UDC 519.172.1: 62-503.57

METHOD OF CREATING A MINIMAL SPANNING TREE ON AN ARBITRARY SUBSET OF VERTICES OF A WEIGHTED UNDIRECTED GRAPH

Batsamut V. M. – Dr. Sc., Professor, The Deputy Head of the Scientific Research Center of the National Academy of the National Guard of Ukraine, Kharkiv, Ukraine.

Hodlevsky S. O. – The Researcher of the Scientific Research Center of the National Academy of the National Guard of Ukraine, Kharkiv, Ukraine.

Babkov Yu. P. – PhD, Associated Professor, Professor of the State Security Department of the National Academy of the National Guard of Ukraine, Kharkiv, Ukraine.

Morkvin D. A. – PhD, Head of the Scientific Research Laboratory of the Scientific Research Center of the National Academy of the National Guard of Ukraine, Kharkiv, Ukraine.

ABSTRACT

Context. The relevance of the article is determined by the need for further development of models for optimal restoration of the connectivity of network objects that have undergone fragmentation due to emergency situations of various origins. The method proposed in this article solves the problematic situation of minimizing the amount of restoration work (total financial costs) when promptly restoring the connectivity of a selected subset of elements of a network object after its fragmentation.

The purpose of the study is to develop a method for creating a minimal spanning tree on an arbitrary subset of vertices of a weighted undirected graph to minimize the amount of restoration work and/or total financial costs when promptly restoring the connectivity of elements that have a higher level of importance in the structure of a fragmented network object.

Method. The developed method is based on the idea of searching for local minima in the structure of a model undirected graph using graph vertices that are not included in the list of base vertices to be united by a minimal spanning tree. When searching for local minima, the concept of an equilateral triangle and a radial structure in such a triangle is used. In this case, there are four types of substructures that provide local minima: first, those with one common base vertex; second, those with two common base vertices; third, those with three common base vertices; fourth, those without common base vertices, located in different parts of the model graph. Those vertices that are not included in the list of basic ones, but through which local minima are ensured, are added to the basic ones. Other vertices (non-basic) along with their incident edges are removed from the structure of the model graph. Then, using one of the well-known methods of forming spanning trees, a minimal spanning tree is formed on the structure obtained in this way, which combines the set of base vertices.

Results. 1) A method for creating a minimal spanning tree on an arbitrary subset of vertices of a weighted undirected graph has been developed. 2) A set of criteria for determining local minima in the structure of the model graph is proposed. 3) The method has been verified on test problems.

Conclusions. The theoretical studies and several experiments confirm the efficiency of the developed method. The solutions developed using the developed method are accurate, which makes it possible to recommend it for practical use in determining strategies for restoring the connectivity of fragmented network objects.

KEYWORDS: network object, weighted undirected graph, connectivity, transitive closure, minimum spanning tree, local optimum, optimization criterion, method.

ABBREVIATIONS

MST is a minimal spanning tree;
TC is a transitive closure.

NOMENCLATURE

G is an undirected weighted graph modeling a network object;

V is a set of vertices of the model graph G;

E is a set of edges of a model graph G;

S_G is a set of edges of a model graph G;

R_G is a matrix of shortest paths of the model graph G;

(u,v) is the graph edge G;

$w(u,v)$ is a weighting coefficient of some edge (u,v);

K is an arbitrarily selected subset of vertices of the model graph G;

$G'_{[K]}$ is an MST, which is created on an arbitrarily selected subset of vertices of the model graph G;

E' is a set of edges that make up the required graph $G'_{[K]}$;

W is a total weight of the constructed tree;

w_{ij} is a weight of the TC between the corresponding vertices of the model graph G;

v_i is a vertex of the model graph G;

n is a number of graph vertices.

INTRODUCTION

Objects with a distributed structure, so-called network objects, have long ago and forever entered the life of mankind. Such facilities include transport networks (road, rail, water transport, air transport); data transmission networks; power grids, water supply networks, gas supply networks and others. A distinctive feature of such objects is the presence in their composition of nodal elements (passenger stations, communication nodes, distribution

DOI 10.15588/1607-3274-2024-1-17

and pumping stations, producers of services and their consumers, etc.) and communication lines between these nodal elements (transport routes, cable facilities, power lines, etc.).

The efficiency of such complex objects depends on the performance of their individual elements, but this dependence is more pronounced on communication lines, since the latter have larger linear dimensions and, therefore, are more often and more exposed to external undesirable influences [1, 2, 3, 4]. The causes of external influences include man-made and natural emergencies. Network facilities undergo particularly significant fragmentation (destruction) because of military (combat) operations in the territories where such facilities are located [5].

As a rule, after the situation in the crisis area is normalized, and in many cases during the emergency response, the issue of conducting restoration work on the destroyed network facility arises to bring the structure of the facility and all its functioning parameters to the design parameters.

In conditions of limited funding or limited time for restoration work, the network facility is usually restored to the spanning structure first [6] as the first stage of restoration work. In such a structure, each nodal element is connected to any other nodal element, although not always by optimal routes. The facility continues to operate, although with some loss of quality of service to end users (subscribers). At the second stage, based on the spanning structure, the network object is restored to its original structure, and even improved, to make it more efficient.

If the nodal elements differ in their degree of importance, the first stage of restoration work involves restoring the connectivity of not all elements of the network object, but the more significant ones selected according to a certain rule (criterion). It should be noted here that when determining the set of such elements, the current situation at the facility due to its fragmentation and the functional purpose of the facility itself are also considered.

Using well-known methods for constructing minimal spanning trees (MSTs) on graphs, such as Prima [7], Kruskala [8], Boravki-Solina [9], and others [10, 11, 12], it is generally possible to construct a spanning tree on an arbitrary subset of the vertices of the initial undirected graph, but in most cases such a tree will not be optimal in terms of the minimum total weight of the weighted edges it is composed of. On network objects with a significant number of node elements and a significant density of communication lines, such an error can be significant and decisive in matters of choosing a strategy for restoring the connectivity of a fragmented network object.

Thus, the article is aimed at minimizing the amount of restoration work (financial costs for such work) aimed at promptly restoring the connectivity of a certain (prescribed) set of node elements of a fragmented network object.

The object of the study is the process of restoring connectivity between an arbitrary subset of node elements of a fragmented network object.

The subject of the study is the method for creating a minimal spanning tree on an arbitrary subset of vertices of a weighted undirected graph.

The purpose of the study is to develop a method for creating a minimal spanning tree on an arbitrary subset of vertices of a weighted undirected graph to minimize the amount of restoration work and/or total financial costs when promptly restoring the connectivity of elements that have a higher level of importance in the structure of a fragmented network object.

1 PROBLEM STATEMENT

The tasks of determining the optimal structures of network objects are, for the most part, formalized and solved using graph theory models and methods [13]. That is why we will model the structure of the network object with some weighted undirected graph $G=(V,E)$, where V is a set containing the vertices of a graph that model the node elements of a network object; E is the set containing the edges of a graph that model the communication lines of a network object.

For each edge $(u,v)\in E$, its weight is known $w(u,v)$. In the plural V the vertices of the initial graph are an arbitrarily selected subset of vertices K, so that $K\subseteq V$, $|K|<|V|$. Vertices that make up a subset K will be called basic. The task is to create MST $G'_{[K]}=\left(K,E'\subseteq E\right)$, connecting a selected subset of base vertices K, namely:

$$w(E')=\sum_{(u,v)\in E'} w(u,v)\to \min\,, \tag{1}$$

under the conditions:

$$\forall <x,y>\in K\ \exists\, x\xrightarrow{\text{TC}}y\,, \tag{2}$$

where $<x,y>$ – an arbitrary pair of vertices from the set K; $x\xrightarrow{\text{TC}}y$ – transitive closure between an arbitrary pair of vertices $<x,y>$.

2 REVIEW OF THE LITERATURE

Currently, the theoretical basis for restoring the connectivity of fragmented (broken) network objects is the graph theory.

A well-known and studied problem of graph theory with numerical practical applications is the problem of creating an initial undirected MST graph on the structure, that is, an acyclic subgraph in which all vertices of the initial graph are transitively closed (there is a path connecting any pair of vertices), and the total weight of the edges of this acyclic subgraph is minimal.

DOI 10.15588/1607-3274-2024-1-17

Currently under creating MST $G_{[+]}$, where “+” – the entire nodal basis of the graph G, as it was mentioned above, the well-known methods of Prim, Kruskal, Boruvka-Solin are used. These methods can also be used to search for minimal covering trees $G'_{[K]}$, by checking at each step whether the tree being built has connectivity between all vertices $v_i \in K$.

Thus, a connected undirected graph is applied to the input of Prim’s algorithm [7]. For each edge, its cost is set. First, an arbitrary vertex is taken and the edge incident to this vertices, which has the lowest cost, is found. The found edge and two vertices connected by it form a tree. Then the edges of the graph are considered, one end of which is a vertex that already belongs to the tree, and the other is not; from these edges, the edge of the lowest cost is selected. The edge selected at each step is joined to the tree. The tree grows until all the vertices of the initial graph are explored. The result of the algorithm is the MST.

If the initial graph is given by the adjacency matrix, the computational complexity of this algorithm is estimated $O(n^2)$.

In Kruskal’s algorithm [8], the current set of edges is initially set to be empty. All the edges of the graph are ordered as the weight increases and are presented in a separate list. An edge of minimum weight is selected from the list and added to the already existing set (the tree being created). A cycle check is performed immediately. If there is no cycle, then the next edge is taken and added to the set. If there is a cycle, the edge that created it is discarded. The process is iteratively repeated until all vertices of the initial graph are included into the required tree. The tree found in this way is the minimum spanning tree of the initial graph.

The computational complexity of this algorithm will be evaluated $O(E\log(E))$, and is mainly determined by the complexity of the process of sorting the edges of the graph.

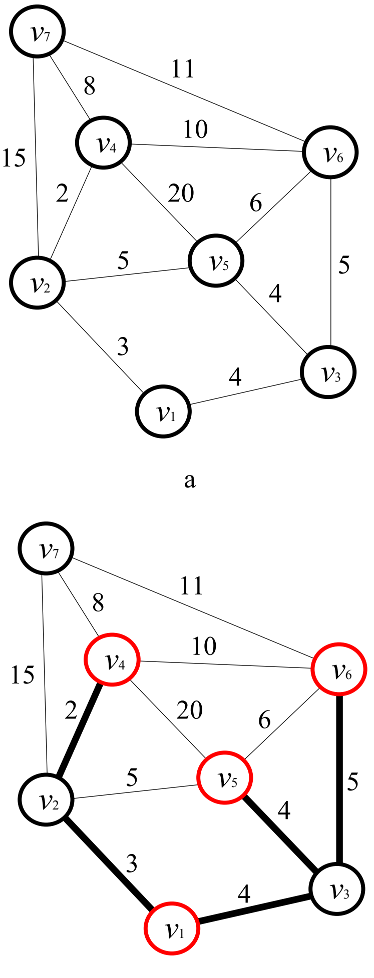
The Boruvka-Solin algorithm [9] is practically no different from Kruskal’s algorithm.

The conducted analysis of the literature shows that the problem in the formal statement (1)-(2) has not been posed or solved by anyone. Our article is dedicated to solving this problem.

3 MATERIALS AND METHODS

The analysis of the Prim, Kruskal, Boruvka-Solin methods on various structures proved that their use for creating trees $G'_{[K]}$, may give some error in the final result, because in the structure of the initial graph G spanning trees may exist $G'_{[K]}$ with less weight. The fact is that at each step of these methods, vertices are needed for transitive linking $v_i \in K$, an edge of minimum weight is added to the structure of the required tree, followed by a check for the presence of a cycle. The total weight of the added edges may exceed the weight of some edge, the weight of which is greater than each of the added ones, but through which the optimal (by the minimum weight criterion) transitive closure of the vertices is carried out $v_i \in K$.

For example, there is some communication network modeled by an undirected weighted graph G, Fig. 1, a. Minimal spanning tree $G'_{[1,\,4,\,5,\,6]}$, built according to the Kruskal’s method, provided by Fig. 1, b bolder lines.



a

b

Figure 1 – Building a minimal spanning tree on an arbitrarily subset of vertices of the initial graph G:
a – the initial undirected graph G;
b – a minimal spanning tree $G'_{[1,4,5,6]}$ of the initial undirected graph G, built according to Kruskal’s method

At the same time, the total weight of five edges (v_1,v_2), (v_2,v_4), (v_1,v_3), (v_3,v_5), (v_3,v_6), which are part of the spanning tree $G'_{[1,4,5,6]}$, is equal to $W = \sum_{(i,j)\in G'_{[1,4,5,6]}} w_{ij} = 18$.

But it can be seen that in fact MST $G'_{[1,4,5,6]}$ consists of four edges (v_1,v_2), (v_2,v_4), (v_2,v_5), (v_5,v_6).

DOI 10.15588/1607-3274-2024-1-17

Herewith $W = \sum_{(i,j)\in G'_{[1,4,5,6]}} w_{ij} = 16$, see Fig. 2. The absolute difference in the weights of these two trees is 2 units.

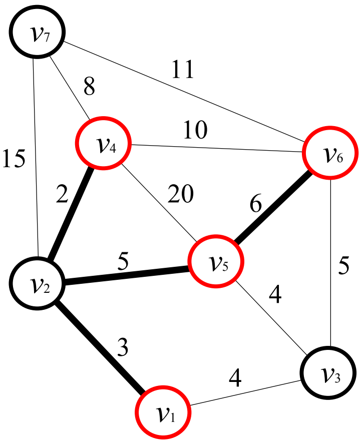


Figure 2 – A minimal spanning tree $G'_{[1,4,5,6]}$ of the initial undirected graph G

Considering the above, we will formulate and prove the following theorem.

Theorem. Let $G = (V, E)$ be an arbitrary weighted undirected graph. Minimal spanning tree $G'_{[K]} = (K, E')$ on a subset of selected vertices $v_i \in K$ of the graph G, where $K \subseteq V$, can be created by adding a subset to the composition K of some vertex $v_i \notin K$, if the optimal (of minimal weight) transitive closure (TC) of some vertices is carried out through it $v_i \in K$.

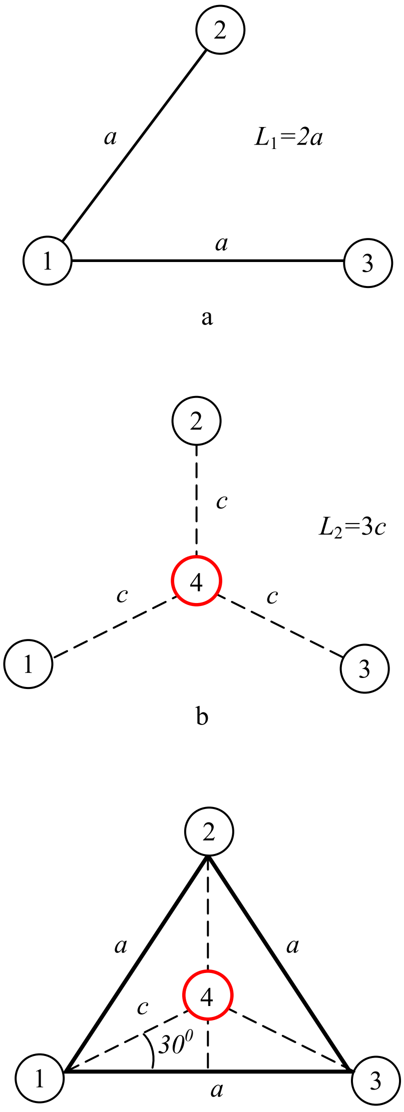
Proof: It is obvious that is a minimal spanning tree on a subset of vertices K, in case $|K| = 2$ is a shortest path connecting these two vertices. If $|K| > 2$, there can be several such paths. Thus, to obtain the connectivity of some vertices s, d, t to the structure of the required $G'_{[s,d,t]}$ can be added $(s,d_1),(d_1,d_2),...,(d_{n-1},d_n),(d_n,d)$ and $(s,t_1),(t_1,t_2),...,(t_{n-1},t_n),(t_n,t)$ edge. Suppose that in the structure of the initial graph G some vertices is present $t_n \notin K$ and edge (t_n,d) for which the following condition is true: $w_{t_n,d} > w_{s,d_1}$, $w_{t_n,d} > w_{d_1,d_2}$,..., $w_{t_n,d} > w_{d_n,d}$ and $w_{t_n,d} < (w_{s,d_1} +...+ w_{d_{n-1},d_n} + w_{d_n,d})$. So, considering the edge (t_n,d) it is possible to reduce the total weight of the required spanning tree $G'_{[s,d,t]}$.

Therefore, the adjacent edges whose weight coefficients are in parentheses can increase the total weight of $G'_{[s,d,t]}$. Thus, the problem should be solved taking into account the possible addition to the structure of the required tree $G_{[K]}$ of additional vertices $v_i \notin K$, the total weight of the transitive closure through which will ensure the minimization of the total weight of the required spanning tree. The theorem is proved.

Let us note an important consequence of the theorem.

Consequence. The weight of transitive closure of vertices $v_i \in K$, can be reduced through some vertices $v_i \notin K$, starting from $|K| = 3$.

Let's explain the mentioned consequence graphically. For example, there are two connected networks with lengths L_1 and L_2, see Fig. 3.



a

b

c

Figure 3 – Geometric comparison of the total weight of TC in networks with $|K| = 3$ and different organization of the structure:
a – without using an additional vertices – a linear substructure.
b – using an additional vertices (v_4) – a radial substructure.
c – geometric interpretation of TC weight based on an equilateral triangle

DOI 10.15588/1607-3274-2024-1-17

The size of an edge c in an equilateral triangle, see Fig. 3, c is determined as follows: $\cos 30^0 = \frac{a/2}{c} \to c = \frac{a}{\sqrt{3}}$. Thus, in an equilateral triangle (or close to it), the inequality $L_1 > L_2$ will always be valid.

Based on the above, the main idea of the method is to check the structure of the initial graph G on the possibility of reducing the weight of TC between threes $v_i \in K$ in their various combinations (sets) due to the addition of some vertices $v_i \notin K$ (see Fig. 3, b). If such a possibility exists, we will speak of the existence of a local minimum, which is ensured by this $v_i \notin K$. Vertices $v_i \notin K$, which do not provide local minima will be removed from the structure of the initial graph G together with the edges, incidental to them, and they will not be considered in the further creating of the MST.

If several radial substructures that provide local minima are found in the graph G structure, they should be analyzed for the extent to which the base vertices $v_i \in K$ are used together. The following options are possible here, see Fig. 4:

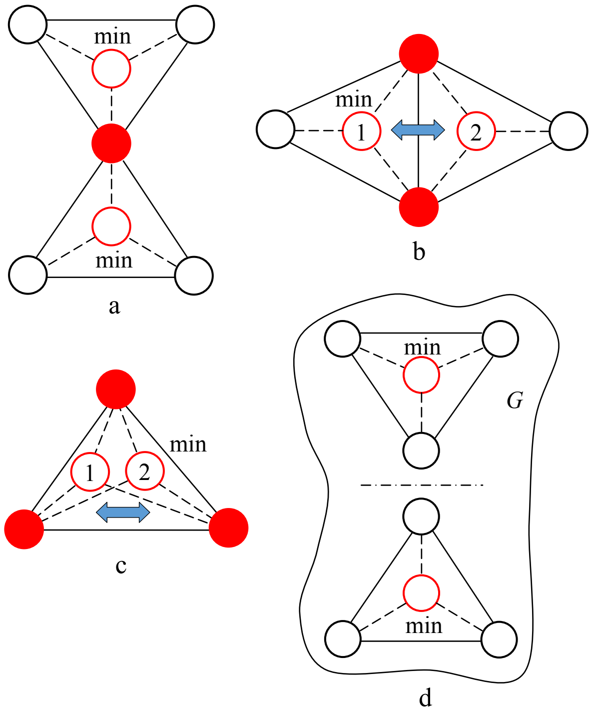


Figure 4 – Degree of compatible use by radial substructures of base vertices $v_i \in K$ (such vertices are marked in solid red):
a – first; b – second; c – third; d – null (no compatible use of base vertices)

Thus, when detected in the structure of the model graph G several radial substructures with the first degree of their joint use of basic vertices (see Fig. 4, a), all vertices $v_i \notin K$, through which such substructures are formed, remain in the structure of the graph G, providing corresponding local minima in it. The same situation occurs with radial substructures with zero degree of use of base vertices (see Fig. 4, d), because substructures are at some distance from each other. In this case, all vertices $v_i \notin K$, due to which such substructures are formed, remain in the structure of the graph G, providing corresponding local minima in it.

The situation will be different in the presence of the second and third degrees of base vertex usage. In such a situation (see Fig. 4, b and Fig. 4, c) you need to find out which of the vertices $v_i \notin K$ (in this case v_1 or v_2) will ensure a lower weight of the TC of the base vertices $v_i \in K$. In the case of the third degree of use of base vertices, competition between vertices $v_i \notin K$ occurs on several parallel radial substructures that connect some triplet of base vertices $v_i \in K$, see Fig. 4, c. The result of such competition is the selection of a single vertex $v_i \notin K$, which ensures the smallest weight of the TC of this trio of vertices. At the same time, the local minimum remains within the three analyzed vertices. In the case of the second degree of using base vertices (see Fig. 4, b), vertices $v_i \notin K$ are the roots of adjacent radial substructures that connect different sets of basic vertices triples $v_i \in K$. In this case, the result of competition between the corresponding vertices $v_i \notin K$ is the choice of the radial substructure that provides the smallest weight of the TC of the corresponding triple of vertices (within the example shown in Fig. 4, b, the local minimum is provided on the triple of vertices, the root of which is the vertex v_1). Both in the first and in the second cases, the vertices $v_i \notin K$, that lost the competition are removed from the structure of the modeling graph G along with the edges, incident to them.

Having considered the general theoretical provisions, we will present the developed method in the form of the following six steps:

Step 1. Based on the modeling weighted undirected graph G, creating its adjacency matrix S_G. According to the matrix S_G creating a matrix of the shortest paths R_G between all pairs of the vertices of graph G. For this purpose, we can use the Warshall-Floyd algorithm [14, 15] or Shimbel [16] and some others.

Step 2. For each $v_i \notin K$ according to the R_G to find the weight of the TC with three base vertices $v_i \in K$, for which the condition $\sum_{j=1}^{n} w_{ij} \to \min$ is valid.

The result of the operation: the column indices (vertices $v_i \in K$) are $idx1$, $idx2$, $idx3$; the total weight of the TC connecting the given trio of base vertices $v_i \in K$

DOI 10.15588/1607-3274-2024-1-17

with $v_i \notin K$, that is analyzed is $\sum w_{v_i \notin K}^{[v_{idx1}, v_{idx2}, v_{idx3}] \in K}$. To save the results of the operation.

Step 3. For each $v_i \in K$ by the column index sets defined in step 2 ($idx1$, $idx2$, $idx3$) according to the matrix R_G to find the weight of the corresponding vertices. The result of the operation: the weight of the TC connecting the base vertices $v_i \in K$ with three base vertices $v_i \in K$ with numbers $idx1$, $idx2$, $idx3$ is $\sum w_{v_i \in K}^{[v_{idx1}, v_{idx2}, v_{idx3}] \in K}$. To save the results of the operation.

Step 4. To remove vertices $v_i \notin K$ for which the condition is valid:

$$\sum w_{v_i \notin K}^{[v_{idx1}, v_{idx2}, v_{idx3}] \in K} \geq \forall \sum w_{v_i \in K}^{[v_{idx1}, v_{idx2}, v_{idx3}] \in K}, \; i = \overline{1, n}, \quad (3)$$

from the structure of the model graph G together with the edges, incident to it. Appropriate changes to the matrix S_G should be made.

Step 5. To carry out a pairwise check of the vertices $v_i \notin K$ remaining after the previous steps for the degree of compatible use by the radial substructures of the base vertices $v_i \in K$:

a) if because of such a check zero or first degree was found (without a match by indices or a match by one index), then such vertices should be left in the structure of the model graph G;

b) in the case of detection of the second or third degree (a match according to two or three indices, respectively), determine the vertices $v_i \notin K$ through which the minimum TC of the corresponding trio of base vertices is ensured $v_i \in K$. To remove the vertices that lost the competition from the structure of the model graph G together with the edges, incident to it. An appropriate changes to the matrix S_G should be made.

Step 6. On the modified in this way graph G, by one of the well-known algorithms for creating the MST the required minimum tree $G'_{[K]} = (K, E' \subseteq E)$ is created.

4 EXPERIMENTS

Let us illustrate the application of the method on the example of the graph provided by Fig. 1, *a*. As before, the MST $G'_{[1,4,5,6]}$ is to be found.

Step 1. A calculated matrix of shortest paths R_G between all pairs of graph vertices has the following form:

$$R_G = \begin{array}{c|ccccccc} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \\ \hline v_1 & 0 & 3 & 4 & 5 & 8 & 9 & 13 \\ v_2 & 3 & 0 & 7 & 2 & 5 & 11 & 10 \\ v_3 & 4 & 7 & 0 & 9 & 4 & 5 & 16 \\ v_4 & 5 & 2 & 9 & 0 & 7 & 10 & 8 \\ v_5 & 8 & 5 & 4 & 7 & 0 & 6 & 15 \\ v_6 & 9 & 11 & 5 & 10 & 6 & 0 & 11 \\ v_7 & 13 & 10 & 16 & 8 & 15 & 11 & 0 \end{array} \quad (4)$$

In expression (4), vertices $v_i \in K$ are marked in red.

Step 2. For each $v_i \notin K$ we define $\sum w_{v_i \notin K}^{[v_{idx1}, v_{idx2}, v_{idx3}] \in K} \to \min$. For the vertex $v_2 \notin K$ it is $\sum w_{v_2}^{[v_1, v_4, v_5]} = 10$. For the vertex $v_3 \notin K$ it is $\sum w_{v_3}^{[v_1, v_5, v_6]} = 13$. For the vertex $v_7 \notin K$ it is $\sum w_{v_7}^{[v_1, v_4, v_6]} = 32$.

Step 3. The results of calculations for this step are presented in Table 1.

Table 1 – The weight of TC of the basic vertices $v_i \in K$ with triples of base vertices $v_i \in K$, having the indices defined in step 2 of the method

$v_i \in K$	$\sum w_{v_i \in K}^{[v_1, v_4, v_5]}$	$\sum w_{v_i \in K}^{[v_1, v_5, v_6]}$	$\sum w_{v_i \in K}^{[v_1, v_4, v_6]}$
1	2	3	4
v_1	13	17	14
v_4	12	22	15
v_5	15	14	21
v_6	25	15	25

Step 4. According to inequality (3), we compare the received sums of weights $\sum w_{v_2}^{[v_1, v_4, v_5]} = 10$, $\sum w_{v_3}^{[v_1, v_5, v_6]} = 13$, $\sum w_{v_7}^{[v_1, v_4, v_6]} = 32$ with sums of weights on the corresponding indices for $v_i \in K$, which are presented in Table 1. The inequality is valid only for the vertex v_7, since $\sum w_{v_7}^{[v_1, v_4, v_6]} = 32$ is greater than any value in column 4, see Table 1. So, the vertex v_7 is removed from the structure of the model graph G with all the edges incident to it. Corresponding changes are also to be made to the matrix S_G.

Step 5. Let us perform a pairwise check of the vertices $v_i \notin K$ remaining after the previous steps for the degree of compatible use by the corresponding radial substructures of the base vertices $v_i \in K$. Such vertices are v_2 and v_3 for which respectively $\sum w_{v_2}^{[v_1, v_4, v_5]} = 10$

DOI 10.15588/1607-3274-2024-1-17

and $\sum w_{v_3}^{[v_1,v_5,v_6]} = 13$. As we can see, the radial substructures, the roots of which are these vertices, jointly use the base vertices v_1 and v_5. Therefore, we got the second degree of joint use of basic vertices by radial substructures $v_i \in K$ (match by two indices). Since the weight of the transitive closure over the vertex v_3 is greater than through the vertex v_2 (13>10), the vertex v_3 is also removed with all its incident edges from the structure of the model graph G. Corresponding changes are also to be made to the matrix S_G.

Step 6. On the modeling graph G modified in this way (Fig. 5) using the Kruskal method, we will create the MST $G_{[1,4,5,6]}^{t}$. It will be identical to the MST presented at Fig. 2.

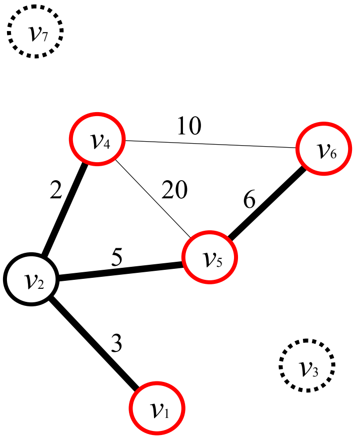


Figure 5 – Modified model graph G and the MST on the selected subset of vertices

5 RESULTS

When applying the developed method to the initial undirected graph G (see Fig. 1), three radial substructures were successively considered, the roots of which were vertices v_2 , v_3, v_7, not included in the set K. During the verification, it was found that the vertex of v_7 does not provide a minimum TC between the specified three base vertices $v_i \in K$. This fact made it possible to modify the initial graph G by removing this vertices and all edges incident to it from its structure. The vertices v_2 and v_3 have provided the minimum TC. At the same time, the radial structures (triplets of vertices), of whose roots they are, intersect along two vertices and have the second degree of joint use of the base vertices $v_i \in K$. This fact led to the need to compare the vertices v_2, v_3, and choose the one that provides the local minimum of TC. This vertex appeared to be the vertex v_2. Consequently, vertex v_3 was also removed from the original graph G structure.

Therefore, the internal tools of the proposed method allow testing the structure for the presence of local minima in the TC of the base vertices $v_i \in K$ through the vertices $v_i \notin K$, and modifying the structure of the initial graph G to further find MST $G_{[K]}^{t}$ in this structure.

6 DISCUSSION

The combination of the approaches proposed in the article allowed us to develop a method by which it is possible to build a MST on an arbitrary subset of vertices of the initial undirected graph. This became possible due to the analysis of radial substructures whose roots are vertices $v_i \notin K$, in terms of the weight of the TC of these substructures, and the search for local minima among them. At the same time, this became possible due to the use of the shortest paths matrix (R_G) between all pairs of vertices of the model graph G. Due to the fact that such a matrix contains information not only about the presence of TC between any pair of vertices, but also quantitatively characterizes this relationship, it became possible to analyze different sets of three basic vertices $v_i \in K$, from different locations of the model graph relative to the root of the current radial substructure. The above allows us to launch a mechanism for revealing local minima in the structure of the model graph G and selecting vertices $v_i \notin K$ that provide this minimum. On the other hand, those vertices $v_i \notin K$, which do not provide local minima are removed from the structure of the model graph G, thereby not increasing the weight of the required MST $G_{[K]}^{t}$.

Several dozen full-scale experiments on various network objects of low density have shown the efficiency of the developed method, and the solutions obtained were optimal. At the same time, the behavior of the method on dense network objects of high dimensionality remains a challenge ($n > 30$). Thus, the method could be considered quasi-optimal at the moment.

The computational complexity of the combinatorial algorithm that implements the developed method will be determined by the computational complexity of its “basic elements” – the algorithm for finding the shortest paths between all pairs of vertices of the model graph and the algorithm for creating the MST. If the Warshall-Floyd algorithm and the Kruskal algorithm are taken as the basic algorithms, respectively, the overall computational complexity of the combinatorial algorithm will be estimated $O(n^3 + E\log(E))$.

The obtained polynomial estimate of the computational complexity is suitable for using such an algorithm in solving relevant management problems in real life.

CONCLUSIONS

The article solves the actual scientific and applied problem of creating the MST $G_{[K]}^{t}$ on an arbitrarily chosen subset of vertices of the initial undirected weighted graph, where K is an arbitrarily chosen subset of vertices of the initial graph G.

The scientific novelty of the developed method is as follows:

1) in the formulation of the consequence that to reduce the weight of the transitive closure of the base vertices $v_i \in K$, through some vertices $v_i \notin K$, starting from $|K| = 3$;

2) in the proposed approach to vertices selection $v_i \notin K$. The essence of the approach is to compare the weights of transitive closures of different radial substructures whose roots are vertices $v_i \notin K$, combining different sets of three basic vertices $v_i \in K$;

3) in the proposed approach to determining the local minimum of the weight of radial substructures, among substructures that are in competition. The core of the approach is to analyze the degree of joint use of base vertices $v_i \in K$ by different radial substructures.

The practical value of the method is when it is applied to large and dense network objects that have undergone fragmentation (destruction due to external influences), it is possible to significantly reduce the amount of restoration work and/or total financial costs while quickly restoring the connectivity of elements that are of higher importance in the structure of such an object.

A promising direction for further research is the final verification of the developed method to determine its optimality class.

ACKNOWLEDGEMENTS

This article highlights one of the results obtained by the authors in 2021-2022 while implementing the research project (state registration number 0120U002173) at the Research Center of the National Academy of the National Guard of Ukraine. The authors are grateful to their colleagues for their support during the research and active participation in the discussion of the results. All authors declare that they have neither financial support nor obligations.

REFERENCES

1. Kerigan-Kyrou D. Critical Energy Infrastructure: Operators, NATO, and Facing Future Challenges, *Connections: The Quarterly Journal 12*, 2013, Vol. 3, pp. 109–117. DOI: 10.11610/Connections.12.3.06.
2. Tichý L., Eichler J. Terrorist Attacks on the Energy Sector: The Case of Al Qaeda and the Islamic State, *Studies in Conflict & Terrorism,* 2018, Vol. 41, No. 6, pp. 450–473. DOI: 10.1080/1057610X.2017.1323469.
3. Dunn S., Wilkinson S. Hazard Tolerance of Spatially Distributed Complex Networks, Re*liability Engineering & System Safety 157,* 2017, pp. 1–12. DOI:10.1016/j.ress.2016.08.010.
4. Varianou Mikellidou C., Shakou L., Boustras G. et al. Energy Critical Infrastructures at Risk from Climate Change: A State-of-the-Art Review, *Safety Science,* 2018, Vol. 110, Part C, pp. 110–120. DOI: 10.1016/j.ssci.2017.12.022.
5. Danyk Yu., Maliarchuk T., Briggs Ch. Hybrid War: High-tech, Information and Cyber Conflicts. Connections, *The Quarterly Journal 16,* 2017, Vol. 2, pp. 5–24. DOI: 10.11610/Connections.16.2.01.
6. Sulema O. K., Lande D. V. Finding the optimal hierarchy in a quasi-hierarchical graph by centrality criteria, *Registration, storage and processing of data,* 2015, Vol. 17, No 4, pp. 3–10. ISSN 1560-9189.
7. Prim R. Shortest connection networks and some generalizations, *Bell System Technical Journal*, 1957, Vol. 36, No. 6, pp. 1389–1401. DOI: 10.1002/j.1538-7305.1957.tb01515.x.
8. Kruskal J. On the Shortest Spanning Subtree of a Graph and the Traveling Salesman Problem, *Proc. AMS,* 1956, Vol 7, No. 1, pp. 48–50. DOI: 10.1090/S0002-9939-1956-0078686-7.
9. Pawan Harish, Narayanan P., Vineet Vibhav, Suryakant Patidar Chapter 7 – Fast Minimum Spanning Tree Computation. GPU Computing Gems Jade Edition, Applications of GPU Computing Series. San Francisco, Morgan Kaufmann, 2012, pp. 77–88. ISBN 9780123859631, DOI: 10.1016/B978-0-12-385963-1.00007-1.
10. Cheriton D., Endre T. Finding minimum spanning trees, *SIAM Journal on Computing,* 1976, Vol. 5 (4), pp. 724–742. DOI:10.1137/0205051.
11. Seth P., Vijaya R. An optimal minimum spanning tree algorithm, *Journal of the ACM,* 2002, Vol. 49 (1), pp. 16–34. DOI:10.1145/505241.505243.
12. Johnson D. Priority queues with update and finding minimum spanning trees, *Information Processing Letters,* 1975, 4 (3), pp. 53–57. DOI:10.1016/0020-0190(75)90001-0.
13. Christofides N. Theory of graphs. Algorithmic approach. Moscow, Mir, 1978, 432 p.
14. Floyd R. Algorithm 97: Shortest Path, *Communications of the ACM,* 1961, Vol. 5 (6), 345 p. DOI:10.1145/367766.368168.
15. Warshall S. Algorithm on Boolean matrices, *Journal of the ACM,* 1962, Vol. 9 (1), pp. 11–12. DOI:10.1145/321105.321107.
16. Shimbel A. Structural parameters of communication networks, *Bulletin of Mathematical Biophysics*, 1953, Vol. 15 (4), pp. 501–507. DOI:10.1007/BF02476438.

Received 11.12.2023.
Accepted 25.02.2024.

УДК 519.172.1: 62-503.57

МЕТОД ПОБУДОВИ МІНІМАЛЬНОГО КІСТЯКОВОГО ДЕРЕВА НА ДОВІЛЬНІЙ ПІДМНОЖИНІ ВЕРШИН ЗВАЖЕНОГО НЕОРІЄНТОВАНОГО ГРАФА

Бацамут В. М. – д-р військ. наук, професор, заступник начальника науково-дослідного центру службово-бойової діяльності Національної гвардії України Національної академії Національної гвардії України, Харків, Україна.

Годлевський С. О. – науковий співробітник науково-дослідного центру службово-бойової діяльності Національної гвардії України Національної академії Національної гвардії України, Харків, Україна.

Бабков Ю. П. – канд. техн. наук, доцент, професор кафедри державної безпеки Національної академії Національної гвардії України, Харків, Україна.

DOI 10.15588/1607-3274-2024-1-17

Морквін Д. А. – д-р філософ., начальник науково-дослідної лабораторії науково-дослідного центру службово-бойової діяльності Національної гвардії України Національної академії Національної гвардії України, Харків, Україна.

АНОТАЦІЯ

Актуальність. Актуальність статті обумовлюється потребою у подальшому розвитку моделей оптимального відновлення зв'язності мережних об'єктів, що зазнали фрагментації внаслідок надзвичайних ситуацій різного характеру походження. Запропонований у статті метод усуває проблемну ситуацію, що полягає у необхідності мінімізації обсягу відновлювальних робіт (загальних фінансових витрат) при оперативному відновленні зв'язності обраної підмножини елементів мережевого об'єкту після його фрагментації.

Мета роботи полягає у розробленні методу побудови мінімального кістякового дерева на довільній підмножині вершин зваженого неорієнтованого графу для мінімізації обсягу відновлювальних робіт і/або загальних фінансових витрат при оперативному відновленні зв'язності елементів, які мають вищий рівень важливості в структурі фрагментованого мережного об'єкту.

Метод. Розроблений метод заснований на ідеї пошуку в структурі модельного неорієнтованого графа локальних мінімумів з використанням вершин графу, що не входять до переліку базових вершин, які потрібно об'єднати мінімальним кістяковим деревом. Під час пошуку локальних мінімумів використовується поняття рівностороннього трикутника та радіальної структури в такому трикутнику. При цьому розрізняються чотири типи підструктур, які забезпечують локальні мінімуми: перші, ті що мають одну спільну базову вершину; другі, ті що мають дві спільні базові вершини; треті, ті що мають три спільні базові вершини; четверті, ті що не мають спільних базових вершин – знаходяться в різних частинах модельного графа. Ті вершини, що не входять до переліку базових, але через які забезпечуються локальні мінімуми, додаються до складу базових. Інші вершини (небазові) разом з інцидентними їм ребрами видаляються з структури модельного графа. Далі, на отриманій таким чином структурі, одним із відомих методів побудови кістякових дерев, будується мінімальне кістякове дерево, яке поєднує набір базових вершин.

Результати. 1) Розроблено метод побудови мінімального кістякового дерева на довільній підмножині вершин зваженого неорієнтованого графа. 2) Запропонована сукупність критеріїв для визначення локальних мінімумів в структурі модельного графа. 3) Виконано верифікацію методу на тестових задачах.

Висновки. Проведені теоретичні дослідження та низка експериментів підтверджують працездатність розробленого методу. Рішення, що виробляються із використанням розробленого методу, є точними, що дозволяє рекомендувати його до практичного використання при визначенні стратегій відновлення зв'язності фрагментованих мережевих об'єктів.

КЛЮЧОВІ СЛОВА: мережевий об'єкт, зважений неорієнтований граф, зв'язність, транзитивне замкнення, мінімальне кістякове дерево, локальний оптимум, критерій оптимізації, метод.

ЛІТЕРАТУРА

1. Kerigan-Kyrou D. Critical Energy Infrastructure: Operators, NATO, and Facing Future Challenges / D. Kerigan-Kyrou // Connections: The Quarterly Journal 12. – 2013. – Vol. 3. – P. 109–117. DOI: 10.11610/Connections.12.3.06.
2. Tichý L. Terrorist Attacks on the Energy Sector: The Case of Al Qaeda and the Islamic State / L. Tichý, J. Eichler // Studies in Conflict & Terrorism. – 2018. Vol. 41, No 6. – P. 450–473. DOI: 10.1080/1057610X.2017.1323469.
3. Dunn S. Hazard Tolerance of Spatially Distributed Complex Networks. / S. Dunn, S. Wilkinson // Reliability Engineering & System Safety 157. – 2017. – P. 1–12. DOI:10.1016/j.ress.2016.08.010.
4. Energy Critical Infrastructures at Risk from Climate Change: A State-of-the-Art Review / [C. Varianou Mikellidou, L. Shakou, G. Boustras et al.] // Safety Science. – 2018. – Vol. 110, Part C. – P. 110–120. DOI: 10.1016/j.ssci.2017.12.022.
5. Danyk Yu. Hybrid War: High-tech, Information and Cyber Conflicts. Connections: / Yu. Danyk, T. Maliarchuk, Ch. Briggs // The Quarterly Journal 16. – 2017. – Vol. 2. – P. 5–24. DOI: 10.11610/Connections.16.2.01.
6. Сулема О. К. Знаходження оптимальної ієрархії у квазіієрархічному графі за критеріями центральності. / О. К. Сулема, Д. В. Ланде // Реєстрація, зберігання і обробка даних. – 2015. – Т. 17. – № 4. – С. 3–10. ISSN 1560-9189.
7. Prim R. Shortest connection networks and some generalizations / R. Prim // Bell System Technical Journal. – 1957. – Vol. 36, No. 6. – P. 1389–1401. DOI: 10.1002/j.1538-7305.1957.tb01515.x.
8. Kruskal J. On the Shortest Spanning Subtree of a Graph and the Traveling Salesman Problem / J. Kruskal // Proc. AMS. – 1956. – Vol 7, No. 1. – C. 48–50. DOI: 10.1090/S0002-9939-1956-0078686-7.
9. Pawan Harish. Chapter 7 – Fast Minimum Spanning Tree Computation. GPU Computing Gems Jade Edition, Applications of GPU Computing Series / Pawan Harish, P. Narayanan, Vibhav Vineet, Suryakant Patidar. – San Francisco: Morgan Kaufmann, 2012. – P. 77–88. ISBN 9780123859631, DOI: 10.1016/B978-0-12-385963-1.00007-1.
10. Cheriton D. Finding minimum spanning trees / D. Cheriton, T. Endre // SIAM Journal on Computing. – 1976. Vol. 5 (4). – P. 724–742. DOI:10.1137/0205051.
11. Seth P. An optimal minimum spanning tree algorithm / P. Seth, R. Vijaya // Journal of the ACM. – 2002. – Vol. 49 (1). – P. 16–34. DOI:10.1145/505241.505243.
12. Johnson D. Priority queues with update and finding minimum spanning trees / D. Johnson. // Information Processing Letters. – 1975. – 4 (3). – P. 53–57. DOI:10.1016/0020-0190(75)90001-0.
13. Кристофидес Н. Теория графов. Алгоритмический подход / Н. Кристофидес. – Москва : Мир, 1978. – 432 с.
14. Floyd R. Algorithm 97: Shortest Path / R. Floyd // Communications of the ACM. – 1961. – Vol. 5 (6). – 345 p. DOI:10.1145/367766.368168.
15. Warshall S. Algorithm on Boolean matrices / S. Warshall // Journal of the ACM. – 1962. – Vol. 9 (1). – P. 11–12. DOI:10.1145/321105.321107.
16. Shimbel A. Structural parameters of communication networks / A. Shimbel // Bulletin of Mathematical Biophysics. – 1953. – Vol. 15 (4). – P. 501–507. DOI: 10.1007/BF02476438.

DOI 10.15588/1607-3274-2024-1-17