METHOD OF CREATING A MINIMAL SPANNING TREE ON AN ARBITRARY SUBSET OF VERTICES OF A WEIGHTED UNDIRECTED GRAPH

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ABSTRACT

Context. The relevance of the article is determined by the need for further development of models for optimal restoration of the connectivity of network objects that have undergone fragmentation due to emergency situations of various origins. The method proposed in this article solves the problematic situation of minimizing the amount of restoration work (total financial costs) when promptly restoring the connectivity of network objects after their fragmentation.

The purpose of the study is to develop a method for creating a minimal spanning tree on an arbitrary subset of vertices of a weighted undirected graph to minimize the amount of restoration work and/or total financial costs when promptly restoring the connectivity of elements that have a higher level of importance in the structure of a fragmented network object.

Method. The developed method is based on the idea of searching for local minima in the structure of a model undirected graph using graph vertices that are not included in the list of base vertices to be united by a minimal spanning tree. When searching for local minima, the concept of an equilateral triangle and a radial structure in such a triangle is used. In this case, there are four types of substructures that provide local minima: first, those with one common base vertex; second, those with two common base vertices; third, those with three common base vertices; fourth, those without common base vertices, located in different parts of the model graph. Those vertices that are not included in the list of base ones, but through which local minima are ensured, are added to the basic ones. Other vertices (non-basic) along with their incident edges are removed from the structure of the model graph. Then, using one of the well-known methods of forming spanning trees, a minimal spanning tree is formed on the structure obtained in this way, which combines the set of base vertices.

Results. 1) A method for creating a minimal spanning tree on an arbitrary subset of vertices of a weighted undirected graph has been developed. 2) A set of criteria for determining local minima in the structure of the model graph is proposed. 3) The method has been verified on test problems.

Conclusions. The theoretical studies and several experiments confirm the efficiency of the developed method. The solutions developed using the developed method are accurate, which makes it possible to recommend it for practical use in determining strategies for restoring the connectivity of fragmented network objects.

KEYWORDS: network object, weighted undirected graph, connectivity, transitive closure, minimum spanning tree, local optimum, optimization criterion, method.

ABBREVIATIONS

MST is a minimal spanning tree;
TC is a transitive closure.

NOMENCLATURE

G is an undirected weighted graph modeling a network object;
V is a set of vertices of the model graph G;
E is a set of edges of a model graph G;
SG is a set of edges of a model graph G;
RG is a matrix of shortest paths of the model graph G;
(u, v) is the graph edge G;
w(u, v) is a weighting coefficient of some edge (u, v);
K is an arbitrarily selected subset of vertices of the model graph G;
G K is an MST, which is created on an arbitrarily selected subset of vertices of the model graph G;
E’ is a set of edges that make up the required graph G K;
W is a total weight of the constructed tree;
w ij is a weight of the TC between the corresponding vertices of the model graph G;
v ij is a vertex of the model graph G;
n is a number of graph vertices.

INTRODUCTION

Objects with a distributed structure, so-called network objects, have long ago and forever entered the life of mankind. Such facilities include transport networks (road, rail, water transport, air transport); data transmission networks; power grids, water supply networks, gas supply networks and others. A distinctive feature of such objects is their presence in their composition of nodal elements (passenger stations, communication nodes, distribution...
The object of the study is the process of restoring connectivity between an arbitrary subset of node elements of a fragmented network object.

The subject of the study is the method for creating a minimal spanning tree on an arbitrary subset of vertices of a weighted undirected graph.

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1 PROBLEM STATEMENT

The tasks of determining the optimal structures of network objects are, for the most part, formalized and solved using graph theory models and methods [13]. That is why we will model the structure of the network object with some weighted undirected graph \( G = (V, E) \), where \( V \) is a set containing the vertices of a graph that model the node elements of a network object; \( E \) is the set containing the edges of a graph that model the communication lines of a network object.

For each edge \((u, v) \in E\), its weight is known \( w(u, v) \). In the plural \( V \) the vertices of the initial graph are an arbitrarily selected subset of vertices \( K \), so that \( K \subseteq V, |K| < |V| \). Vertices that make up a subset \( K \), will be called basic. The task is to create MST \( G'_K = (K, E' \subseteq E) \), connecting a selected subset of base vertices \( K \), namely:

\[
w(E') = \sum_{(u, v) \in E'} w(u, v) \rightarrow \min, \quad (1)
\]

under the conditions:

\[
\forall \langle x, y \rangle \in K \exists x - TC \rightarrow y, \quad (2)
\]

where \( \langle x, y \rangle \) – an arbitrary pair of vertices from the set \( K \); \( x - TC \rightarrow y \) – transitive closure between an arbitrary pair of vertices \( \langle x, y \rangle \).

2 REVIEW OF THE LITERATURE

Currently, the theoretical basis for restoring the connectivity of fragmented (broken) network objects is the graph theory.

A well-known and studied problem of graph theory with numerical practical applications is the problem of creating an initial undirected MST graph on the structure, that is, an acyclic subgraph in which all vertices of the initial graph are transitively closed (there is a path connecting any pair of vertices), and the total weight of the edges of this acyclic subgraph is minimal.
Currently under creating MST $G'_C$, where “$C$” – the entire nodal basis of the graph $G$, as it was mentioned above, the well-known methods of Prim, Kruskal, Boruvka-Solin are used. These methods can also be used to search for minimal covering trees $G'_C$, by checking at each step whether the tree being built has connectivity between all vertices $v_j \in K$.

Thus, a connected undirected graph is applied to the input of Prim’s algorithm [7]. For each edge, its cost is set. First, an arbitrary vertex is taken and the edge incident to this vertex, which has the lowest cost, is found. Then the edges of the graph are considered, one end of which is a vertex that already belongs to the tree, and the other is not; from these edges, the edge of the lowest cost is selected. The edge selected at each step is joined to the tree. The tree grows until all the vertices of the initial graph are explored. The result of the algorithm is the MST.

If the initial graph is given by the adjacency matrix, the computational complexity of this algorithm is estimated $O(n^2)$.

In Kruskal’s algorithm [8], the current set of edges is initially set to be empty. All the edges of the graph are ordered as the weight increases and are presented in a separate list. An edge of minimum weight is selected from this list and added to the already existing set (the tree being created). A cycle check is performed immediately. If there is no cycle, then the next edge is taken and added to the set. If there is a cycle, the edge that created it is discarded. The process is iteratively repeated until all vertices of the initial graph are included into the required tree. The tree found in this way is the minimum spanning tree of the initial graph.

The computational complexity of this algorithm will be evaluated $O(E \log(E))$, and is mainly determined by the complexity of the process of sorting the edges of the graph.

The Boruvka-Solin algorithm [9] is practically no different from Kruskal’s algorithm.

The conducted analysis of the literature shows that the problem in the formal statement (1)-(2) has not been posed or solved by anyone. Our article is dedicated to solving this problem.

### 3 MATERIALS AND METHODS

The analysis of the Prim, Kruskal, Boruvka-Solin methods on various structures proved that their use for creating trees $G'_C$, may give some error in the final result, because in the structure of the initial graph $G$ spanning trees may exist $G'_C$ with less weight. The fact is that at each step of these methods, vertices are needed for transitive linking $v_j \in K$, an edge of minimum weight is added to the structure of the required tree, followed by a check for the presence of a cycle. The total weight of the added edges may exceed the weight of some edge, the weight of which is greater than each of the added ones, but through which the optimal (by the minimum weight criterion) transitive closure of the vertices is carried out $v_i \in K$.

For example, there is some communication network modeled by an undirected weighted graph $G$, Fig. 1, a. Minimal spanning tree $G'[1, 4, 5, 6]$, built according to the Kruskal’s method, provided by Fig. 1, b bolder lines.

At the same time, the total weight of five edges $(v_1, v_2), (v_2, v_4), (v_1, v_3), (v_3, v_5), (v_3, v_6)$, which are part of the spanning tree $G'[1, 4, 5, 6]$, is equal to $W = \sum_{(i,j) \in G'[1, 4, 5, 6]} w_{ij} = 18$.

But it can be seen that in fact MST $G'[1, 4, 5, 6]$ consists of four edges $(v_1, v_2), (v_2, v_4), (v_2, v_5), (v_5, v_6)$.
Herewith $W = \sum_{(i,j) \in G_{[4,5,6]}} w_{ij} = 16$, see Fig. 2. The absolute difference in the weights of these two trees is 2 units.

Figure 2 – A minimal spanning tree $G'_{[4,5,6]}$ of the initial undirected graph $G$

Considering the above, we will formulate and prove the following theorem.

**Theorem.** Let $G = (V, E)$ be an arbitrary weighted undirected graph. Minimal spanning tree $G[K] = (K, E')$ on a subset of selected vertices $v_i \in K$ of the graph $G$, where $K \subseteq V$, can be created by adding a subset to the composition $K$ of some vertex $v_i \notin K$, if the optimal (of minimal weight) transitive closure (TC) of some vertices is carried out through it $v_i \in K$.

**Proof:** It is obvious that is a minimal spanning tree on a subset of vertices $K$, in case $|K| = 2$ is a shortest path connecting these two vertices. If $|K| > 2$, there can be several such paths. Thus, to obtain the connectivity of some vertices $s, d, t$ to the structure of the required $G[{s,d,t}]$ can be added $(s,d_1), (d_1,d_2), ..., (d_{n-1}, d_n), (d_n,d)$ and $(s,t_1), (t_1,t_2), ..., (t_{n-1}, t_n), (t_n,t)$ edge. Suppose that in the structure of the initial graph $G$ some vertices is present $t_n \notin K$ and edge $(t_n, d)$ for which the following condition is true: $w_{t_n,d} > w_{s,d_1}, w_{t_n,d} > w_{d_1,d_2}, ..., w_{t_n,d} > w_{d_n,d}$ and $w_{t_n,d} < (w_{s,d} + ... + w_{d_1,d_2} + ... + w_{d_n,d})$. So, considering the edge $(t_n, d)$ it is possible to reduce the total weight of the required spanning tree $G[{s,d,t}]$.

Therefore, the adjacent edges whose weight coefficients are in parentheses can increase the total weight of $G[{s,d,t}]$. Thus, the problem should be solved taking into account the possible addition to the structure of the required tree $G[K]$ of additional vertices $v_i \notin K$, the total weight of the transitive closure through which will ensure the minimization of the total weight of the required spanning tree. The theorem is proved.

Let us note an important consequence of the theorem.

**Consequence.** The weight of transitive closure of vertices $v_i \in K$, can be reduced through some vertices $v_i \notin K$, starting from $|K| = 3$.

Let’s explain the mentioned consequence graphically. For example, there are two connected networks with lengths $L_1$ and $L_2$, see Fig. 3.

Figure 3 – Geometric comparison of the total weight of TC in networks with $|K| = 3$ and different organization of the structure:

- a – without using an additional vertices – a linear substructure.
- b – using an additional vertices (nu) – a radial substructure.
- c – geometric interpretation of TC weight based on an equilateral triangle.
The size of an edge $c$ in an equilateral triangle, see Fig. 3, c is determined as follows:

$$\cos 30^\circ = \frac{a/2}{c} \Rightarrow c = \frac{a}{\sqrt{3}}.$$  Thus, in an equilateral triangle (or close to it), the inequality $L_1 > L_2$ will always be valid.

Based on the above, the main idea of the method is to check the structure of the initial graph $G$ on the possibility of reducing the weight of TC between threes $v_i \in K$ in their various combinations (sets) due to the addition of some vertices $v_j \notin K$ (see Fig. 3, b). If such a possibility exists, we will speak of the existence of a local minimum, which is ensured by this $v_i \notin K$. Vertices $v_j \notin K$, which do not provide local minima will be removed from the structure of the initial graph $G$ together with the edges, incidental to them, and they will not be considered in the further creating of the MST.

If several radial substructures that provide local minima are found in the graph $G$ structure, they should be analyzed for the extent to which the base vertices $v_i \in K$ are used together. The following options are possible here, see Fig. 4:

- a – first;
- b – second;
- c – third;
- d – null (no compatible use of base vertices)

Thus, when detected in the structure of the model graph $G$ several radial substructures with the first degree of their joint use of basic vertices (see Fig. 4, a), all vertices $v_j \notin K$, through which such substructures are formed, remain in the structure of the graph $G$, providing corresponding local minima in it. The same situation occurs with radial substructures with zero degree of use of base vertices (see Fig. 4, d), because substructures are at some distance from each other. In this case, all vertices $v_j \notin K$, due to which such substructures are formed, remain in the structure of the graph $G$, providing corresponding local minima in it.

The situation will be different in the presence of the second and third degrees of base vertex usage. In such a situation (see Fig. 4, b and Fig. 4, c) you need to find out which of the vertices $v_j \notin K$ (in this case $v_1$ or $v_2$) will ensure a lower weight of the TC of the base vertices $v_i \in K$. In the case of the third degree of use of base vertices, competition between vertices $v_j \notin K$ occurs on several parallel radial substructures that connect some triplet of base vertices $v_i \in K$, see Fig. 4, c. The result of such competition is the selection of a single vertex $v_j \notin K$, which ensures the smallest weight of the TC of this trio of vertices. At the same time, the local minimum remains within the three analyzed vertices. In the case of the second degree of using base vertices (see Fig. 4, b), vertices $v_j \notin K$ are the roots of adjacent radial substructures that connect different sets of basic vertices triples $v_i \in K$. In this case, the result of competition between the corresponding vertices $v_j \notin K$ is the choice of the radial substructure that provides the smallest weight of the TC of the corresponding triple of vertices (within the example shown in Fig. 4, b, the local minimum is provided on the triple of vertices, the root of which is the vertex $v_j$). Both in the first and in the second cases, the vertices $v_j \notin K$, that lost the competition are removed from the structure of the modeling graph $G$ along with the edges, incident to them.

Having considered the general theoretical provisions, we will present the developed method in the form of the following six steps:

Step 1. Based on the modeling weighted undirected graph $G$, creating its adjacency matrix $S_G$. According to the matrix $S_G$ creating a matrix of the shortest paths $R_G$ between all pairs of the vertices of graph $G$. For this purpose, we can use the Warshall-Floyd algorithm [14, 15] or Shimbel [16] and some others.

Step 2. For each $v_j \notin K$ according to the $R_G$ to find the weight of the TC with three base vertices $v_i \in K$, for which the condition $\sum_{j=1}^{n} w_{ij} \rightarrow \text{min}$ is valid.

The result of the operation: the column indices (vertices $v_i \in K$) are $idx1, idx2, idx3$; the total weight of the TC connecting the given trio of base vertices $v_i \in K$

Figure 4 – Degree of compatible use by radial substructures of base vertices $v_i \in K$ (such vertices are marked in solid red):

- a – first;
- b – second;
- c – third;
- d – null (no compatible use of base vertices)
with $v_i \notin K$, that is analyzed is $\sum_{v_i \notin K} w_{v_i}^{k_{i,j}}$. To save the results of the operation.

Step 3. For each $v_i \in K$ by the column index sets defined in step 2 ($idx1$, $idx2$, $idx3$) according to the matrix $R_{G_i}$ to find the weight of the corresponding vertices. The result of the operation: the weight of the TC connecting the base vertices $v_i \in K$ with three base vertices $v_j \in K$ with numbers $idx1$, $idx2$, $idx3$ is $\sum_{v_i \in K} w_{v_i}^{k_{i,j}}$. To save the results of the operation.

Step 4. To remove vertices $v_i \notin K$ for which the condition is valid:

$$\sum_{v_i \notin K} w_{v_i}^{k_{i,j}} \geq \forall \sum_{v_i \in K} w_{v_i}^{k_{i,j}}, i=1,n, (3)$$

from the structure of the model graph $G$ together with the edges, incident to it. Appropriate changes to the matrix $S_G$ should be made.

Step 5. To carry out a pairwise check of the vertices $v_i \notin K$ remaining after the previous steps for the degree of compatible use by the radial substructures of the base vertices $v_i \in K$:

a) if because of such a check zero or first degree was found (without a match by indices or a match by one index), then such vertices should be left in the structure of the model graph $G$;

b) in the case of detection of the second or third degree (a match according to two or three indices, respectively), determine the vertices $v_i \notin K$ through which the minimum TC of the corresponding trio of base vertices is ensured $v_i \in K$. To remove the vertices that lost the competition from the structure of the model graph $G$ together with the edges, incident to it. An appropriate changes to the matrix $S_G$ should be made.

Step 6. On the modified in this way graph $G$, by one of the well-known algorithms for creating the MST the required minimum tree $G_{(K)} = (K, E' \subseteq E)$ is created.

**4 EXPERIMENTS**

Let us illustrate the application of the method on the example of the graph provided by Fig. 1, a. As before, the MST $G_{(K)} = (K, E' \subseteq E)$ is to be found.

Step 1. A calculated matrix of shortest paths $R_{G_i}$ between all pairs of graph vertices has the following form:

$\begin{array}{|c|c|c|c|c|c|c|}
\hline
v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \\
\hline
v_1 & 0 & 3 & 4 & 5 & 8 & 9 & 13 \\
\hline
v_2 & 3 & 0 & 7 & 2 & 5 & 11 & 10 \\
\hline
v_3 & 4 & 7 & 0 & 9 & 4 & 5 & 16 \\
\hline
v_4 & 5 & 2 & 9 & 0 & 7 & 10 & 8 \\
\hline
v_5 & 8 & 5 & 4 & 7 & 0 & 6 & 15 \\
\hline
v_6 & 9 & 11 & 5 & 10 & 6 & 0 & 11 \\
\hline
v_7 & 13 & 10 & 16 & 8 & 15 & 11 & 0 \\
\hline
\end{array}$

In expression (4), vertices $v_i \in K$ are marked in red.

Step 2. For each $v_i \notin K$ we define $\sum_{v_i \notin K} w_{v_i}^{k_{i,j}} \rightarrow \min$. For the vertex $v_2 \notin K$ it is $\sum_{v_i \notin K} w_{v_i}^{k_{i,j}} = 10$. For the vertex $v_3 \notin K$ it is $\sum_{v_i \notin K} w_{v_i}^{k_{i,j}} = 13$. For the vertex $v_7 \notin K$ it is $\sum_{v_i \notin K} w_{v_i}^{k_{i,j}} = 32$.

Step 3. The results of calculations for this step are presented in Table 1.

<table>
<thead>
<tr>
<th>$v_i \in K$</th>
<th>$\sum_{v_i \notin K} w_{v_i}^{k_{i,j}}$</th>
<th>$\sum_{v_i \notin K} w_{v_i}^{k_{i,j}}$</th>
<th>$\sum_{v_i \notin K} w_{v_i}^{k_{i,j}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_1$</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$v_4$</td>
<td>12</td>
<td>22</td>
<td>15</td>
</tr>
<tr>
<td>$v_5$</td>
<td>15</td>
<td>14</td>
<td>21</td>
</tr>
<tr>
<td>$v_6$</td>
<td>25</td>
<td>15</td>
<td>25</td>
</tr>
</tbody>
</table>

Step 4. According to inequality (3), we compare the received sums of weights $\sum_{v_i \notin K} w_{v_i}^{k_{i,j}} = 10$, $\sum_{v_i \notin K} w_{v_i}^{k_{i,j}} = 13$, $\sum_{v_i \notin K} w_{v_i}^{k_{i,j}} = 32$ with sums of weights on the corresponding indices for $v_i \in K$, which are presented in Table 1. The inequality is valid only for the vertex $v_7$, since $\sum_{v_i \notin K} w_{v_i}^{k_{i,j}} = 32$ is greater than any value in column 4, see Table 1. So, the vertex $v_7$ is removed from the structure of the model graph $G$ with all the edges incident to it. Corresponding changes are also to be made to the matrix $S_G$.

Step 5. Let us perform a pairwise check of the vertices $v_i \notin K$ remaining after the previous steps for the degree of compatible use by the corresponding radial substructures of the base vertices $v_i \in K$. Such vertices are $v_2$ and $v_3$ for which $\sum_{v_i \notin K} w_{v_i}^{k_{i,j}} = 10$. The remaining after the previous steps for the degree of compatible use by the corresponding radial substructures of the base vertices $v_i \in K$.
and $\sum_{v_1}^{v_6} = 13$. As we can see, the radial substructures, the roots of which are these vertices, jointly use the base vertices $v_1$ and $v_3$. Therefore, we got the second degree of joint use of basic vertices by radial substructures $v_i \in K$ (match by two indices). Since the weight of the transitive closure over the vertex $v_3$ is greater than through the vertex $v_2 (13>10)$, the vertex $v_3$ is also removed with all its incident edges from the structure of the model graph $G$. Corresponding changes are also to be made to the matrix $S_G$.

Step 6. On the modeling graph $G$ modified in this way (Fig. 5) using the Kruskal method, we will create the MST $G'[1,4,5,6]$. It will be identical to the MST presented at Fig. 2.

Figure 5 – Modified model graph $G$ and the MST on the selected subset of vertices

5 RESULTS

When applying the developed method to the initial undirected graph $G$ (see Fig. 1), three radial substructures were successively considered, the roots of which were vertices $v_2$, $v_3$, $v_7$, not included in the set $K$. During the verification, it was found that the vertex of $v_7$ does not provide a minimum TC between the specified three base vertices $v_i \in K$. This fact made it possible to modify the initial graph $G$ by removing this vertices and all edges incident to it from its structure. The vertices $v_2$ and $v_3$ have provided the minimum TC. At the same time, the radial structures (triplets of vertices), of whose roots they are, intersect along two vertices and have the second degree of joint use of the base vertices $v_i \in K$. This fact led to the need to compare the vertices $v_2$, $v_3$, and choose the one that provides the local minimum of TC. This vertex appeared to be the vertex $v_2$. Consequently, vertex $v_3$ was also removed from the original graph $G$ structure.

Therefore, the internal tools of the proposed method allow testing the structure for the presence of local minima in the TC of the base vertices $v_j \in K$ through the vertices $v_j \notin K$, and modifying the structure of the initial graph $G$ to further find MST $G'[K]$ in this structure.

6 DISCUSSION

The combination of the approaches proposed in the article allowed us to develop a method by which it is possible to build a MST on an arbitrary subset of vertices of the initial undirected graph. This became possible due to the analysis of radial substructures whose roots are vertices $v_j \notin K$, in terms of the weight of the TC of these substructures, and the search for local minima among them. At the same time, this became possible due to the use of the shortest paths matrix ($R_G$) between all pairs of vertices of the model graph $G$. Due to the fact that such a matrix contains information not only about the presence of TC between any pair of vertices, but also quantitatively characterizes this relationship, it became possible to analyze different sets of three basic vertices $v_i \in K$, from different locations of the model graph relative to the root of the current radial substructure. The above allows us to launch a mechanism for revealing local minima in the structure of the model graph $G$ and selecting vertices $v_j \notin K$ that provide this minimum. On the other hand, those vertices $v_j \notin K$, which do not provide local minima are removed from the structure of the model graph $G$, thereby not increasing the weight of the required MST $G'[K]$.

Several dozen full-scale experiments on various network objects of low density have shown the efficiency of the developed method, and the solutions obtained were optimal. At the same time, the behavior of the method on dense network objects of high dimensionality remains a challenge ($n>30$). Thus, the method could be considered quasi-optimal at the moment.

The computational complexity of the combinatorial algorithm that implements the developed method will be determined by the computational complexity of its “basic elements” – the algorithm for finding the shortest paths between all pairs of vertices of the model graph and the algorithm for creating the MST. If the Warshall-Floyd algorithm and the Kruskal algorithm are taken as the basic algorithms, respectively, the overall computational complexity of the combinatorial algorithm will be estimated $O(n^3 + E \log(E))$.

The obtained polynomial estimate of the computational complexity is suitable for using such an algorithm in solving relevant management problems in real life.

CONCLUSIONS

The article solves the actual scientific and applied problem of creating the MST $G'[K]$ on an arbitrarily chosen subset of vertices of the initial undirected weighted graph, where $K$ is an arbitrarily chosen subset of vertices of the initial graph $G$. © Batsamat V. M., Hodlevskyi S. O., Babkov Yu. P., Morkvin D. A., 2024 DOI 10.15588/1607-3274-2024-1-17

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The scientific novelty of the developed method is as follows:

1) in the formulation of the consequence that to reduce the weight of the transitive closure of the base vertices $v_i \in K$, through some vertices $v_j \not\in K$, starting from $|K| = 3$;

2) in the proposed approach to vertices selection $v_i \in K$. The essence of the approach is to compare the weights of transitive closures of different radial substructures whose roots are vertices $v_i \not\in K$, combining different sets of three basic vertices $v_j \in K$;

3) in the proposed approach to determining the local minimum of the weight of radial substructures, among substructures that are in competition. The core of the approach is to analyze the degree of joint use of base vertices $v_i \in K$ by different radial substructures.

The practical value of the method is when it is applied to large and dense network objects that have undergone fragmentation (destruction due to external influences), it is possible to significantly reduce the amount of restoration work and/or total financial costs while quickly restoring the connectivity of elements that are of higher importance in the structure of such an object.

A promising direction for further research is the final verification of the developed method to determine its optimality class.

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АНОТАЦІЯ
Актуальність. Актуальність статті обумовлюється потребою у подальшому розвитку моделей оптимального відновлення зв'язності мережних об'єктів, що зазнали фрагментації внаслідок надзвичайних ситуацій різного характеру походження. Запропонований у статті метод усуне проблемну ситуацію, що полягає у необхідності мінімізації обсягу відновлювальних робіт (загальних фінансових витрат) при оптимальному відновленні зв'язності обраної підмножини елементів мережевого об'єкту після його фрагментації.

Мета роботи полягає у розробленні методу побудови мінімального кістякового дерева на довільній підмножині вершин зваженого неорієнтованого графа для мінімізації обсягу відновлювальних робіт і/або загальних фінансових витрат при оптимальному відновленні зв'язності елементів, які мають нижчі рівні важливості в структурі фрагментованого мережевого об'єкту.

Метод. Розроблений метод заснований на ідеї пошуку в структурі модельного неорієнтованого графа локальних мінімумів використання вершин графу, що не входять до переліку базових вершин, які потрібно об'єднати мінімальним кістяковим деревом. Під час пошуку локальних мінімумів використовується поняття рівністі вершин графу, яка визначає ієрархію якість вершин графу. При цьому розбиваються вершини, що відповідають критеріям використання рівністі вершин, які забезпечують локальні мінімуми:

1. Відповідають одному вершинному графу, що відповідає двом вершинам базових вершин графу і базових вершин графу,
2. Відповідають двом вершинам базових вершин графу і трьом вершинам базових вершин графу.

Мінімальне кістякове дерево визначається на основі відмінності рівністі вершин графу різного характеру походження.

Результати. Результати були вибіркові з використанням розробленого методу в різних фрагментах мережевого об'єкту. Результати містять в собі відповідність відомим методам в різних сферах розвитку.

Висновки. Проведені теоретичні дослідження та низка експериментальних досліджень впливають на розуміння розробленого методу. Результати, що виробляються, є зрозумілими для використання розробленого методу, і є відповідними до висновків рекомендованих в роботах різних науковців. Результати, що виробляються, є відповідними до висновків рекомендованих в роботах різних науковців.

КЛЮЧОВІ СЛОВА: мережевий об'єкт, зважений неорієнтований граф, зв'язність, транзитивне замкнення, мінімальне кістякове дерево, локальний оптимум, критерії оптимізації, метод.