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SCATTERING OF ELECTROMAGNETIC WAVES ON FLAT GRID TWO-PERIODIC STRUCTURES

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ABSTRACT

Context. One of the scientific hypotheses for the creation of nonreciprocal optical metasurfaces is based on the use of a wave channel in which rays of the direct and reverse diffraction scenarios are realized on two-periodic flat structures with nonlinear elements. Such processes in the nanometer wavelength range of electronic devices require precise calculations of the interaction of waves and microstructures of devices. It is also important to describe the behavior of antenna devices in mobile communications. Expanding the wavelength range of stable communication is achieved by using prefractal structures in antenna devices in combination with periodic structuring. Similar modeling problems arise when electromagnetic waves penetrate materials with a crystalline structure (radio transparency).

Objective. To test this hypothesis, it is necessary to carry out mathematical modeling of the process of scattering of electromagnetic waves by metasurfaces under conditions of excitation of several diffraction orders. It is known that among two-periodic flat lattices of different structures there are five types that fill the plane. These are the Bravais grilles. The problem of scattering of an incident monochromatic TE polarized wave on a metal screen with recesses in two-periodic structures filled with silicon was considered.

Method. The paper builds mathematical models for the study of spatial-amplitude spectra of metasurfaces on Brave lattices and gives some results of their numerical study. The condition for determining the diffraction orders propagating over the grating is proposed. Scattered field amplitudes are from the solution of the boundary value problem for the Helmholtz equation in the COMSOL Multiphysics 5.4 package. Similar problem formulations are possible when studying the penetration of an electromagnetic field into a crystalline substance.

Results. Obtained relations for diffraction orders of electromagnetic waves scattered by a diffraction grating. The existence of wavelengths incident on a two-periodic lattice for which there is no reflected wave is shown for different shapes (rectangular, square, hexagonal) of periodic elements in the center of which a depression filled with silicon was made. Distributions of reflection coefficients for different geometric sizes of colored elements and recesses are given. The characteristics of the electric field at resonant modes in the form of modulus isolines show the nature of the interaction of the field over the periodic lattice and the scatterers-depressions. At the resonant wavelengths of the incident waves, standing waves appear in the scatterers.

Conclusions. A mathematical model of the set of diffraction orders propagating from a square and hexagonal lattice into half-space is proposed $z \geq 0$. It has been shown that flat periodic lattice with square or hexagonal periodicity elements and resonant scatterers in the form of cylindrical recesses filled with silicon can produce a non-mirrored scattered field in metal. The response of the lattices to changes in the wavelength of the incident field by the structure of diffraction orders of the scattered field and high sensitivity to the rotation of the incident plane were revealed. The two-periodic lattices have prospects for creating anti-reflective surfaces of various devices. Two-periodic lattices have prospects for creating anti-reflective surfaces for various devices, laser or sensor electronic devices, antennas in mobile communication elements, and radio transparency elements. They have more advanced manufacturing technologies in relation to spatial crystal structures.

KEYWORDS: Maxwell's equation, periodic lattice elements, diffraction, diffraction orders, non-reciprocity of diffraction spots, numerical tracking methods, resonant metasurface, non-specular reflection.

NOMENCLATURE

EMW is a electro-magnetic wave;

$\vec{E}(x,y,z,t)$ is a vectors of electrical intensity field;

$\vec{H}(x,y,z,t)$ is a vectors of magnetically intensity field;

$\rho(x,y,z,t)$ is a density of distribution of electric charges;

ε_α is an absolute electrical permeability of the medium;

ε_0 is an electric constant (or vacuum permittivity);

μ_α is an absolute magnetic permeability of the medium;

μ_0 is a magnetic constant (or vacuum permeability);

σ is a conductivity of the medium;

ω is a cyclic frequency;

$\vec{U}(x,y,z)$ is a total field amplitude;

S_{mn} is a modes (harmonics);

Γ_{mn} is a constant spreading for modes S_{mn};

θ is an EMF incidence angle;

φ is an azimuthal angle of the plane of the wave vector of the incident wave;

α is an angle between the coordinate axes;

λ is a wavelength;

ℓ_1, ℓ_2 are lattice parameters;

a_1, h_1 are the radius of the cylinder;

h_1 is a parameters of a cylindrical diffuser.

INTRODUCTION

In cases where the characteristic dimensions of the obstacles to light propagation are proportional to the wavelength, adequate models of light diffraction on them are

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based on the Maxwell's system of equations. Problems based on them are solved using the following numerical methods:

– finite and boundary element method for integral equations;

– modal methods for differential and integral equations;

– difference methods for systems of differential and integral equations;

– a method for solving problems on eigenvalues and eigenfunctions for differential and integral operators, which implements the projection method on the basis of the eigenfunctions of the problem operator.

Modal methods have a significant drawback - an increase in numerical complexity with an increase in the number of modes. The time-domain unfolding along orthogonal modes in the time domain for transversely inhomogeneous structures remains problematic.

The object of study is the diffraction of electromagnetic waves on biperiodic lattices with recesses.

The subject of study is the subject of the study is the conditions on diffraction orders scattered above the lattice and their amplitudes.

The purpose of the work is to separate the wavelengths of the incident field at which there is no reflected beam for different forms of periodic elements with a depression on a biperiodic lattice.

1 PROBLEM STATEMENT

To study the optical properties of complex periodic structures, it is necessary to overcome the limit of numerical complexity. This is possible when using numerical methods. Variation methods (for example, the Galerkin's method) applied to the Helmholtz equation with respect to the spatial amplitudes of the full field include the choice of the sampling scheme, construction and minimization. The obtained relations are transformed into a system of linear algebraic equations to which discrete analogues of boundary conditions are added.

Maxwell's equations for the electromagnetic field in a conducting medium are as follows:

$$\begin{cases} \varepsilon_\alpha \dfrac{\partial \vec{E}}{\partial t} + \sigma \vec{E} = rot \vec{H}, \\ -\mu_\alpha \dfrac{\partial \vec{H}}{\partial t} = rot \vec{E}, \\ div \vec{E} = \dfrac{\rho}{\varepsilon_\alpha}, \\ div \vec{H} = 0. \end{cases}$$

The system can be used to obtain wave equations for electric and magnetic tension vectors of field in the space-time representation or frequency range. Thus, for the vector $\vec{E}(x,y,z,t) = \vec{U}(x,y,z)e^{i\omega t}$, the amplitude of the total field $\vec{U}(x,y,z)$ in the frequency domain is obtained from the solution of the following boundary value problem for the system of equations in a three-dimensional periodic element over the plane of the structure.

$$rot\left[\frac{1}{\mu_r} rot \vec{U}\right] - k_0^2 \left(\varepsilon_r - \frac{i\sigma}{\omega\varepsilon_0}\right)\vec{U} = 0 .$$

Substituting the expansion of the field amplitude vector components along the basis from piecewise linear functions into the upper equation, we obtain residual for minimization together with additional boundary conditions on the perfectly conducting basis of the plane structure by projecting it onto the basic elements. We calculate the necessary characteristics of the scattered field above the lattice (for example, reflection coefficients of diffraction modes, intensity of scattered rays (modes), etc.) from the found field parameters.

Let us consider some rectangular and square periodic lattices with scatterers of different depth located in their centers, and forms in the plane, a fragment of which is shown in Fig. 1.

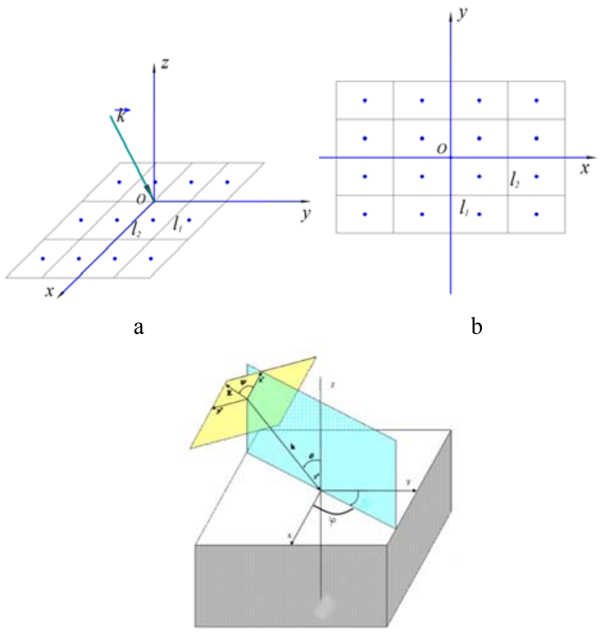

Figure 1 – Rectangular and square ($l_1 = l_2$) lattice: a – spatial position; b – in-plane configuration of the lattice; c – angular characteristics of the falling field

The lattice formed by the periodic translation of the parallelogram ($l_1 = l_2$) are also of practical interest (Fig. 2).

Hexagonal lattices are widely used in antenna technology. For example, the structure of graphene is a periodic combination of hexagonal elements (Fig. 3).

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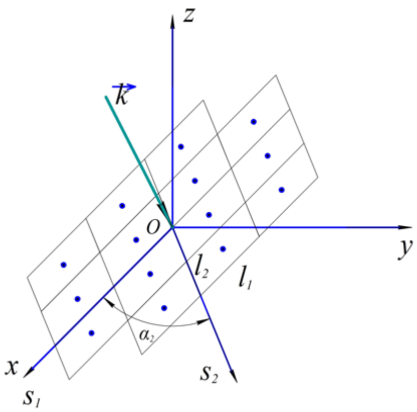

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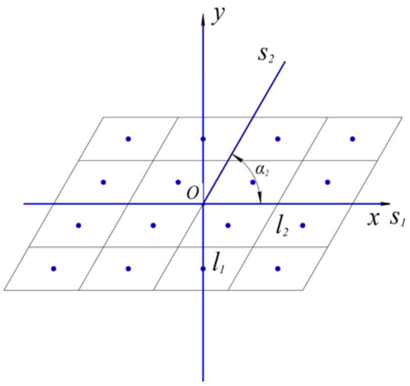
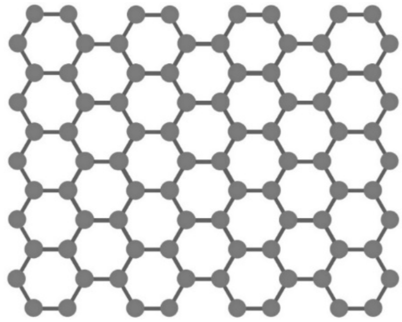


b

Figure 2 – Lattices made of parallelograms and rhombuses: a – spatial position; b – in-plane configuration of the grating

Fig. 3 shows the hexagonal elements and their associated three sets of rhombuses-shaped elements that fill the unbounded lattice plane and the required coordinate systems (xoy – the original coordinate system associated with hexagons, $x'oy'$ – a coordinate system rotated by an angle α, s_1os_2 – an oblique coordinate system associated with associates rhombuses, for example, type I).

Transmissive and reflective diffraction gratings are used to spatially separate electromagnetic waves (EMW) into a spectrum. We focus on the study of some of its regularities and methods of controlling the characteristics for two-periodic grilles-screens in the xoy plane. It is assumed that in the centers of the periodic elements there are depressions – scatterers of various shapes. For the study, we choose the first square grille (Fig. 1) with scatterers in the form of cylindrical recesses with radius a_1 and depth h_1 (Fig. 3b, c) that can be filled with different materials.



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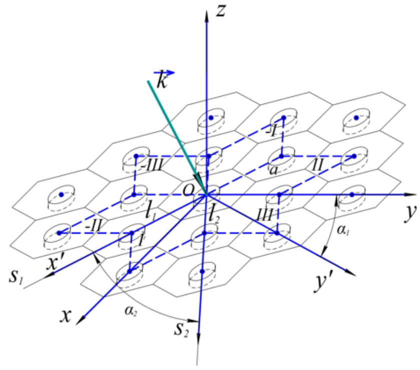

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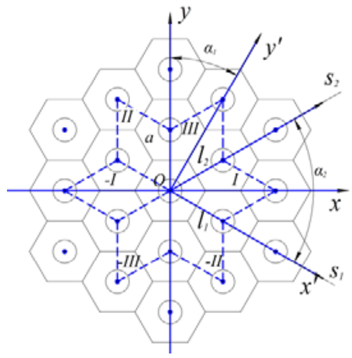

c

Figure 3 – The structure of grapheme: a – hexagonal lattice of graphene, carbon layer in a honeycomb packing, lattice of regular ($l_1 = l_2$) hexagons; b – spatial position of the hexagonal lattice with cylindrical diffusers; c – a fragment of the lattice in the plane and possible rhombuses to fill it

2 REVIEW OF THE LITERATURE

The possibilities of using metamaterials in acoustics, optics and radiophysics are related to the results of research in the theory of electromagnetic wave diffraction on biperiodic flat structures. The characteristic size of the geometric periodicity is of the same order as the wavelength of the incident field. The parameters of dielectric and magnetic permeability also have corresponding periodic changes. Such structures have an advantage over spacious three-period structures in terms of their affordability. Flat biperiodic lattices in the radio and optical wavelength ranges allow obtaining zones of non-reciprocity (absence of reflected rays) in scattered or (deviations in the law of ray direction) penetrating rays (orders) of electromagnetic or sound waves that are not typical for natural materials. The similarity of the mechanisms of nonre-

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ciprocal reflection and penetration into the material stimulated the development of their mathematical modeling based on Maxwell's equations. Among them, we note the tasks of studying the practical application of photonic crystals [1–3]. The study of the nonreciprocal scattering and the peculiarities of metamaterials in the form of biperiodic lattices with various shapes (Brave lattices) of a periodic element was performed in [4–7]. The presence of a scattered field without reflected rays is noted for some frequencies of the incident field. Numerical methods are used to study this phenomenon numerically [8–10]. A numerical study of the non-reciprocity phenomenon allows us to obtain detailed information about the polarization features of the scattered field and the diffraction orders of the rays in it.

3 MATERIALS AND METHODS

The scatterers can be located on two families of parallel lines with periods l_1 and l_2. In the lattices plane, we introduce two coordinate systems – a rectangular one with the basis orths $\overrightarrow{e_x}, \overrightarrow{e_y}$, and an oblique one $s_1 o s_2$ with the basis orths $\overrightarrow{e_1}, \overrightarrow{e_2}$. The position of the centers of the elements-scattering is determined by the radius-vector

$$\overrightarrow{\rho_{v_1 v_2}} = v_1 l_1 \overrightarrow{e_1} + v_2 l_2 \overrightarrow{e_2} .$$

Then the component of the wave vector incident field $\overrightarrow{k^l}$ in the grille plane is

$$\overrightarrow{k^l} = k_1 \overrightarrow{e_1} + k_2 \overrightarrow{e_2}, \quad \overrightarrow{k^l} = k_x \overrightarrow{e_x} + k_y \overrightarrow{e_y}.$$

The complete system of solutions of the scalar Helmholtz equation, in accordance with the requirements of Bloch's theorem (known as Floquet's theorem in one-dimensional problems), in the domain $z > 0$ (over a periodic lattice) can be represented as [9–10].

$$S_{mn} = e^{i\Gamma_{mn} z} e^{i[(\overrightarrow{k^l}\vec{S}) - \frac{2\pi m}{\ell_1} s_1 - \frac{2\pi n}{\ell_2} s_2]} .$$

Considering the obliquity (0< $\alpha \le \pi/2$) of the coordinate system $s_1 o s_2$ (Fig. 2) and its connection with the rectangular coordinate system xoy, where $\alpha = \alpha_2$

$$\begin{pmatrix} \overrightarrow{e_1} \\ \overrightarrow{e_2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \cos\alpha & \sin\alpha \end{pmatrix} \begin{pmatrix} \overrightarrow{e_x} \\ \overrightarrow{e_y} \end{pmatrix} = A \begin{pmatrix} \overrightarrow{e_x} \\ \overrightarrow{e_y} \end{pmatrix},$$

$$\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} 1 & -ctg\alpha \\ 0 & \dfrac{1}{\sin\alpha} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = B \begin{pmatrix} x \\ y \end{pmatrix},$$

then $B = A^{-1}$ we will get

$$S_{mn} = e^{i\Gamma_{mn} z} e^{i(k_x - \frac{2\pi m}{\ell_1})x} e^{i(k_y + \frac{2\pi m}{\ell_1 \mathrm{tg}\alpha} - \frac{2\pi n}{\ell_2 \sin\alpha})y} .$$

For each mode (harmonic) S_{mn}, the propagation constant along the z-axis is

$$\Gamma_{mn} = \sqrt{k^2 - \chi_x^2 - \chi_y^2} ,$$

where $\chi_{x,m,n} = k_x - \dfrac{2\pi m}{\ell_1}, \chi_{y,m,n} = k_y + \dfrac{2\pi m}{\ell_1 tg\alpha} - \dfrac{2\pi n}{\ell_2 \sin\alpha}$, $k_x = k\sin\theta\cos\varphi, k_y = k\sin\theta\sin\varphi, k = \dfrac{2\pi}{\lambda}$.

Each spatial mode S_{mn} for which

$$\mathrm{Im}\,\Gamma_{mn} \ge 0, \tag{1}$$

meets the condition of energy transfer from the grille plane. The mode (m = 0, n = 0) is a mirror-reflected electromagnetic wave. From condition (1) we obtain

$$k^2 - \chi_{x,m,n}^2 - \chi_{y,m,n}^2 \ge 0.$$

We will study the condition for wave propagation in the region above the lattice as a set of admissible parameters of the incident wave and the geometric characteristics of the periodicity element of the lattices.

Let us introduce the characteristic function of parameters of the lattice and incident wave

$$F\left(m,n,\theta,\varphi,\frac{\lambda}{\ell_1},\frac{\lambda}{\ell_2},\alpha\right) = \left(\sin\theta\cos\varphi - m\frac{\lambda}{\ell_1}\right)^2 + \left(\sin\theta\sin\varphi - n\frac{\lambda}{\ell_2\sin\alpha} + m\frac{\lambda}{\ell_1}\mathrm{ctg}\alpha\right)^2 - 1,$$

then, the limit of the set of diffraction orders (m, n) propagating from the lattice to the region $z > 0$ is determined by equation.

$$F\left(m,n,\theta,\varphi,\frac{\lambda}{\ell_1},\frac{\lambda}{\ell_2},\alpha\right) = 0, \tag{2}$$

and the set of orders of propagating modes satisfies the condition

$$F\left(m,n,\theta,\varphi,\frac{\lambda}{\ell_1},\frac{\lambda}{\ell_2},\alpha\right) \le 0 . \tag{3}$$

In the case of a rectangular coordinate system from (2) with $\alpha = \pi/2$, we have

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$$F\left(m,n,\theta,\varphi,\frac{\lambda}{\ell_1},\frac{\lambda}{\ell_2},\frac{\pi}{2}\right)=\left(\sin\theta\cos\varphi-m\frac{\lambda}{\ell_1}\right)^2+ \left(\sin\theta\sin\varphi-n\frac{\lambda}{\ell_2}\right)^2-1=0. \quad (4)$$

The region of propagation orders (Fig. 4) at $\theta=\frac{\pi}{2.5}$, $\varphi=\frac{\pi}{6}$, $\lambda=400$ nm, $\ell_1=500$ nm, $\ell_2=500$ nm looks like a circle, which includes a certain set of diffraction orders.

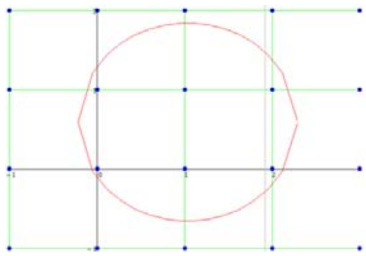


Figure 4 – Scattered diffraction orders for a square grating $\theta=\frac{\pi}{2.5}$, $\varphi=\frac{\pi}{6}$, $\lambda=400$ nm, $\ell_1=500$ nm, $\ell_2=500$ nm

The region of propagation orders in the case $\theta=\frac{\pi}{3}$, $\varphi=\frac{\pi}{6}$, $\lambda=500$ nm, $\ell_1=500$ nm, $\ell_2=500$ nm(Fig. 5) includes only four diffraction orders (0, 0), (0, 1), (1, 0), (1, 1)

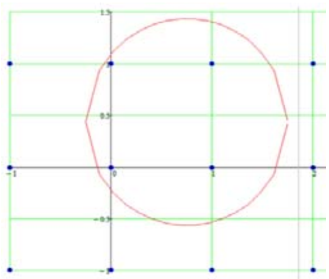


Figure 5 – Scattered diffraction orders for a square grating $\theta=\frac{\pi}{3}$, $\varphi=\frac{\pi}{6}$, $\lambda=500$ nm, $\ell_1=500$ nm, $\ell_2=500$ nm

Parameters at which the two-beam scattering mode (0.0), (1.0) (Fig. 6) is realized, for example, can be $\theta=\frac{\pi}{3}$, $\varphi=0$, $\lambda=500$ nm, $\ell_1=500$ nm, $\ell_2=500$ nm.

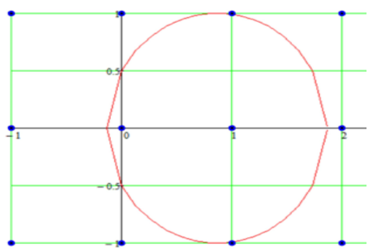
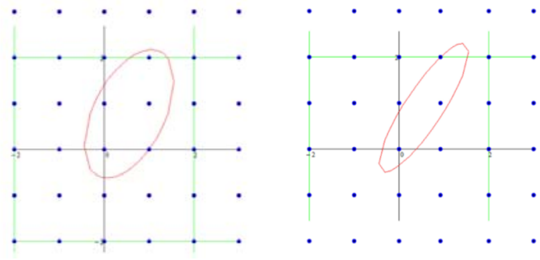


Figure 6 – Scattered diffraction orders for a square grating $\theta=\frac{\pi}{3}$, $\varphi=0$, $\lambda=500$ nm, $\ell_1=500$ nm, $\ell_2=500$ nm

Choosing the angles θ, φ, α and wavelength λ and the lattice parameters ℓ_1, ℓ_2, we obtain the condition on the diffraction orders (*m*, *n*) of scattered waves in the region z > 0 for parallelogram lattices. Thus, with $\theta=\frac{\pi}{4.5}$, $\varphi=\frac{\pi}{6}$, $\lambda=500$ nm, $\ell_1=500$ nm, $\ell_2=500$ nm, $\alpha=\frac{\pi}{3}$, we have a set of diffraction orders (Fig. 7a). The two-beam regime of scattering of orders (0, 0) and (1, 1) in such a grille is realized, for example, at $\theta=\frac{\pi}{4.5}$, $\varphi=0$, $\lambda=500$ nm, $\ell_1=500$ nm, $\ell_2=500$ nm, $\alpha=\frac{\pi}{3}$, and with a further change of α to π/6, the diffraction orders are localized (Fig. 7b).



a b

Figure 7 – Diffraction orders of a rhombus grille: a – ion orders for a square grating $\theta=\frac{\pi}{4.5}$, $\varphi=\frac{\pi}{6}$, $\lambda=500$ nm, $\ell_1=500$ nm, $\ell_2=500$ nm, $\alpha=\frac{\pi}{3}$; b – $\alpha=\frac{\pi}{6}$, $\varphi=0$

In the case of a hexagonal grille (Fig. 3) (side of the hexagon *a* = 500nm), the propagation condition for the associated diamond-shaped grille for it

$$\Phi\left(m,n,\theta,\varphi,\frac{\lambda}{\ell_1},\frac{\lambda}{\ell_2},\alpha_1,\alpha_2\right)= =\left(\sin\theta\cos\varphi-m\frac{\lambda}{\ell_1}\left[\cos(\alpha_1)+\frac{\sin(\alpha_1)}{tg(\alpha_1)}\right]+n\frac{\lambda}{\ell_2}\frac{\sin(\alpha_1)}{\sin(\alpha_2)}\right)^2+ \quad (5)$$
$$+\left(\sin\theta\cos\varphi-m\frac{\lambda}{\ell_1}\left[\sin(\alpha_1)+\frac{\cos(\alpha_1)}{\mathrm{tg}(\alpha_2)}\right]+n\frac{\lambda}{\ell_2}\frac{\cos(\alpha_1)}{\sin(\alpha_2)}\right)^2- -1\le 0,$$

where α_1 – angle of rotation of the coordinate system relative to the original, orthogonal one; α_2 – the angle between the axes of the oblique coordinate system associated with the rhombus $l_1=l_2$.

4 EXPERIMENTS

Of practical interest is the problem of increasing the power of some orders in the scattered field and minimizing the intensity of other orders, for example, (0,0). To

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investigate this possibility, we performed a comparative analysis of the propagation sets for the studied gratings, controlled the composition of the propagation mode orders by the plane wave incidence angles, and estimated their intensity.

At this stage, we considered the linear problem of the electrodynamics of the incidence of a plane monochromatic wave with TE polarization on a metal screen with periodically arranged scatterers (Fig. 2). The size of the side of the square cell is a = 750nm, the dimensions of the cylindrical recess filled with silicon are $\alpha_1 = 155$ nm(cylinder radius), $h_1 = 75$ nm (depth). The incident wave is characterized by the angles of the wave vector $\theta = 40^0$, $\varphi = 0^0$. The plane of incidence of the wave, with such data, coincides with the coordinate plane xoz.

At such values of the interaction parameters, the propagation condition (Fig. 8) gives a two-beam scattering mode (0.0), (1.0) at a certain range of incident wavelengths. The red curve is the boundary of the region of diffraction orders of propagation. Inside, the dots indicate the orders of modes that propagate from the grille to the region z > 0.

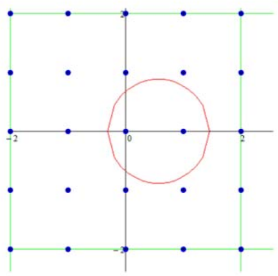


Figure 8 – Diffraction orders of grille with a quare-cell at a two-beam composition of the scattered field

A direct numerical study of the diffraction of electromagnetic waves, solving the initial boundary value problem by the finite difference method for unsteady Maxwell's equations on a spatially periodic grille element with C-shaped strips of a given length of perfectly conductive inclusions, was performed in [11].

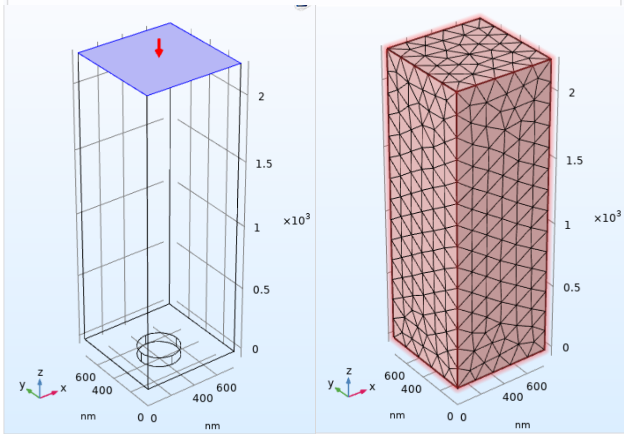
COMSOL Multiphysics 5.4 provides the ability to create and study various models of the interaction of electromagnetic waves with objects with a periodic structure. The package allows you to supplement the user interface with your own models. With the help of built-in physical interfaces and support from material properties libraries, it is possible to create adequate mathematical models to study the patterns and numerical characteristics of the interaction process.

The research used a section on the physics of beam optics. At the first stage, we considered a linear problem for the wave equations of electrodynamics when a plane monochromatic wave with a given polarization is incident on a two-periodic screen of scatterers (Fig. 2, 3). The problem for system (2) in the region (Fig. 9) is solved by the finite element method.

Fig. 9 shows a periodic fragment of the computational domain over a flat grille (a) and a variant of its discrete elemental breakdown (b) with cylindrical scatterers filled with silicon. The size of the side of the square is a = 750nm, the dimensions of the cylinder are $\alpha_1 = 155$ nm, $h_1 = 75$ nm. The incident wave is characterized by the angles of the wave vector $\theta = 40^0$, $\varphi = 0^0$ and the power $E_0 = 1V/m$, $H_0 = 1A/m$. The plane of incidence of the wave, with such data, coincides with the coordinate plane xoz.

A set of input data has been prepared for the calculations

Name	Expression	Value	Description
a	750[nm]	7.5E-7 m	square side
lda0	2*a	1.5E-6 m	Wave lehgth
h0	3*lda0	4.5E-6 m	air heigth
a1	155[nm]	1.55E-7 m	Radius diska
h1	75[nm]	7.5E-8 m	Disk heigth
theta	40[deg]	0.69813 rad	Elevation angle
phi	0[deg]	0.20944 rad	Azimut angle
E0	1[V/m]	1 V/m	Electric amplitude
H9	1[A/m]	1 A/m	Magnetic amplitude



a b

Figure 9 – Periodic element of the computational domain with an input boundary (a) and their finite element partition (b)

Zero amplitude of the mirror wave (0, 0) is realized by choosing the geometric parameters of a cylindrical diffuser – a recess ($\alpha_1 = 155$ nm, and $h_1 = 75$ nm) filled with homogeneous silicon. The search for resonant wavelengths was performed in the range $\lambda \in [800,900]$nm.

Fig. 10 shows the presence of an incident wavelength at which the reflection coefficient of the reflected wave (curve 1), mode (0, 0), is zero, and for the other beam (curve 2), which corresponds to mode (1, 0), Kr = 1.

Fig. 10 shows that for a wave of $\lambda^* \cong 848$ nm, the specular reflection coefficient is zero and the energy is emitted in modes of other orders. The cylindrical recess, which is filled with silicon, works as a waveguide connected to a free half-space and can be resonant. Analysis of the distribution of the modulus of the electric component of the field above the grille shows its moderate interference character. Under resonance conditions, the main field energy is localized in a cylindrical waveguide.

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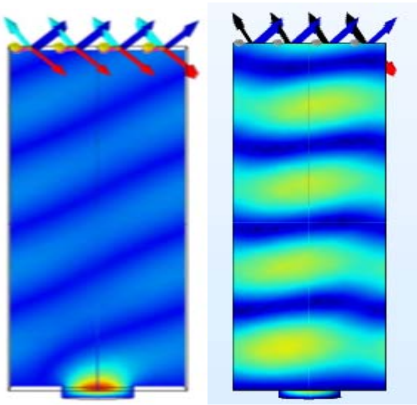
a



b c

Figure 10 – Full field characteristics over a square lattice:
a – reflection coefficient of the incident wavelengths for the following parameters $\alpha = 750$ nm; $\alpha_1 = 155$ nm, $h_1 = 75$ nm; $\theta = 40^0$, $\varphi = 0^0$.; b – isolines of the modulus of the electric field component in the xoz plane for $\lambda^* \cong 848$ nm; c – isolines of the modulus of the electric field component in the xoz plane for wavelengths other than resonant ones

The choice of the geometric dimensions of the cylindrical recess $\alpha_1 = 300$ nm, $h_1 = 300$ nm, gives rise to a more complex dependence of the reflection coefficients on the incident wavelength (Fig. 11). In a cylindrical deepening-waveguide, a standing wave with two maxima is excited, which gives rise to the shown reflection distribution.

When comparing the isolines of the modulus of the amplitude of the electric component of the field, a change in the polarization of the full-field waves over the periodic grille is noticeable at different wavelengths of the incident field (Fig. 10 b, c).

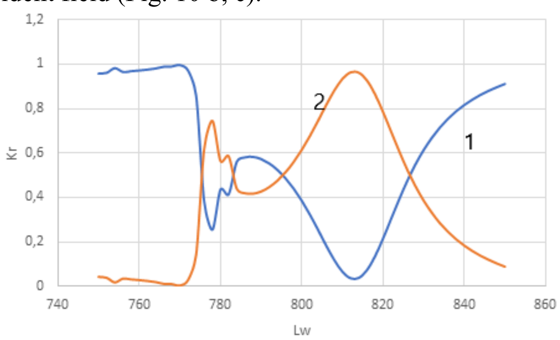
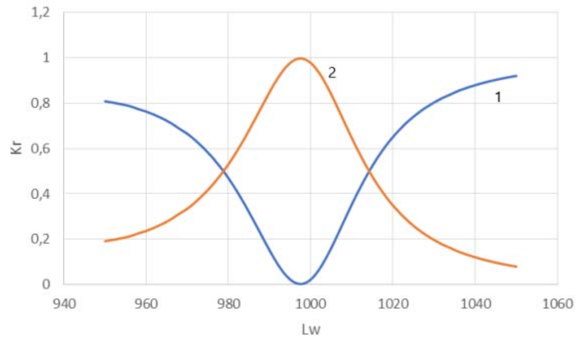
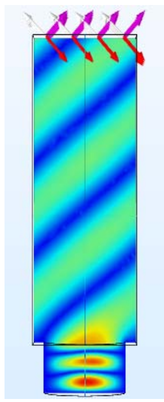


Figure 11 – Reflectance as a function of wavelength for the following parameters $\alpha = 750$ nm, $\alpha_1 = 300$ nm, $h_1 = 300$ nm, $\theta = 40^0$, $\varphi = 0^0$

A similar phenomenon of energy redistribution between waves of different orders for a given grille with square cells of periodicity can be found for other sizes of a cylindrical scatterer filled with silicon. Thus, Fig. 12 shows the results of calculations of the reflective properties of a periodic grille at the dimensions of the scatterer $\alpha_1 = 300$ nm. $h_1 = 375$ nm. Resonant complete energy redistribution is realized at $\lambda^* \cong 998$ nm.



a



b

Figure 12 – Full field characteristics over a square lattice:
a – distribution of reflection coefficients (1-mirror beam, 2-non-mirror beam) along the incident wave length for parameters $\alpha = 750$ nm, $\alpha_1 = 300$ nm, $\theta = 40^0$, $\varphi = 0^0$, $h_1 = 375$ nm;
b – isolines of the modulus of the electric field component in the xoz plane for $\lambda^* \cong 998$ nm

More complicated reflection phenomena can be observed when the size of the waveguides or the angle of the wave incidence plane is changed, when a resonant standing wave with many maxima and a nonuniform distribution of electric field parameters at the boundary of the waveguide and free half-space z≥0 is excited in the waveguide (Fig. 13).

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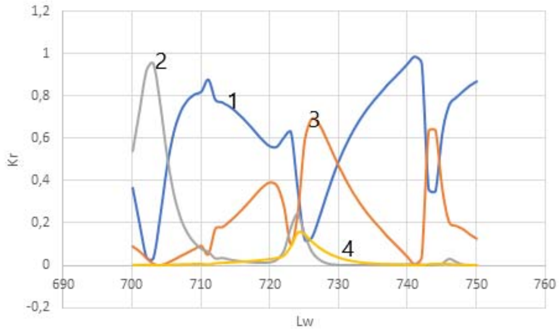
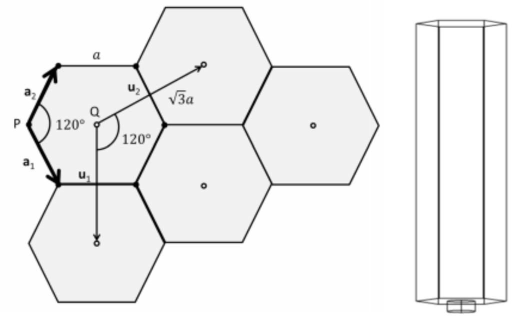


Figure 13 – Distribution of reflection coefficients (1-mirror beam, 2,3,4-nonmirror beams) by wavelength for parameter $\alpha = 750$ nm, $\alpha_1 = 300$ nm, $h_1 = 375$ nm, $\theta = 40^0$, $\varphi = 45^0$

In the wave range (Fig. 13) $\lambda \in [700,750]$nm of the incident field, there are zones where the nature of the scattered electromagnetic field and the composition of the diffraction orders that create it change. At $\lambda^* \approx 703$ nm we have a practically single-beam scattered field (curve 2), and at $\lambda^* \approx 724$ nm the field is created by four beams, which changes to a single-beam field (curve 1) reflected at $\lambda^* \approx 742$ nm. Thus, the realization of single-beam scattering is possible for a grille with not square-cell not only at angles $\varphi = 0^0$ (Fig. 12), but also at other angles, for example, $\varphi = 45^0$.

We also investigated the case of a periodic grille with hexagonal elements in the period (Fig. 14). Fig. 15 shows the reflection intensity of different modes over a hexagonal grating.

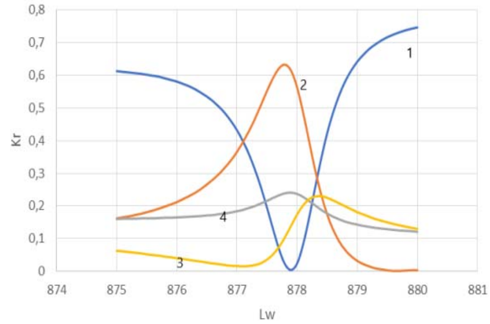
The analysis of diffraction orders for a grille of regular hexagons and a grille of associated rhombuses provides different information about the composition of the set of propagation orders. Thus, applying condition (5) and condition (3), we obtain a different composition of the set of diffraction scattering orders.



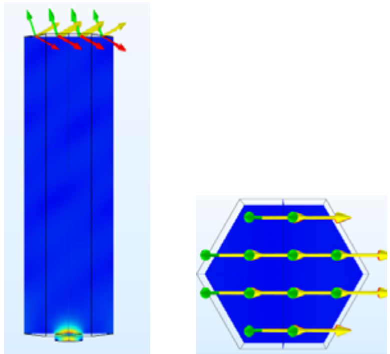
a b

c

Figure 14 – Elements of periodicity of the geometry of the computational domain of the hexagonal lattice: a – general scheme, b – spatial image, c – projection on xoy



a



b c

Figure 15 – Full field characteristics over a hexagonal lattice: a – distribution of reflection coefficients along the incident wavelength for parameters $\alpha = 500$ nm, $\alpha_1 = 155$ nm, $h_1 = 75$ nm, $\theta = \frac{\pi}{3}$, $\varphi = 60^0$; b – isolines of the modulus of the electric field vector above the grating; c – wave vectors of modes at the input boundary of the computational domain (plane parallel to xoy), λ=877.8 nm

It can be seen from Fig. 15 that the scattered field of a periodic grille made of hexagonal elements with cylindrical scatterers, the size of which is similar to a square grille, almost does not have geometric reflection at a wavelength of λ = 877.8 nm. The energy from the incident mode is transformed into several scattered modes.

In the case $\alpha = 500$ nm, $\alpha_1 = 155$ nm, $h_1 = 75$ nm, $\theta = \frac{\pi}{3}$, $\varphi = 0^0$, we have a two-beam case of the scattered field above the grille in a certain range of incident wavelengths (Fig. 16).

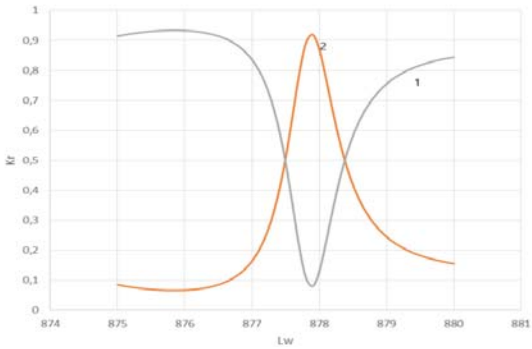


Figure 16 – Distribution of reflection coefficients (1-mirrored beam, 2-non-mirrored beam) along the incident wavelength for the parameters $\alpha = 500$ nm, $\alpha_1 = 155$ nm, $h_1 = 75$ nm, $\theta = \frac{\pi}{3}$, $\varphi = 0^0$

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We found (Fig. 16) a regime for a grille of regular hexagons where two-beam scattering with a minimum intensity of the reflected beam is realized in the main field.

It is further found that the distribution of the scattering coefficient at $\varphi = -60^0$ coincides with the distribution at φ=0.

5 RESULTS

A mathematical model of the set of diffraction orders propagating from a square and hexagonal lattice into half-space is proposed $z \geq 0$.

It is shown that flat periodic lattices with square or hexagonal periodicity elements and resonant scatterers in the form of cylindrical recesses filled with silicon in a metal can create a non-mirrored scattered field.

The response of gratings to a change in the wavelength of the incident field by the structure of the diffraction orders of the scattered field and a high sensitivity to the rotation of the plane of incidence have been revealed.

6 DISCUSSION

The behavior of the electromagnetic field in photonic crystals and near biperiodic lattices is explained by the possibility of influencing the Bregovian scattering by periodic inhomogeneity of the medium.

The existence of metamaterials for which there are continuous intervals of wavelengths of the incident field with a nonreciprocal scattered field is important.

Some researchers consider metamaterials formed by hexagonal, square cells with symmetry breaking out of plane or in plane. Such designs have a free space in the middle of the cell where you can place some controls. Then a strong electric field concentration in the near field can be achieved. This field behavior can be useful in laser or sensor devices.

CONCLUSIONS

The scientific novelty is the analysis of the diffraction orders of electromagnetic field rays over a biperiodic lattice, which allows us to determine the ray composition of the scattered field.

The amplitude characteristics of the rays are determined from the solution of the boundary value problem for the Helmholtz equation in the frequency domain. Numerical methods for Maxwell's equations in time-space coordinates are promising [11].

The proposed analysis of the orders of diffraction of electromagnetic field rays on a biperiodic lattice on a perfectly conducting substrate allows us to determine the ray composition of the scattered field.

The amplitude characteristics of the beams are determined from the solution of the boundary value problem for the Helmholtz equation in the frequency domain. We show the existence of wavelengths of the incident field at which only non-mirrored rays with nonzero amplitude exist. This makes it possible to use such lattices in a variety of electronic devices.

In the case of arbitrary time-dependence of the parameters of the passive field, numerical methods for Maxwell's equations in time-space coordinates are promising [11].

The practical significance of this work is detection of features of the behavior of the scattered field over a biperiodic lattice can be used to create non-reflective surfaces, creating anti-reflective coatings in various devices.

The revealed features of the scattered field behavior over a byperiodic grating can be used to create non-reflective surfaces, creating anti-reflective coatings in various devices, broadband mobile antennas, and sensor devices

Prospects for further research are studies on the influence of boundary conditions in the mathematical model on the composition of scattered diffraction orders and the relationship between diffraction orders of the scattered field and characteristic directions (eigenvalues and eigenvectors of Jacobi matrices).

Two-periodic gratings have prospects for creating anti-reflective surfaces for various devices.

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РОЗСІЮВАННЯ ЕЛЕКТРОМАГНІТНИХ ХВИЛЬ НА ПЛОСКИХ РЕШІТЧАТИХ ДВОХПЕРІОДИЧНИХ СТРУКТУРАХ

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АНОТАЦІЯ

Актуальність. Одна з наукових гіпотез створення невзаємних контрольованих оптичних метаповерхонь є використання хвильового каналу, який базується на променях прямого та зворотного сценаріїв дифракції на двоперіодичних плоских структурах з нелінійними елементами. Такі процеси в нанометровому діапазоні хвиль електронних пристроїв вимагають точних розрахунків процесів взаємодії хвиль і мікроструктур приладів. Важливо також описати поведінку антенних пристроїв в засобах мобільного зв'язку. Розширення діапазону довжин хвиль стабільного зв'язку досягається на дофрактальних структурах в антенних пристроях у поєднанні із періодичним структуруванням. Схожі проблеми моделювання виникають і при проникненні електромагнітних хвиль через матеріали із кристалічною структурою (радіопрозорість).

Мета. Для перевірки цієї гіпотези необхідно провести математичне моделювання процесу розсіяння електромагнітних хвиль метаповерхнями в умовах збудження декількох дифракційних порядків. Як відомо, серед двохперіодичних плоских решіток різних структур є п'ять типів, які покривають площину. Це є решітки Браве. Розглядалась задача розсіювання падаючої монохроматичної ТЕ поляризованої хвилі на металевий екран із заглибленнями в двохперіодичних структурах, заповнених кремнієм.

Метод. В роботі побудовані математичні моделі для вивчення просторових амплітудних спектрів метаповерхонь на решітках Браве та наведені деякі результати їх чисельного дослідження. Запропонована умова визначення дифракційних порядків які розповсюджуються над решіткою. Амплітуди розсіяного поля знаходяться із розв'язання крайової задачі для рівняння Гельмгольця в пакеті COMSOL Multiphysics 5.4. Аналогічні постановки задач можливі і при дослідженні проникнення електромагнітного поля в кристалічну речовину.

Результати. Отримані співвідношення для дифракційних порядків розсіяних електромагнітних хвиль дифракційною решіткою. Показано існування довжин падаючих хвиль на двохперіодичну решітку для яких відсутня віддзеркалена хвиля при різних формах (прямокутна, квадратна, шестикутна) періодичних елементів в центрі яких було виконане заглиблення, наповнене кремнієм. Приведені розподіли коефіцієнту віддзеркалення при різних геометричних розмірах періодичних елементів і заглиблення. Характеристики електричного поля на резонансних режимах у вигляді ізоліній його модуля показують характер взаємодії поля над періодичною решіткою і розсіювачами-заглибленнями. На резонансних довжинах падаючих хвиль виникають стоячі хвилі в розсіювачах.

Висновки. Запропонована математична модель множини дифракційних порядків які розповсюджуються від квадратної та шестикутної решітки в півпростір. Показано, що плоскі періодичні решітки із квадратними або шестикутними елементами періодичності та резонансними розсіювачами у вигляді циліндричних заглиблень, заповнених кремнієм, у металі можуть створювати недзеркальне розсіяне поле. Виявлена реакція решіток на зміну довжини хвилі падаючого поля структурою дифракційних порядків розсіяного поля та висока чутливість до повороту площини падіння. Двохперіодичні решітки мають перспективу при створенні антиблікових поверхонь різних пристроїв. The two-periodic lattices have prospects for creating anti-reflective surfaces of various devices. Двоперіодичні решітки мають перспективу при створенні антиблікових поверхонь різних пристроїв, лазерних чи сенсорних радіоелектронних пристроїв, антен в елементах мобільного зв'язку, радіопрозорорихелементів. Вони мають більш розвинені технології виготовлення по відношенню до просторових кристалічних структур.

КЛЮЧОВІ СЛОВА: рівняння Максвела, періодичні елементи решітки, дифракція, дифракційні порядки, невзаємність дифракційних явищ, чисельні методи досліджень, резонансна метаповерхня, невзаємне віддзеркалення.

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