\times m} \\ O_{m\times n} & S'_{rr} + S'_{\varphi_0\varphi_0} \end{bmatrix}; \tag{8}$$

$O_{m\times n}$ is a zero matrix of size $m\times n$; R is a positively defined weight matrix.

The transposed matrices of spectral densities S'_{rr} and cross spectral densities $S'_{\zeta x}$ from expressions (7), (8) can be found as a result of approximating the estimates of these matrices obtained, for example, using the CPSD function of the Matlab package [20], on the class of fractional rational functions of complex argument [16]. The transposed spectral density matrix of the fictitious noise vector φ_0, obtained by using the Wiener-Khinchin theorem applied to the vector (5), can be represented as

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$$S'_{\varphi_0\varphi_0} = W_1 P_1^{-1} S'_{\psi\psi} P_{1*}^{-1} W_{1*} + S'_{\varphi\varphi} . \qquad (9)$$

The expression for calculating the transposed cross spectral densities matrix $S'_{\varphi_0 x_n}$, is determined as a result of the structural scheme (Fig. 3) transformations by the following equation

$$S'_{\varphi_0 x_n} = S'_{u_n x_n} + S'_{x_n x_n} M_{1*} P_{1*}^{-1} W_{1*} - S'_{r x_n} ; \qquad (10)$$

where the transposed matrices of spectral and cross spectral densities $S'_{x_n x_n}$, $S'_{u_n x_n}$, $S'_{r x_n}$, as well as the matrix $S'_{\zeta x}$ can be estimated from the records of the components of the vectors r, u_n, x_n.

The only difficulty is to find the equation of the relationship between the transposed matrix $S'_{\delta x_n}$ and the original data of the identification task. The analysis of the block diagram in Fig. 3 shows the equation

$$x_n = W_n \left(r + \varphi_0 - P_1^{-1} M_1 x_n \right) + W_p \delta . \qquad (11)$$

In addition, the control systems statistical dynamics postulates [21] allow proving the following identity for the transfer function matrix W_p (Fig. 1):

$$S'_{\delta x_n} \left(S'_{\delta\delta} \right)^{-1} S'_{x_n \delta} = W_p S'_{\delta\delta} W_{p*} . \qquad (12)$$

Thus, as a result of applying Wiener-Khinchin theorem to the left and right sides of equation (11), taking into account identity (12), the following matrix coupling equation is determined

$$S'_{\delta x_n} \left(S'_{\delta\delta} \right)^{-1} S'_{x_n \delta} = S'_{x_n x_n} - S'_{r x_n} \left(S'_{rr} \right)^{-1} S'_{x_n r} - - A \left(S'_{\varphi_0\varphi_0} \right)^{-1} A_* , \qquad (13)$$

where

$$A = S'_{u_n x_n} + S'_{x_n x_n} M_{1*} P_{1*}^{-1} W_{1*} - S'_{r x_n} .$$

Factoring the right-hand side of equation (13) on the left [22] along with taking into account the known form of the spectral density matrix of uncorrelated white noises of single intensity $S'_{\delta\delta}$, allows finding the transposed matrix $S'_{\delta x_n}$. The materials presented in [14] prove that there is a connection between matrices Φ_{11}, Φ_{12} and other matrices from the structural diagram (Fig. 3). It is formalised by the following equations:

$$\Phi_{11} = F_1^{-1} W_p , \qquad (14)$$

$$\Phi_{12} = F_1^{-1} W_n , \qquad (15)$$

where

$$F_1 = E_n + W_n W_1 P_1^{-1} . \qquad (16)$$

Equations (14)–(16), given the known matrices Φ_{11} and Φ_{12}, allow uniquely finding the matrices of the transfer functions W_n and W_p. Thus, the problem of optimal identification of the closed-loop system elements is reduced to finding the transfer functions matrix Φ corresponding to a physically possible system while the functional (6) being minimal, given the known matrices of spectral and cross spectral densities $S'_{x_n x_n}$ and $S'_{\zeta x}$.

The solution to the problem was found by the well-known Wiener-Kolmogorov method of minimising the quadratic functional (6) on the class of transfer function matrices of physically possible systems Φ in the frequency domain. In accordance with the chosen method, the first variation of the functional (6) was found in the following form

$$\delta J = \frac{1}{j} \int_{-j\infty}^{j\infty} tr \left\{ - R S'_{\zeta x} + R \Phi S'_{\zeta\zeta} \right\} \delta \Phi_* ds + + \frac{1}{j} \int_{-j\infty}^{j\infty} tr \delta \Phi \left\{ - S'_{x\zeta} R + S'_{\zeta\zeta} \Phi_* R \right\} ds. \qquad (17)$$

The search for physically possible Lyapunov variational matrices requires factorisation of the weight matrix R on the right and factorisation of the transposed spectral density matrix of the generalised input vector (8) on the left [22]. As a result of these operations, the stable matrices Γ and D, together with their inverses, are found to satisfy by the equations

$$\Gamma_* \Gamma = R , \quad D D_* = S'_{\zeta\zeta} . \qquad (18)$$

Substituting the obtained equations (18) into the variation of (17) allows representing the latter as

$$\delta J = \frac{1}{j} \int_{-j\infty}^{j\infty} tr \left\{ \Gamma_* \left[- \Gamma S'_{\zeta x} D_*^{-1} + \Gamma \Phi D \right] D_* \right\} \delta \Phi_* ds + + \frac{1}{j} \int_{-j\infty}^{j\infty} tr \delta \Phi \left\{ D \left[- D^{-1} S'_{x\zeta} \Gamma_* + D_* \Phi_* \Gamma_* \right] \right\} ds. \qquad (19)$$

As a result of the separation (splitting) of the product of matrices $-\Gamma S'_{\zeta x} D_*^{-1}$ is represented as the sum of three matrices:

$$N_0 + N_+ + N_- = -\Gamma S'_{\zeta x} D_*^{-1} , \qquad (20)$$

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where N_0 is the matrix of the results of dividing the polynomials of the numerators by the polynomials of the denominators of the product on the right side of the expression (20); N_+ is the matrix of fractional rational functions of the complex argument $s = j\omega$, with the poles located in the left half-plane of the complex plane; $N\_$ is the matrix of fractional rational functions with poles in the right half-plane.

Substituting the result (20) into the variation of (19) allows obtaining the following result:

$$\delta J = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\{\Gamma_*[N_0 + N_+ + \Gamma\Phi D]D_*\}\delta\Phi_* ds + \frac{1}{j}\int_{-j\infty}^{j\infty} tr\{\Gamma_* N\_ D_*\}\delta\Phi_* ds + \frac{1}{j}\int_{-j\infty}^{j\infty} tr\delta\Phi\{D[N_{0*} + N_{+*} + D_*\Phi_*\Gamma_*]\Gamma\}ds + \frac{1}{j}\int_{-j\infty}^{j\infty} tr\{DN_{-*}\Gamma\}\delta\Phi ds. \quad (21)$$

According to the Cauchy residual theorem, the second and fourth integrals in expression (21) are zero, and the condition for ensuring the minimum of the functional (6) on the class of stable and minimally phase variations of Φ is as follows:

$$\Gamma\Phi D = -(N_0 + N_+). \quad (22)$$

The solution of equation (22) with respect to the transfer function matrix Φ:

$$\Phi = -\Gamma^{-1}(N_0 + N_+)D^{-1}, \quad (23)$$

allows determining the blocks Φ_{11} and Φ_{12} on the basis of relation (3) and proceed to the search for the matrices of transfer functions W_n and W_p. Substituting expression (16) into relation (15) allows writing the following rule for identifying the transfer function matrix W_n:

$$W_n = \Phi_{12}\left(E_m + W_1 P_1^{-1} M_1 \Phi_{12}\right)^{-1}. \quad (24)$$

The transformation of equations (14) and (16) defines the rule for identifying the transfer function matrix W_p in the following form:

$$W_p = \left(E_m + W_n W_1 P_1^{-1} M_1\right)\Phi_{11}. \quad (25)$$

Thus, equations (7)–(10) and (13) have been defined, which allow forming a set of a posteriori information about signal vectors in the control paths of a closed-loop system necessary for identification. In addition, the rules (23)–(25) are substantiated, which define an algorithm for identifying two-level closed-loop control system's one of the elements dynamics model based on passive experiment data, on the condition that the identified dynamics model delivers an extremum of the quadratic quality index (6). Thus, it is optimal [23].

4 EXPERIMENTS

The basis for checking the correctness of the new identification rules is the principle of comparing the given transfer functions W_{n0} and W_{p0} of the form:

$$W_{n0} = \frac{0.3(s+2)}{s}, \quad W_{p0} = \frac{0.032}{s+0.053}, \quad (26)$$

with the transfer functions W_n and W_p calculated as a result of applying algorithms (24), (25). As the initial data for identification, we used the structural diagram of the system (Fig. 3) with the following dynamic characteristics of it's known elements:

$$M_1 = 2, \quad P_1 = 10(s+0.2), \quad W_1 = 1, \quad (27)$$

as well as the following spectral densities

$$S'_{rr} = \frac{0.01}{-s^2 + 0.01}, \quad S'_{\psi\psi} = \frac{0.1}{-s^2+1}, \quad S'_{\varphi\varphi} = 0.001, \quad S'_{\delta\delta} = 1. \quad (28)$$

Let us assume that the following spectral and cross spectral densities are obtained as a result of processing the statistical data on the records of signals r, u_n, x_n:

$$S'_{x_n x_n} = \frac{\left|(s+0.35)(s+0.057)(s^2+5.3s+7.1)\right|^2}{\left|(s+1)(s+0.053)(s^2+0.26s+0.12)\right|^2} \times 0.9\cdot 10^{-5}, \quad (29)$$

$$S'_{r x_n} = \frac{0.003(s+2)(s+0.2)}{|s+0.1|^2(s^2+0.26s+0.12)}, \quad (30)$$

$$S'_{u_n x_n} = \frac{0.0003(-s+0.08)|s+0.21|^2(s+1.99)}{\left|(s+0.053)(s+0.1)(s^2+0.13s+0.06)\right|^2} \times |s+3.16|^2(s^2+0.11s+0.004). \quad (31)$$

In this case, to form the transposed matrix (8), the following spectral density $S'_{\varphi_0\varphi_0}$ was found as a result of substituting the corresponding data from (27) and (28) into expression (9):

$$S'_{\varphi_0\varphi_0} = \frac{0.001\left|s^2+1.43s+1\right|^2}{\left|(s+0.1)(s+0.2)\right|}. \quad (32)$$

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Thus, the transposed spectral density matrix of the generalised input vector, compiled in accordance with expression (9), taking into account the result of (32), is as follows:

$$S'_{\zeta\zeta} = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{0.001\left|(s+3.148)(s+0.38)\right|^2}{\left|(s+0.1)(s+0.2)\right|^2} \end{bmatrix}. \quad (33)$$

From expression (7), it follows that to determine the transposed matrix of mutual spectral densities between the generalised input vector ζ and the vector x_n, it is necessary to find the spectral densities $S'_{\varphi_0 x_n}$ and $S'_{\delta x_n}$. Substitution of the corresponding matrices from relations (27), (29)–(31) into expression (10) allows us to determine that

$$S'_{\varphi_0 x_n} = \frac{0.0003(s+2)\left|s^2+1.43s+1\right|^2}{(-s+0.2)\left(s^2+0.26s+0.12\right)\left|s+0.1\right|^2}. \quad (34)$$

The calculation of the right-hand side of the coupling equation (13) and taking into account the value of $S'_{\delta\delta}$, from expressions (28) substantiates the following relationship:

$$S'_{\delta x_n} S'_{x_n\delta} = \frac{0.000997\left|s+0.2\right|^2}{\left|(s+0.053)\left(s^2+0.26s+0.12\right)\right|^2}. \quad (35)$$

Factoring the right-hand side of equation (35) from the left provided the spectral density $S'_{\delta x_n}$ in the following form

$$S'_{\delta x_n} = \frac{0.032(s+0.2)}{(s+0.053)\left(s^2+0.26s+0.12\right)}. \quad (36)$$

Thus, the transposed matrix of mutual spectral densities between the generalised input vector ζ and the vector x_n, taking into account relations (7), (30), (34) and (36), is represented as

$$S'_{\zeta x} = \left[\frac{0.032(s+0.2)}{(s+0.053)\left(s^2+0.26s+0.12\right)} \quad \frac{0.0003(s+2)\left|(s+3.148)(s+0.38)\right|^2}{(-s+0.2)\left(s^2+0.26s+0.12\right)\left|s+0.1\right|^2}\right]. \quad (37)$$

Since vector x_n has only one component, the weighting coefficient is used instead of the weighting matrix R:

$$R=1. \quad (38)$$

From expression (38) and the definition of the factorisation operation of the fractional rational function on the right [22], it follows that

$$\Gamma=1. \quad (39)$$

The left factorisation of matrix (33) allowed defining the following fractional rational matrix D, all features of which are located in the left half-plane of the complex variable $s=j\omega$,

$$D = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{0.032(s+3.148)(s+0.38)}{(s+0.1)(s+0.2)} \end{bmatrix}. \quad (40)$$

The initial data for separation were determined by expression (20), taking into account the results of (37), (39), (40) in the following form

$$N_0 + N_+ + N_- = \left[\frac{-0.032(s+0.2)}{(s+0.053)\left(s^2+0.26s+0.12\right)} \quad \frac{-0.0095(s+3.148)(s+2)(s+0.38)}{(s+0.1)\left(s^2+0.26s+0.12\right)}\right]. \quad (41)$$

Since the fractional rational functions on the right-hand side of equation (41) have features with a negative real part, the result of the separation coincides with the initial data for it, namely

$$N_0 + N_+ = N_0 + N_+ + N_-. \quad (42)$$

Substituting the results (39), (40), (42) into rule (23) allows finding the following matrix Φ, which ensures the minimum of the functional (6),

$$\Phi = \left[\frac{0.032(s+0.2)}{(s+0.053)\left(s^2+0.26s+0.12\right)} \quad \frac{0.3(s+0.2)(s+2)}{\left(s^2+0.26s+0.12\right)}\right]. \quad (43)$$

The analysis of the right-hand side of equation (43) shows that two relations are fulfilled:

$$\Phi_{11} = \frac{0.032(s+0.2)}{(s+0.053)\left(s^2+0.26s+0.12\right)}; \quad (44)$$

$$\Phi_{12} = \frac{0.3(s+0.2)(s+2)}{\left(s^2+0.26s+0.12\right)}. \quad (45)$$

They provide possibility of finding optimal estimates of the transfer functions W_n and W_p. Using equation (24), taking into account the given data (27) and the result (45), we prove that

$$W_n = \frac{0.3(s+2)}{s}. \quad (46)$$

At the same time, substituting data from (27), (44) and (46) into the relation (25) allowed identifying the second transfer function in the form of

$$W_p = \frac{0.032}{s+0.053}. \quad (47)$$

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Comparison of the obtained transfer functions (46) and (47) with the given transfer functions (26) proves the correctness of the rules justified in the problem solution process.

Thus, the research goal has been achieved. The rules that extend the effect of the indirect identification method to the case of estimating the two-level closed-loop control system's one of the elements dynamics model based on passive experiment data have been determined.

5 RESULTS

As a result of the research, the identification problem was formalized, equations (7)–(10) and (13) which allow forming a set of a posteriori information about the input-output signals necessary for identification are defined,.

The rules (23)–(25) for identifying the two-level closed-loop control system's one of the elements dynamics model, which minimizes the sum of the identification errors variances in the frequency domain (6), are obtained and verified.

6 DISCUSSION

The conditions and restrictions on the use of the new frequency method for closed-loop system elements optimal identification were determined as follows:

– the system operates under the influence of one-dimensional or multidimensional centred stationary useful signals, disturbances and measurement noises, the dynamics of which may differ from white noise;

– models of dynamics of system elements that are not subject to identification should be known in advance;

– models of the dynamics of external influences on the system that act during the identification experiment must be specified;

– it is necessary to ensure the possibility of measuring the input-output signals of the closed-loop system element to be identified.

The signals in the control paths of closed-loop systems analysis proves the possibility of these signals mathematical expectations changing effect, even under the conditions of existence only centred stationary input influences on the system. On this basis, further development of research can be directed at overcoming such effects.

CONCLUSIONS

The urgent problem of mathematical support development is solved for identifying the two-level closed-loop control system's one of the elements dynamics model with the minimum of the identification errors variances sum.

The application of the Wiener-Kolmogorov ideas allows overcoming the contradictions between the assumptions made in the formulation of the identification method and a two-level closed-loop system design and operating conditions by developing and applying new rules for optimal identification of this system elements.

The scientific novelty of obtained results is that a result of the study, a new method for identifying a complex multidimensional element of a two-level closed-loop control system was determined for the first time. The justified method has two main distinguishing features. The first feature is that to solve the identification problem it is enough to measure one of the two vectors at the inputs of the identification object and the vector of signals at its outputs. The second distinctive feature is that as a result of solving the identification problem, two transfer function matrices are determined. The first characterizes the influence of the vector of controlled input signals on the output signals of the object, and the second determines the influence of the vector of uncontrolled input signals on the output of the object.

The practical significance of the results lies in the fact that the a priori conditions for obtaining initial data for identifying the elements of a closed-loop control system are substantiated. This allows for effective planning of an identification experiment, as well as justifying the list of signals that are to be recorded during this experiment.

Prospects for further research are to develop methods and rules for overcoming the effect of violation of stationarity in closed-loop control systems, which occurs even under the action of centered stationary random influences, when carrying out identification.

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ЧАСТОТНИЙ МЕТОД ОПТИМАЛЬНОЇ ІДЕНТИФІКАЦІЇ ЕЛЕМЕНТІВ ЗАМКНЕНОЇ СИСТЕМИ

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АНОТАЦІЯ

Актуальність. Стаття присвячена подоланню протиріч між припущеннями, прийнятими при формулюванні методу ідентифікації, та конструкцією і умовами функціонування замкненої системи керування. У статті здійснено приведення архітектури дворівневої замкненої систем керування до структурної схеми, яка має дві частини. Перша частина поєднує у собі головний контролер та систему зв'язку з головними сенсорами. Друга частина складається з локальної систему керування, кінематичної ланки та головних сенсорів.

Мета роботи. Метою дослідження, результати якого представлені у цій статті, є поширення дії непрямого методу ідентифікації на випадок оцінювання моделі динаміки головного контролера дворівневої замкненої системи керування за даними пасивного експерименту.

Метод. У статті використано метод ідентифікації в частотній області багатовимірних стохастичних систем стабілізації рухомих об'єктів з довільною динамікою. Початкова інформація про зміни сигналів «вхід-вихід» отримана за даними пасивного експерименту під час натурних випробувань, яка спотворена недосконалістю вимірювальних приладів та системи реєстрації.

Результати. Визначено новий метод ідентифікації елементів дворівневої замкненої системи керування, яка функціонує в умовах дії багатовимірних стаціонарних центрованих випадкових впливів.

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Висновки. Обґрунтовані правила дозволяють коректно визначати матриці передатних функцій обраного елемента замкненої системи при виконанні визначеного переліку умов. Проведений аналіз сигналів контурів керування замкнутими системами доводить можливість існування ефекту зміни цих сигналів статистичними засобами навіть за умов дії на систему лише зосереджених стаціонарних вхідних впливів. Виходячи з цього, подальший розвиток досліджень може бути спрямований на подолання таких ефектів.

КЛЮЧОВІ СЛОВА: ідентифікація, матриця передавальних функцій, спектральна щільність, дисперсія похибки, функціонал якості.

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