

# УПРАВЛІННЯ У ТЕХНІЧНИХ СИСТЕМАХ

## CONTROL IN TECHNICAL SYSTEMS

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### STEWART PLATFORM DYNAMICS MODEL IDENTIFICATION

**Zozulya V. A.** – PhD, Associate Professor of the Department of Digital Economy and System Analysis, State University of Trade and Economics, Kyiv, Ukraine.

**Osadchy S. I.** – Dr. Sc., Professor, of the Department of Aircraft Construction, Aircraft Engines, and Airworthiness Maintenance, Flight Academy of the National Aviation University, Kropyvnytskyi, Ukraine.

**Nedilko S. N.** – Dr. Sc., Professor, Acting Director of the Flight Academy of the National Aviation University, Kropyvnytskyi, Ukraine.

#### ABSTRACT

**Context.** At the present stage, with the current demands for the accuracy of motion control processes for a moving object on a specified or programmable trajectory, it is necessary to synthesize the optimal structure and parameters of the stabilization system (controller) of the object, taking into account both real controlled and uncontrolled stochastic disturbing factors. Also, in the process of synthesizing the optimal controller structure, it is necessary to assess and consider multidimensional dynamic models, including those of the object itself, its basic components, controlled and uncontrolled disturbing factors that affect the object in its actual motion.

**Objective.** The aim of the research, the results of which are presented in this article, is to obtain and assess the accuracy of the Stewart platform dynamic model using a justified algorithm for the multidimensional moving object dynamics identification.

**Method.** The article employs a frequency-domain identification method for multidimensional stochastic stabilization systems of moving objects with arbitrary dynamics. The proposed algorithm for multidimensional moving object dynamics model identification is constructed using operations of polynomial and fractional-rational matrices addition, multiplication, Wiener factorization, Wiener separation, and determination of dispersion integrals.

**Results.** As a result of the conducted research, the problem of identifying the dynamic model of a multidimensional moving object is formalized, illustrated by the example of a test stand based on the Stewart platform. The outcomes encompass the identification of the dynamic model of the Stewart platform, its transfer function, and the transfer function of the shaping filter. The verification of the identification results confirms the sufficient accuracy of the obtained models.

**Conclusions.** The justified identification algorithm allows determining the order and parameters of the linearized system of ordinary differential equations for a multidimensional object and the matrix of spectral densities of disturbances acting on it under operating conditions approximating the real functioning mode of the object prototype. The analysis of the identification results of the dynamic models of the Stewart platform indicates that the primary influence on the displacement of the center of mass of the moving platform is the variation in control inputs. However, neglecting the impact of disturbances reduces the accuracy of platform positioning. Therefore, for the synthesis of the control system, methods should be applied that enable determining the structure and parameters of a multidimensional controller, considering such influences.

**KEYWORDS:** Identification, transfer function matrix, spectral density, quality functional, Stewart platform.

#### ABBREVIATIONS

AHRS is an attitude and heading reference systems;  
IKP is an inverse kinematics problem;  
IMU is an inertial measurement unit;  
LMS is a linear movement system;  
ONS are sensors of orientation and navigation;  
RT is a target Real-Time;  
SLM are sensors of linear movements;  
TIG is a trajectory and interpolation generator;  
WS is a working surface.

#### NOMENCLATURE

$D$  is a result of the Wiener factorization of the transposed matrix  $S'_{\zeta\zeta}$ ;

$E_n$  is a  $n \times n$  unit matrix;  
 $J$  is a quality functional;  
 $M$  is a matrix of dimension  $m \times n$ , the elements of which are polynomials from the differentiation operator  $s$ ;  
 $m$  is a number of signals at the output of the control system;  
 $n$  is a local system inputs number;  
 $O_{m \times n}$  is a zero matrix of size  $m \times n$ ;  
 $P$  is a polynomial matrix of dimension  $m \times m$ ;  
 $R$  is an additionally defined weight matrix;  
 $r_0$  is a vector of program signals;  
 $S'_{r_0 r_0}$  is a transposed spectral density matrix of the vector  $r_0$ ;

$S'_{x_1 x_1}$  is a transposed spectral density matrix of the vector  $x_1$ ;

$S'_{\zeta x}$  is a transposed matrix of mutual spectral densities between the generalised input vector  $\zeta$  and the vector  $x_1$ ;

$S'_{x \zeta}$  is a transposed matrix of mutual spectral densities between the vectors  $x_1$  and  $\zeta$ ;

$S'_{\Delta \Delta}$  is a transposed matrix of spectral densities of uncorrelated white noise of single intensity;

$S'_{\varepsilon \varepsilon}$  is a transposed matrix of spectral densities of the vector of identification errors  $\varepsilon$ ;

$S'_{\Psi_{ob} \Psi_{ob}}$  is a transposed matrix of spectral densities of the disturbing influence;

$S'_{\zeta \zeta}$  is a transposed spectral density matrix of the generalized input vector;

$T_0$  is a matrix of results of dividing the polynomials of the numerators by the polynomials of the denominator of the product on the right side of the expression;

$T_+$  is a matrix of fractional rational functions whose poles are located in the left half-plane of the complex plane;

$T_-$  is a matrix of fractional rational functions with poles in the right half-plane;

$W_i$  are controllers;

$W_1$  is an optimal structure of the matrix is the transfer function of the identification object;

$W_2$  is an optimal structure of the matrix is the transfer function of the shaping filter  $\Psi$ ;

$x_0$  is a vector of movement of the working surface in the working space;

$x_1$  is a vector of signals at the output of the control system;

$x_{id}$  is an estimate of the vector of the WS movement, obtained through the identification process;

$\Delta$  is an uncorrelated white noise of single intensity;

$\Phi$  is a block matrix of transfer functions of size  $n \times (n+m)$ ;

$\varepsilon$  is a vector of identification errors;

$\varphi_i$  is a vector of measurement noise;

$\Psi$  is a transfer function of the shaping filter;

$\Psi_{ob}$  is a vector of centred stationary random disturbances in the control object;

$\Psi_{WS}$  is a vector of centred stationary random disturbances in the working surface;

$\zeta$  is a generalised vector of input influences.

## INTRODUCTION

At the present stage of creating and operating moving objects of various purposes, spatial mechanisms with parallel structures, and a range of complex and responsible controlled technological processes, the issues of ensuring the competitiveness of products being created have become crucial. Their competitiveness is mainly influenced by the extent to which they achieve high quality and efficient utilization, as well as the ultimate goals of functioning in responsible operating modes. As stated in refer-

ences [1, 2], the operation of these products is affected by a multitude of stochastic factors, both deterministic and random, which considerably complicate the processes of attaining set goals and the ultimate results in each specific responsible operating mode of moving objects. Determining the dynamics models of input-output stochastic signal vectors of a control object or its prototype in respective operating modes allows for structural identification of the dynamics models of such an object. The practical methods and algorithms for structural identification should enable the determination of dynamic models for both the control object itself in the mode of interest and the uncontrolled stochastic disturbances acting on the object under these conditions.

This relevance is driven by the practical requirements to align identification procedures with the conditions of designing closed-loop control systems. Modern methods and algorithms should be based on new approaches in creating computational procedures that exhibit enhanced accuracy and reliability in computation, reducing the growth of orders in the results. This enables the determination of the order and parameters of the linearized system of ordinary differential equations for a multidimensional object and the matrix of spectral densities of disturbances acting on it in conditions approximating the real operating mode of the experimental object.

**The object of study** in this paper is the Stewart platform working surface motion closed-loop control system. The Stewart platform is a spatial mechanism with a parallel kinematic structure, consisting of six identical kinematic chains (actuators) [3]. Such mechanisms can be used as machining centers (machines), coordination-measuring centers, vibration platforms (test stands), motion simulators, and stabilization platforms. The Stewart platform has six degrees of freedom for the motion of the mobile platform. By programmatically adjusting the Stewart platform drives lengths, it is possible to control the moving base position, move it in vertical and horizontal directions, as well as rotate it in three planes.

**The subject of study** is the algorithm for identifying the Stewart platform dynamic model, its transfer function, and the shaping filter transfer function.

**The purpose of the work** is to obtain the Stewart platform dynamic model using a justified algorithm for the multidimensional moving object dynamics identification.

## 1 PROBLEM STATEMENT

The basis of the theory of constructing mathematical models (identification) is the information-algorithmic approach. Under conditions of a priori uncertainty, the information component begins to play a dominant role, as its analysis largely determines the application of certain algorithmic procedures and formalization methods that allow the object under study mathematical description synthesis [4].

Structural identification allows establishing the interaction of system individual components in the process of

forming reactions. In this case, the system configuration is considered known, or assumptions are made about the class of the functional description relative to it, while the parameters that characterize the system are treated as unknown. The identification task boils down to search for solutions in the space of the sought parameters of the system [1, 5].

The analysis carried out made it possible to propose a structural diagram of the Stewart platform motion control system, which is built according to the principle of two-loop tracking systems (Fig. 1).

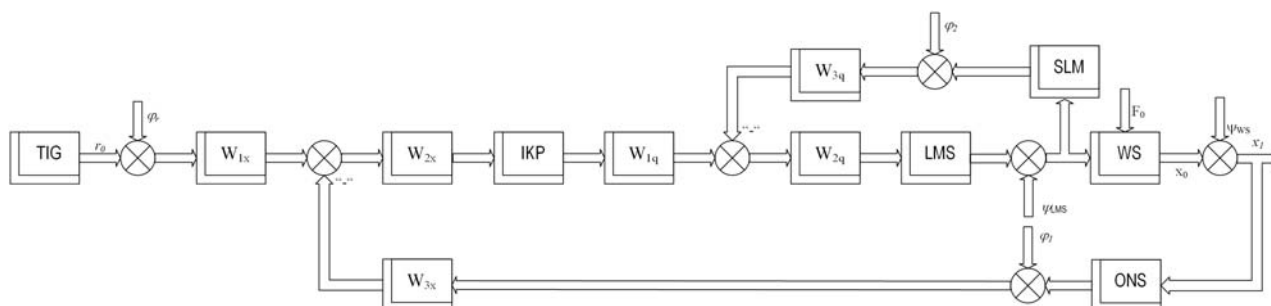


Figure 1 – Structural diagram of the motion control system of the working surface of Stewart platform

The programme signal vector  $r_0$  is received from the trajectory and interpolation generator block, i.e. the set trajectory of movement of the working surface:

$$r_0 = [\xi_0 \quad \eta_0 \quad \zeta_0 \quad \psi_0 \quad \vartheta_0 \quad \gamma_0]^T,$$

where  $\xi_0, \eta_0, \zeta_0$  – given coordinates of the rotation center of the working surface relative to the coordinate system associated with the base of Stewart platform;  $\psi_0$  is a specified yaw angle,  $\vartheta_0$  is a specified pitch angle,  $\gamma_0$  is a specified roll angle of the working surface; index  $j$  is a transposition sign.

The position of the working surface relative to the base characterizes the vector of movement of the working surface  $x_0$  in the working space, of the form:

$$x_0 = [\xi_{out} \quad \eta_{out} \quad \zeta_{out} \quad \psi_{out} \quad \vartheta_{out} \quad \gamma_{out}]^T,$$

where  $\xi_{out}, \eta_{out}, \zeta_{out}$  – the output coordinates of the rotation center of the working surface relative to the coordinate system associated with the base of Stewart platform;  $\psi_{out}$  is a yaw output angle,  $\vartheta_{out}$  is an output pitch angle,  $\gamma_{out}$  is and output roll angle of the working surface.

Vector of real values of the movement of the working surface  $x_0$ :

$$x_1 = x_0 + \psi_{WS},$$

where  $\psi_{WS}$  is the vector of disturbances affecting the working surface, or

$$x_1 = [\xi_1 \quad \eta_1 \quad \zeta_1 \quad \psi_1 \quad \vartheta_1 \quad \gamma_1]^T.$$

As a result of the structural transformation of the WS motion control scheme of Stewart platform in Fig. 1 and considering that we can measure the  $m$ -dimensional vec-

tor of control signals  $r_0$  and the vector of actual values of the motion of the working surface  $x_1$ , the scheme can transform to the form depicted in Fig. 2 in cases where feedback cannot be established.

Furthermore, it is common for the origin of disturbances  $\Delta$  and the vector of control signals  $r_0$  to have distinct physical natures. Therefore, the assumption of their independence is typically adopted.

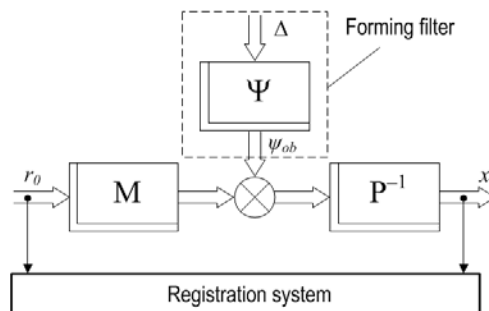


Figure 2 – Structural diagram of the identification object

Fig. 2 shows  $P, M$  – polynomial matrices from the differentiation operator  $s$  of the appropriate size, which characterize the dynamics of a closed system,  $\psi_{ob}$  is a vector of centered stationary random disturbances, which characterizes the action of all disturbances and noises in the control system (Fig. 1) and  $n$  is a measurable stationary random process with zero mathematical expectation and the unknown matrix of spectral densities  $S_{\psi_{ob}\psi_{ob}}$ .

Reviewing the types of tasks addressed by mechanisms utilizing the Stewart platform [3], one can affirm that the working surface undergoes minor movements around the center of rotation. Subsequently, in the first approximation, it is possible to formulate an ordinary differential equation representing the dynamics of the control object. This equation reflects the correlation between the system inputs  $r_0, \psi_{ob}$ , and the output  $x_1$  (Fig. 2):

$$Px_1 = Mr_0 + \psi_{ob}.$$

The output  $x_1$  and one of the inputs  $r_0$  can be measured, but the input  $\psi_{ob}$  cannot be measured.

The vector  $\psi_{ob}$  is formed from the noise vector  $\Delta$  by a linear stable filter with a matrix of transfer functions  $\Psi$  and can be represented as:

$$\psi_{ob} = \Psi\Delta.$$

The task of obtaining dynamic characteristics of a multidimensional moving object and the disturbance vector involves the following: based on the measured input vector  $r_0$  and output vector  $x_1$ , it is necessary to determine the order and parameters of the matrices  $P$ ,  $M$ , and  $\Psi$  that minimize the following identification quality criterion.

$$J = \frac{1}{j} \int_{-j\infty}^{j\infty} \text{tr}(S'_{\varepsilon\varepsilon} R) ds, \quad (1)$$

where  $R$  is a symmetric positive definite weight matrix;  $S'_{\varepsilon\varepsilon}$  is the transposed matrix of spectral densities of the vector of identification errors  $\varepsilon$ .

The identification error vector, denoted as  $\varepsilon$ , is defined as the difference between the vector of measured actual values of the movement of the WS  $x_1$  and the estimate of the vector of the WS movement,  $x_{id}$ , obtained through the identification process:

$$\varepsilon = x_1 - x_{id}.$$

Similarly, the identification error vector  $\varepsilon$ , can be expressed as:

$$\varepsilon = x_1 - \Phi\zeta. \quad (2)$$

where  $\Phi$  represents the block matrix comprising the transfer functions of the identification object, defined as

$\Phi = [W_1, W_2]; \zeta = \begin{bmatrix} r_0 \\ \Delta \end{bmatrix}$  extended vector of input signals.

Then the task of identifying the dynamics model of the Stewart platform is simplified to minimizing either the error vector  $\varepsilon$  (2) or the identification quality indicator (1) by determining the two transfer function matrices  $W_1$  and  $W_2$ .

The solution to the task was found as a result of three stages of work:

- development of the algorithm for structural identification of a multidimensional dynamic object with stochastic input signals;
- gathering and processing experimental data of vectors  $r_0$  and  $x_1$ ;
- assessing variations in the variance of the output coordinates vector of the working surface of the Stewart platform  $x_1$  with random alterations in vectors  $r_0$  and  $\psi_{ob}$ .

## 2 REVIEW OF THE LITERATURE

Given the modern requirements for the accuracy of motion control processes of a moving object along a specified or programmable trajectory, it is necessary to synthesize optimal structures and parameters for the object stabilization system (controller). This synthesis should take into account the real controlled and uncontrollable stochastic disturbing factors that act on the object in each specific mode of its operation. In today's demand for precise control processes of a moving object along a specified or programmable trajectory, there is a need to design the optimal structure and parameters of the object's stabilization system (controller). This involves considering real controllable and uncontrollable stochastic disturbing factors, which operate on the object in each specific operational mode. Furthermore, in the process of synthesizing the optimal controller structure, it is also necessary to evaluate and consider multidimensional dynamic models of the object itself, its basic components, as well as controlled and uncontrollable disturbing factors that influence the object in its actual motion [6].

As a rule, the mentioned dynamic models of moving objects, corresponding to real operational modes of motion, are either unknown or known very imprecisely [7, 8]. Such situations in the modern stage of technological development do not meet the requirements for the competitiveness of motion control systems for existing or newly created objects. Due to the lack of knowledge of the required methods and algorithms for processing and the target application of the results of processing stochastic information that can be obtained during testing, convenient and necessary 'real' models of the dynamics of objects, their parts, and disturbing factors are practically absent at the present time. For instance, considering the Stewart platform as an object controlled reveals several peculiarities, the main of which is that, for many technological tasks, the parameters of disturbing influences applied to the working part, individual axes, are not predetermined, and there is complexity in constructing an adequate analytical mathematical model [7, 9].

However, special full-scale (semi-full-scale) studies of prototypes of future mobile objects allow solving problems of structural identification [1] of the dynamics models of complex objects.

Today, there is a fairly wide range of methods for identifying dynamic models of control objects that operate under conditions of uncontrolled concentrated stationary random influences [1, 4, 5, 10], based on the 'input-output' data. The 'input-output' method [10] involves combining signals acting at the input and output of the system into a single signal vector. This vector is considered the output of an imaginary dynamic system, to the input of which a virtual test signal with known dynamics, such as "white noise", is applied. However, almost all of them are designed for use in conditions where there is no mutual influence between external disturbance and control signal. At the same time, the presence of feedback that cannot be unlocked during identification makes it impossible to accept the hypothesis even for different

sources of disturbances and control signals. In such cases, specialized, sophisticated identification technologies are required.

The method of identifying the dynamics of multidimensional control objects, as described in sources [11, 12], overcomes these drawbacks but limits the class of useful signals, disturbances, and interferences acting during the experiment. All the mentioned signals must belong to centered stationary random processes or to the additive mixture of a stationary random process and a deterministic time function.

The article in [13] presents a method for identifying multidimensional stochastic stabilization systems for moving objects with arbitrary dynamics in the frequency domain. Initial information about changes in the “input-output” signals is obtained from passive experiments during field trials, which is distorted by the imperfections of measuring instruments and the recording system. This method is employed with the requirement that the dynamic models of external influences on the system, which come into play during the identification experiment, need to be explicitly defined.

In such conditions, the range of identification methods is significantly narrowed. For instance, in the article [14], algorithms for identifying the dynamics of elements in a multidimensional stabilization system are presented. It asserts that under the conditions where the signals in the control loop fall within the category of centered stationary random processes and sensor noises stem from diverse sources introducing disturbances to the system, it becomes feasible to distinctly identify the matrices of fractional-rational functions associated with the disturbance generator. Moreover, it allows determining a system of ordinary differential equations of minimal order that characterizes the dynamics of the controlled object. Nevertheless, there is no empirical verification for the application of this identification algorithm through real-world experiments on either a dynamic object or its prototype.

### 3 MATERIALS AND METHODS

To attain the objective, as outlined in reference [1], the task of developing an identification algorithm for the dynamic model of the multi-dimensional moving object – the Stewart platform – was formulated and solved.

Using expression (2) and the Wiener-Khinchin theorem in vector form [15], it is possible to form the transposed matrix of spectral densities of the vector of random identification errors  $S'_{\varepsilon\varepsilon}$ :

$$S'_{\varepsilon\varepsilon} = S'_{x_1x_1} - S'_{\zeta x_1} \Phi^* - \Phi S'_{x_1\zeta} + \Phi S'_{\zeta\zeta} \Phi^*,$$

where  $S'_{\zeta\zeta}$  is a transposed matrix of spectral densities of the extended vector of input signals:

$$S'_{\zeta\zeta} = \begin{bmatrix} S_{r_0r_0} & O_{6 \times 6} \\ O_{6 \times 6} & S_{\Delta\Delta} \end{bmatrix}, \quad (3)$$

where  $S'_{x_1\zeta}$  is a transposed matrix of mutual spectral densities between vector random processes  $\zeta$  and  $x_1$ :

$$S'_{\zeta x_1} = \begin{pmatrix} S'_{r_0x_1} & S'_{\Delta x_1} \end{pmatrix}. \quad (4)$$

To solve the task, it is necessary to express the matrix of mutual spectral densities  $S_{\Delta x_1}$  in terms of the output data. For this purpose, taking into account matrices (3), (4), and the system structure, and performing some transformations, the relationship equation is obtained:

$$S_{x_1\Delta} S_{\Delta\Delta}^{-1} S_{\Delta x_1} = S_{x_1x_1} - S_{x_1r_0} S_{r_0r_0}^{-1} S_{r_0x_1}. \quad (5)$$

To find the function  $S_{\Delta x_1}$ , it is necessary to factorize the matrix (5), taking into account the specificity of the vector  $\Delta$  as a vector of unit “white” noises [16]. Knowing the matrix  $S_{\Delta x_1}$ , we begin to solve the identification problem, which is equivalent to minimizing the functional (1) over the class of functions  $\Phi$  physically realizable and having analytical variation  $\delta\Phi$  only in the right half-plane.

The task of finding the function  $\Phi$  that minimizes the functional is solved using the Wiener-Kolmogorov method. The first variation of the identification quality functional  $\delta J$  is expressed as:

$$\delta J = \frac{1}{j} \int_{-j\infty}^{j\infty} tr \left[ R_{0*} \left( -R_0 S'_{\zeta x_1} D_*^{-1} + R_0 \Phi D \right) D_* \delta\Phi_* + R_0 \left( -S'_{x_1\zeta} R_0 D^{-1} + S'_{\zeta\zeta} \Phi_* R_{0*} D_* \right) \delta\Phi D \right] ds. \quad (6)$$

The condition of identical equality to zero of the variation (6) under the assumption of only stable variations of the functions is as follows:

$$\Phi = R_0^{-1} [T_0 + T_+] D^{-1}, \quad (7)$$

where  $R_0^{-1}$  is the result of Wiener factorization [16, 17] of a positively definite polynomial matrix  $R$ , which can be expressed as the product of Hermitian conjugate polynomial matrices:

$$R = R_0 R_{0*}, \quad (8)$$

where the determinant of the matrix  $R_0$  has only zeros with negative real parts;  $T_0 + T_+$  are a matrix with poles in the left half-plane of the complex variable, which is the result of the separation [16] of the following product

$$T = T_0 + T_+ + T_- = S'_{\zeta x_1} D_*^{-1}, \quad (9)$$

where  $D$  is the result of the Wiener factorization [17] of the transposed matrix  $S'_{\zeta\zeta}$ :

$$DD_* = S'_{\zeta\zeta}, \quad (10)$$

$$D = \begin{bmatrix} d_u & O_{6 \times 6} \\ O_{6 \times 6} & E_{6 \times 6} \end{bmatrix}. \quad (11)$$

Using algorithm (7), optimal structures of the transfer function matrices of the identification object  $W_1$  and the filter  $\Psi$ , which shapes the dynamic characteristics of the disturbance brought to the system output, are determined –  $W_2$ .

Determining the dynamic properties of matrix  $P$  by applying the single-side pole removal operation to the matrices  $W_1$  and  $W_2$  [16, 18], after which the matrix  $M$  is found:

$$M = \tilde{P}W_1; \quad (12)$$

Calculation of the matrix of spectral densities of disturbing influence:

$$S'_{\Psi_{ob}\Psi_{ob}} = \tilde{P}W_1W_2\tilde{P}^*. \quad (13)$$

To determine the value of the minimum identification error variance, it is necessary to substitute the matrix  $D$  from expression (11) and matrices (7) and  $R_0$  from (8) into the functional (6).

The algorithm for identifying the dynamics model of a multidimensional moving object mentioned above is developed through addition operations, multiplication of polynomial and fractional-rational matrices, Wiener factorization, Wiener separation of fractional-rational matrices, and determination of variance integrals.

#### 4 EXPERIMENTS

The authors in [19] developed a research prototype of a machine tool based on the Stewart platform (Fig. 3), applicable for the physical modeling of the movement of

various technical objects in space. During the research, a software and hardware system was developed to collect experimental data for identifying the dynamics model of the Stewart platform.

The software and technical components of the experimental data collection system were developed using LabVIEW, utilizing FPGA, SoftMotion, and Real-Time modules. To ensure determinism, the time-critical task is off-loaded from Windows and transferred to the real-time kernel or the target Real-Time (RT) system. Therefore, two systems are in use. The first system with the Windows OS is called the Host. Application development takes place on the Host system. The developed application is loaded into the processor of the second system, referred to as the Target RT system. The Target RT system executes the software code, manages input/output devices, and exchanges data with the Host system. The Host PC and the Target Real-Time platform are connected via an Ethernet network but operate independently. The hardware chosen for the target RT system includes a personal computer with a multifunctional reconfigurable I/O device based on the NI PCI-7833R with Virtex-II 3M Gate FPGA. On this personal computer, the Venturcom Phar Lap Embedded Tool Suite was installed to create the target Real-Time platform. It is a 32-bit real-time operating system based on x86 architecture and built upon the Win32 API by Microsoft. The PCI-7833R board enables the creation of comprehensive measurement or control systems, featuring up to 8 channels of analog input (AI), 8 channels of analog output (AO), and 96 digital channels (DIO) (Fig. 3).

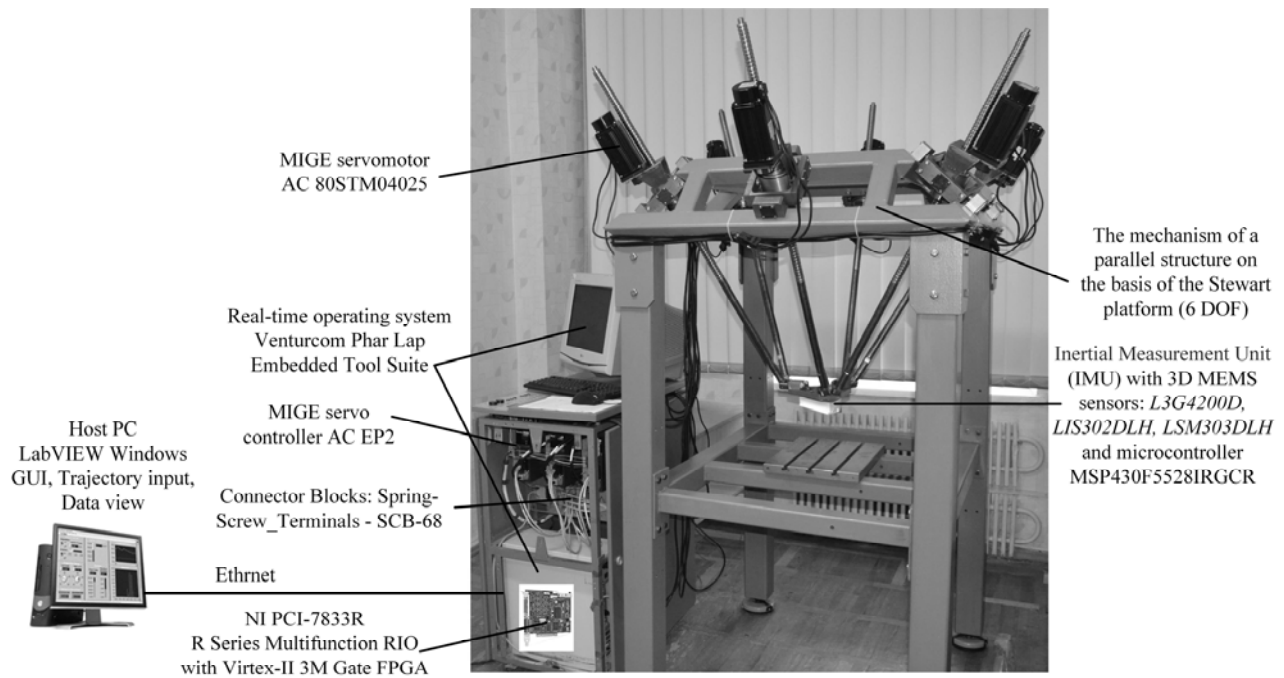


Figure 3 – Technical implementation of the motion control system for the working surface of the Stewart platform using National Instruments technology

To determine the current coordinates, velocity, and acceleration of the working surface of the Stewart platform, an orientation and navigation sensor (ONS) was developed based on an inertial measurement unit (IMU) with MEMS sensors. The unit includes the following set of primary information sources: a three-axis microelectromechanical gyroscope L3G4200D, a three-axis microelectromechanical accelerometer LIS302DLH, a three-axis magnetic field sensor (magnetic compass) LSM303DLH, and a computational block based on the MSP430F5528IRGCR microcontroller (Fig. 4). The information obtained from the primary information sources is processed using the Attitude and Heading Reference Systems (AHRS) position determination algorithm implemented in LabView.

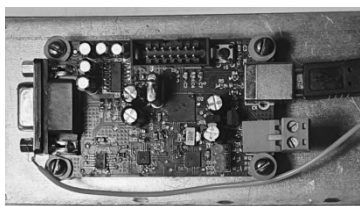


Figure 4 – Technical implementation of the inertial measurement unit based on 3D MEMS sensors

The electric drive and IMU were connected to the NI PCI-7833R board using Spring-Screw Terminals – SCB-68. A multifunctional AC servo drive from the EP2 series with an 80ST-M04025 servo motor from HANGZHOU MIGE ELECTRIC CO., LTD is used as the electric drive (Fig. 3).

To conduct an active experiment, with such identification, the formation of a vector of programmed control signal for moving the center of rotation of the Stewart platform WP is possible with a multidimensional filter based on the standard Dryden model as a universal tool for forming a stochastic external influence [20]. Based on this, a vector of the programmed signal  $r_0$ , was generated (Fig. 5) using the Dryden model implemented in Matlab/Simulink [21]. The range of this signal was constrained by the size of the working zone of the physical prototype of the Stewart platform [17].

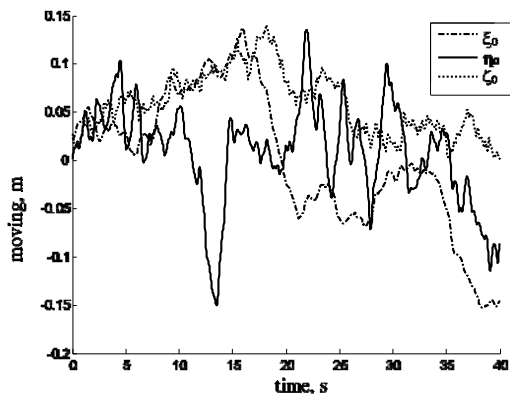


Figure 5 – Input program signal  $r_0$  values

The experiment was constructed as follows: a realization of the vector of program signals  $r_0$ , simulating the change in the position of the center of rotation of the Stewart platform, was applied to the inputs of the control system of the physical prototype. The acquisition of the displacement vector of the Stewart platform's working surface, based on a full-scale experiment, was performed using a physical prototype of the Stewart platform with an IMU installed at the center of rotation. Before this, the transfer function matrix and spectral densities of measurement disturbances of the developed IMU were estimated using structural identification methods [22]. This contributed to improved accuracy through optimal stochastic signal filtration [23]. After the preliminary processing using the AHRS algorithm, an array of records of the displacement vector signal for the center of rotation of the working surface of the Stewart platform  $x_1$ , was obtained, the graph of which is depicted in Fig. 6.

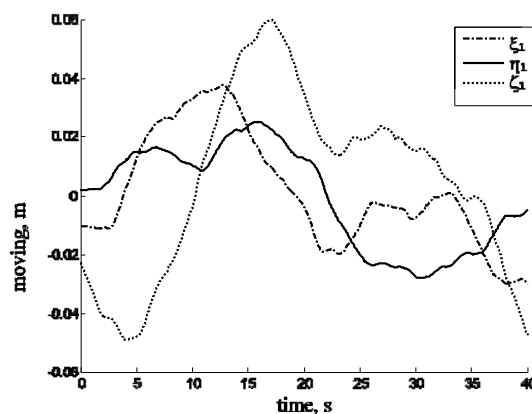


Figure 6 – Output signal  $x_1$  values

As a result of the full-scale experiment, two arrays of points were obtained: one for the input signal  $r_0$  and the other for the reaction of the Stewart platform to this signal  $x_1$ . These arrays were input into the Matlab program for calculating the dynamic model.

## 5 RESULTS

As a result of the conducted research, the task of identifying the dynamics model of a multidimensional moving object was formalized, illustrated by the example of a test specimen of a mechanism based on the Stewart platform (Fig. 3). Vectors of useful signals and interference are multidimensional stationary random processes with zero mathematical expectations and fractional-rational matrices of spectral and mutual spectral densities  $S_{x_1x_1}$ ,  $S_{r_0r_0}$ ,  $S_{x_1r_0}$ ,  $S_{r_0x_1}$  which have already been obtained as a result of experimental processing. Measurement disturbances are uncorrelated with each other and with the useful signal, while the disturbance  $\psi_{ob}$  is uncorrelated with the control signal  $r_0$ . As an example, here are some of the fractional-rational matrices of spectral and mutual spectral densities:

– spectral density of the input signal  $S_{r_0r_0}$  :

$$S'_{r_0 r_0} = \begin{bmatrix} \frac{0.049}{|(s+0.6)(s+0.7)|^2} & \frac{0.03}{(-s+0.6)(s+0.7)(s+1.7)(s+2)} & \frac{-0.01(s+2)}{(s+0.6)(s+0.7)(s-0.68)(s-0.85)} \\ \frac{0.03}{(-s+2)(-s+1.7)(-s+0.7)(s+0.6)} & \frac{0.68}{|(s+2)(s+1.7)|^2} & \frac{0.005}{(s+1.7)(-s+0.68)(-s+0.85)} \\ \frac{-0.01(-s+2)}{(s-0.7)(s-0.6)(s+0.68)(s+0.85)} & \frac{0.005}{(-s+1.7)(s+0.68)(s+0.85)} & \frac{-0.005|s+2|^2}{|(s+0.68)(s+0.85)|^2} \end{bmatrix};$$

– mutual spectral density of the signal  $S_{r_0 x_1}$  :

$$S'_{r_0 x_1} = \begin{bmatrix} \frac{9^{-5}(s-1.37)(s+0.16)(s^2+7.9s+54.14)}{|(s+0.7)(s+0.6)|^2(s-1.7)z_1} \\ \frac{-4.8^5(s+2.22)(s-16.75)(s^2+1.36s+0.93)(s^2-2.02s+1.84)}{|(s+0.7)(s+0.6)|^2(s-1.7)(s-2)z_1} \\ \frac{-29.9^5(s+0.95)(s-3.22)(s-8.92)(s^2+0.04s+0.25)}{|(s+0.7)(s+0.6)|^2(s-2)(s^2+0.11s+0.04)(s^2+2s+1.09)} \\ \frac{3.46^{-5}(s+0.14)(s+0.02)(s-331.8)}{|s+1.7|^2(s+2)(s^2+0.11s+0.04)(s^2+0.63s+0.15)} \\ \frac{-1.84^{-5}(s+8.36)(s+142.1)}{|(s+1.7)(s+2)|^2(s^2+0.63s+0.15)} \\ \frac{-11.48^{-5}(s+1.1)(s+0.87)(s+3.08)(s-28.41)(s^2+0.61s+0.09)(s^2+1.37s+1.37)}{|(s+1.7)(s+2)|^2(s+0.7)z_2} \\ \frac{3.99^{-5}(s+0.16)(s^2+3.38s+2.96)(s^2-2.08s+31.39)}{(s+0.6)(s+1.7)(s-0.85)(s-0.68)z_1} \\ \frac{-2.12^{-5}(s+1.42)(s-6.81)(s^2+1.36s+0.89)(s^2+3.87s+4.77)}{|s+0.85|^2(s+0.6)(s+1.7)(s-0.68)z_1} \\ \frac{-13.24^{-5}(s+1.78)(s-6.87)(s^2+0.5s+0.14)(s^2+0.005s+0.2)}{(s+0.6)(s-0.85)(s-0.68)(s^2+0.11s+0.04)(s^2+2s+1.09)} \end{bmatrix},$$

where  $z_1 = (s^2 + 0.63s + 0.15)(s^2 + 2s + 1.09)$ ,  
 $z_2 = (s^2 + 0.11s + 0.04)z_1$ .

Matrices  $S_{x_1 x_1}$ ,  $S_{r_0 r_0}$ ,  $S_{x_1 r_0}$ ,  $S_{r_0 x_1}$  were subjected to reduction using the method of typical logarithmic frequency characteristics [8]. As a result, estimates of these densities were determined.

The transposed matrix of spectral densities of the extended vector of input signals  $S'_{\zeta \zeta}$  is obtained according to the expression (3) of the identification algorithm, using Wiener factorization of polynomial matrices, substituting the spectral density of the input signal  $S_{r_0 r_0}$ , and assuming  $S_{\Delta \Delta} = 1$ .

Accordingly, with the expression (5), we obtained the matrices  $S_{x_1 \Delta} S_{\Delta \Delta}^{-1} S_{\Delta x_1}$ , and by performing its factorization, we obtained:

$$S'_{\Delta \zeta x_1} = \begin{bmatrix} \frac{0.03}{z_3} & 0 & 0 \\ 0.018 & \frac{0.0245}{z_3} & 0 \\ \frac{0.01}{z_3} & \frac{0.02}{z_3} & \frac{0.054}{z_3} \end{bmatrix},$$

where  $z_3 = (s+0.45)(s+0.55)$ .

Subsequently, by inserting  $S'_{\Delta x_1}$  and  $S'_{r_0 x_1}$  into equation (4), we derive the matrix containing mutual spectral



densities  $S'_{\zeta\zeta_1}$ . Using the expression (10) of the identification algorithm and executing the factorization of the matrix  $S'_{\zeta\zeta}$ , we have achieved:

$$D = \begin{bmatrix} d_u & O_{3 \times 3} \\ O_{3 \times 3} & E_{3 \times 3} \end{bmatrix},$$

where

$$d_u = \begin{bmatrix} \frac{0.22(s^2 + 3.49s + 3.28)}{(s + 0.7)z_4} & \frac{0.24819(s + 0.78)}{(s + 0.7)z_4} & \frac{0.008(s + 1.14)}{(s + 0.7)z_4} \\ \frac{-0.15(s + 0.29)}{z_4} & \frac{0.81(s + 0.61)}{z_4} & \frac{-0.027}{z_4} \\ \frac{-0.054(s - 1.92)}{(s + 0.68)(s + 0.85)} & \frac{0.005}{(s + 0.68)(s + 0.85)} & \frac{-0.005|s + 2|^2}{(s + 0.68)(s + 0.85)} \end{bmatrix},$$

where  $z_4 = (s + 0.6)(s + 1.7)(s + 2)$ .

Next, in accordance with expression (9) of the identification algorithm, by multiplying the fractional-rational

matrices  $S'_{\zeta\zeta_1}$  and  $D$  and performing the Wiener separation, the resulting matrix is obtained:

$$T_0 + T_+ = \begin{bmatrix} \frac{0.012(s + 1.15)(s^2 + 4.03s + 4.12)}{(s + 0.7)z_4} & \frac{0.013(s + 2.27)(s^2 + 2.56s + 1.7)}{(s + 0.7)z_4} & \frac{0.015(s + 1)(s^2 + 4.09s + 4.4)}{(s + 0.7)z_4} \\ \frac{0.0079(s + 0.73)(s + 2.23)}{z_4} & \frac{-0.0034(s + 0.45)(s + 3.52)}{z_4} & \frac{0.01(s + 0.67)(s + 2.9)}{z_4} \\ \frac{0.004(s - 1.96)}{(s + 0.68)(s + 0.85)} & \frac{0.0075(s - 1.34)}{(s + 0.68)(s + 0.85)} & \frac{0.004(s - 2.47)}{(s + 0.68)(s + 0.85)} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Entering the notation  $R_0=1$  and following expression (7) of the identification algorithm, multiplying the fractional-rational matrices  $(T_0 + T_+)$  and  $D^{-1}$ , the matrix  $\Phi = [W_1 \ W_2]$  is obtained, where  $W_1$  is the optimal structure

of the matrix of the transfer function of the identification object:

$$W_1 = \frac{10^{-2}}{z_2} \begin{bmatrix} 1.3(s + 5.4)(s + 0.83)(s + 0.15)(s^2 + 0.3s + 0.067) & -1.6(s - 2.12)(s + 0.14)(s - 0.029)(s^2 + 1.89s + 1.04) \\ (s + 2.1)(s^2 + 0.22s + 0.03)(s^2 + 1.35s + 0.83) & -0.4(s + 9.3)(s^2 + 1.95s + 0.99)(s^2 + 0.066s + 0.09) \\ 4.9(s + 0.96)(s^2 + 0.5s + 0.14)(s^2 - 0.018s + 0.2) & 0.6(s + 0.19)(s^2 + 1.43s + 0.54)(s^2 + 1.23s + 4.09) \\ -0.8(s - 2.1)(s + 0.76)(s + 0.19)(s^2 + 0.25s + 0.08) \\ 0.4(s + 1.96)(s^2 + 0.22s + 0.055)(s^2 + 1.06s + 0.67) \\ 2.5(s + 0.94)(s^2 - 0.2s + 0.068)(s^2 + 0.59s + 0.17) \end{bmatrix};$$

$W_2$  is the optimal structure of the matrix of transfer function of the filter, which forms the dynamic characteristics of the disturbance brought to the output of the system:

$$W_2 = \begin{bmatrix} \frac{0.03}{z_3} & 0 & 0 \\ 0.018 & \frac{0.0245}{z_3} & 0 \\ \frac{0.01}{z_3} & \frac{0.02}{z_3} & \frac{0.054}{z_3} \end{bmatrix}.$$

Further, applying the operation of one-sided removal of the poles of matrices  $W1$  and  $W2$ , a polynomial matrix  $P$  is obtained which is equal to:

$$P = \begin{bmatrix} z_2 & 0 & 0 \\ 0 & z_2 & 0 \\ 0 & 0 & z_2 \end{bmatrix}.$$

After that, there is matrix  $M$  according to (12):

$$M = \begin{bmatrix} 0.013(s+5.4)(s+0.83)(s+0.15)(s^2+0.3s+0.067) & -0.016(s-2.1)(s+0.14)(s-0.027)(s^2+1.89s+1) \\ 0.01(s+2.1)(s^2+0.22s+0.03)(s^2+1.35s+0.83) & -0.004(s+9.3)(s^2+1.95s+1)(s^2+0.067s+0.09) \\ 0.05(s+0.96)(s^2+0.5s+0.14)(s^2-0.018s+0.2) & 0.006(s+0.19)(s^2+1.4s+0.54)(s^2+1.23s+4.09) \\ -0.008(s-2.1)(s+0.76)(s+0.19)(s^2+0.24s+0.085) \\ 0.004(s+1.95)(s^2+0.22s+0.055)(s^2+1.06s+0.67) \\ 0.025(s+0.94)(s^2-0.2s+0.067)(s^2+0.59s+0.17) \end{bmatrix}.$$

According to expression (13) of the identification algorithm, applying the operation of entering zeros to the left, the matrix of spectral densities of the disturbing influence was obtained:

$$S'_{\Psi_{ob}\Psi_{ob}} = 10^{-4} z_5 \begin{bmatrix} 9 & 5.3 & 3.2 \\ 5.3 & 9 & 6.7 \\ 3.2 & 6.7 & 34.4 \end{bmatrix},$$

where

$$z_5 = \left| \frac{(s^2+0.1s+0.04)(s^2+0.6s+0.15)(s^2+2s+1)}{z_3} \right|^2.$$

Thus, the research goal has been achieved. The results include the identification of the dynamic model of the Stewart platform, its transfer function and the transfer function of the shaping filter.

## 6 DISCUSSION

To validate the identification results, simulations were conducted to assess the accuracy of the dynamic model identification of the Stewart platform. It was done using the Simulink simulation tool in the Matlab 6.5 environment.

The principle of checking the accuracy of identification consists of comparing the vector of the change in the coordinates of the rotation center of WS of Stewart platform  $x_1$  (Fig. 6) measured during the full-scale experiment with the sum of the vector formed when the software control signal vector  $r_0$  (Fig. 5) passes through the transfer function of the identification object  $W_1$  and vector of sta-

tionary random disturbances  $\Psi_{ob}$ , which is formed when “white noise” signals pass through the transfer function of the forming disturbance filter  $W_2$ .

A number of relevant blocks are presented on the diagram of the simulation model (Fig. 7), which implements this principle. Block  $r_1$  is designed to form a set of changes in the control signal  $r_0$ . At the output of block  $x_1$ , the vector  $x_l$  of the values of the coordinates of the center of rotation WS of the Stewart platform recorded during the live experiment is formed. To generate the vector of stationary random disturbances  $\Psi_{ob}$ , blocks of “white noise” generators with unit intensity were used. These generators are combined into the  $\Psi_{ob}$  block. All data necessary for the operation of these blocks are presented in the workspace of the Matlab engineering calculation system in the format of *iddata* structures. Also, all components of these vectors were centred using the Constant block. Blocks  $W1$  and  $W2$  are designed to store matrices of transfer functions  $W_1$  and  $W_2$ .

The diagram of the simulation model (Fig. 7) also presents demultiplexers for extracting vector components and multiplexers for combining data into vectors, as well as Scope oscilloscopes used to display simulation results for viewing and in the workspace.

As a result of the simulation to assess the accuracy of the identification of the dynamic model of the Stewart platform, in the *ksi\_delta*, *ita\_delta*, *sigma\_delta* blocks comparison plots were obtained. These plots depict the vector formed at the output of the identification object  $x_{id}$  compared to the output vector  $x_l$  in linear coordinates  $\xi$ ,  $\eta$ ,  $\zeta$  (Fig. 8). Also, graphs in the delta block illustrating the change in the identification error were obtained (Fig. 9).

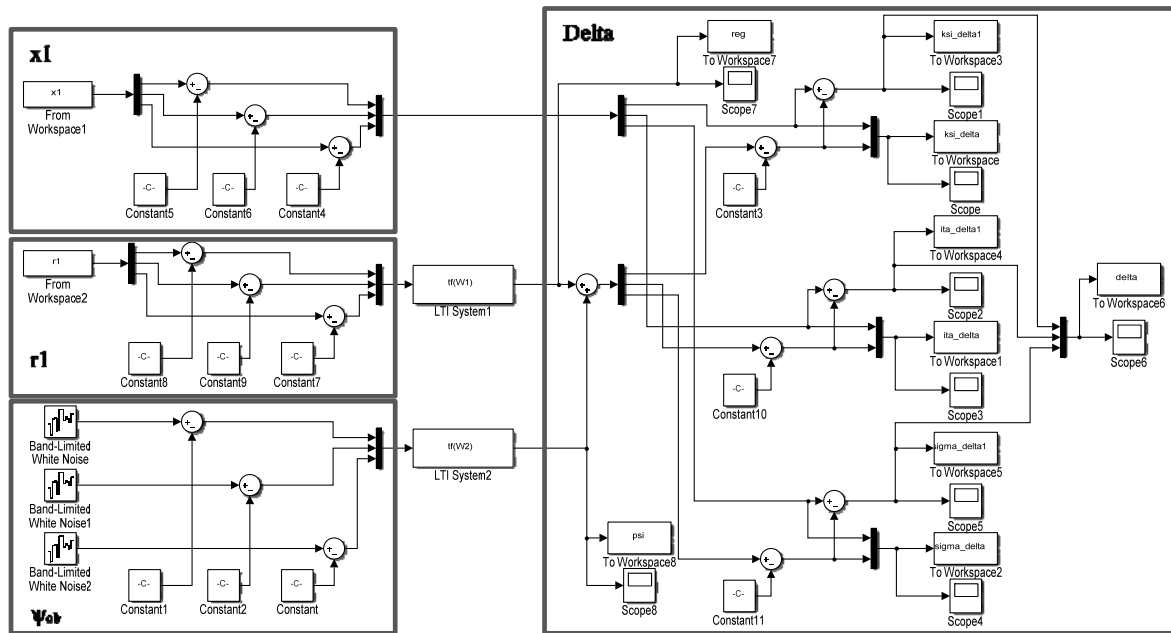


Figure 7 – Schematic of a simulation model for verifying the accuracy of mechanism identification based on Stewart platform

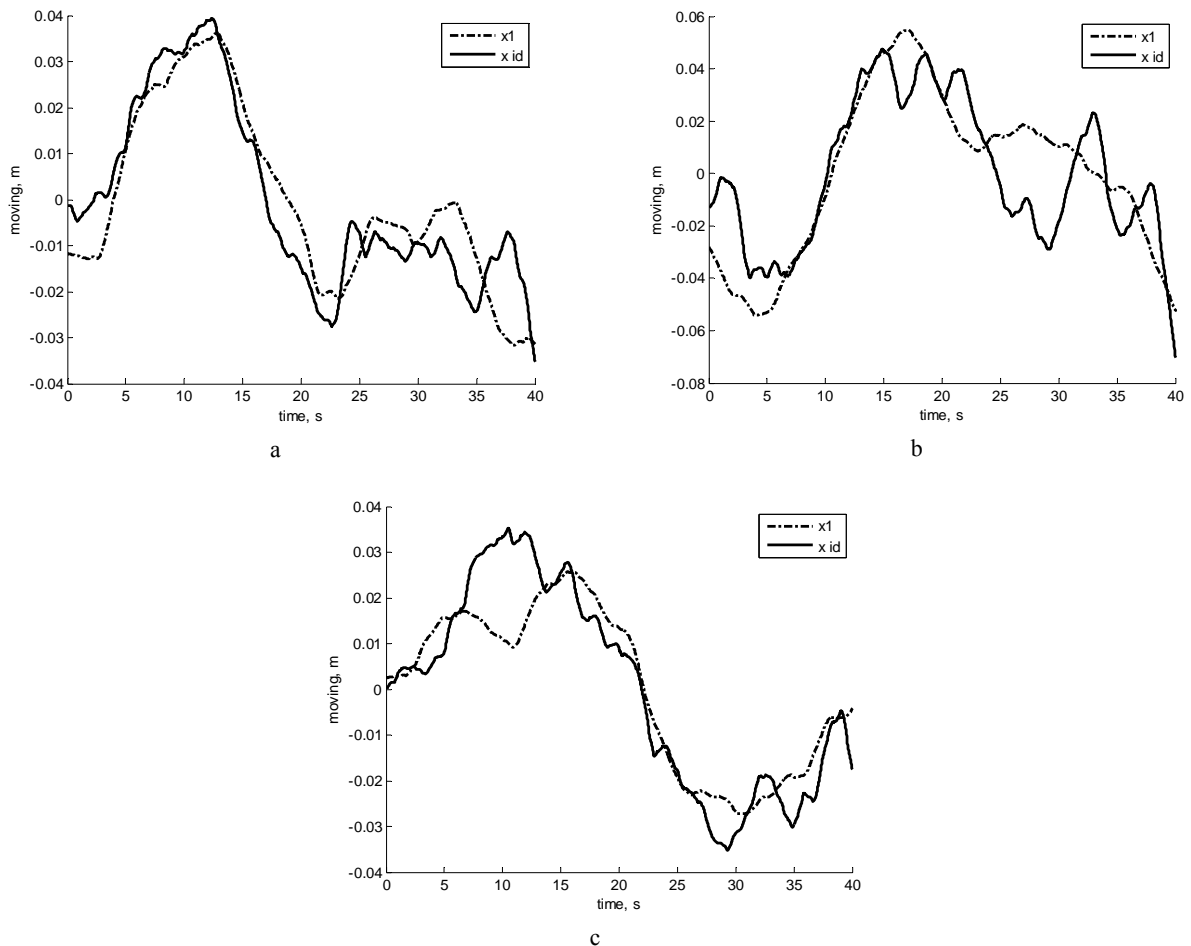


Figure 8 – Graphs of change in the coordinates of the rotation center of WS according to the results of the full-scale experiment  $x_1$  and the estimates obtained as a result of the identification  $x_{id}$ , according to linear coordinates: a –  $\xi$ , b –  $\eta$ , c –  $\zeta$

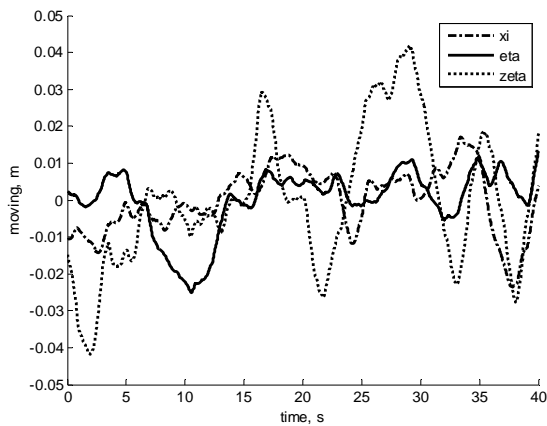


Figure 9 – Graphs of change in the identification error by linear coordinates  $\xi$ ,  $\eta$ ,  $\zeta$

On the basis of the analysis of the graphs of the identification error (Fig. 9), it is possible to estimate the root mean square deviation of the error relative to its mathematical expectation based on the unbiased estimate of its dispersion along the linear coordinates  $\xi$ ,  $\eta$ ,  $\zeta$ , which are respectively equal to:  $\sigma_{\xi}^{id} = 0.0086$ ,  $\sigma_{\eta}^{id} = 0.0085$ ,  $\sigma_{\zeta}^{id} = 0.00188$  and  $D_{\xi}^{id} = 7.485 \cdot 10^{-5}$ ,  $D_{\eta}^{id} = 7.3389 \cdot 10^{-5}$ ,  $D_{\zeta}^{id} = 3.5458 \cdot 10^{-4}$ .

As a result, the analysis of simulation results confirms the sufficient accuracy of identifying dynamic models of mechanisms based on the Stewart platform. Additionally, the main influence on the movement of the center of mass of the moving platform has changes in control actions. However, neglecting the influence of disturbances reduces the precision of platform positioning. Therefore, for the synthesis of the control system, methods should be applied that allow determining the structure and parameters of a multi-dimensional controller, taking into account such influences.

### CONCLUSIONS

In the paper, an algorithm for identifying the dynamics model of a Stewart platform is substantiated based on known frequency algorithms of structural identification. It allows finding the order and parameters of the linearized system of ordinary differential equations for a multidimensional object and the matrix of spectral densities of disturbances acting on it under operating conditions close to the real mode of operation of the experimental object.

**The scientific novelty** of the obtained results lies in the fact that the above-justified algorithm for identifying the dynamics model of a multidimensional moving object demonstrates enhanced accuracy and reliability in computational performance. These improvements were achieved through the introduction of a novel approach to the processes of polynomial matrix factorization. At its core is the improvement of algorithms for multiplying polynomial matrices to minimize the loss of significant digits. This was achieved through the appropriate ordering and rank-

ing of elementary operands, and the multiplication of fractional-rational matrices to mitigate the increase in order within the results.

**The practical significance** of the obtained results lies in the fact that the verification of the identification results confirms sufficient accuracy in identifying dynamic models of the mechanism based on the Stewart platform and the forming filter. Root-mean-square deviations of the error relative to its mathematical expectation were obtained based on an unbiased estimate of its variance along linear coordinates. Additionally, it was determined that the primary influence on the displacement of the center of mass of the moving platform is the variation in control inputs.

**Prospects for further research** involve the development of a method and algorithm for synthesizing the control system of the motion of the WS of the Stewart platform. These should allow for determining the structure and parameters of a multidimensional controller, taking into account that the primary influence on the displacement of the center of mass of the moving platform is the variation in control inputs.

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## ІДЕНТИФІКАЦІЯ МОДЕЛІ ДИНАМІКИ ПЛАТФОРМИ СТЮАРТА

**Зозуля В. А.** – канд. техн. наук доцент кафедри цифрової економіки та системного аналізу Державного торговельно-економічного університету, Кропивницький, Україна.

**Осадчий С. І.** – д-р техн. наук, професор кафедри конструкції повітряних суден, авіадвигунів та підтримання льотної придатності Льотної академії Національного авіаційного університету, Кропивницький, Україна.

**Неділько С. М.** – д-р техн. наук, професор, виконуючий обов'язки директора Льотної академії Національного авіаційного університету, Кропивницький, Україна.

### АНОТАЦІЯ

**Актуальність.** За сучасних вимог до точності процесів керування рухом рухомого об'єкта на заданій або програмованій траєкторії руху необхідно синтезувати оптимальні структуру та параметри системи стабілізації (регулятора) об'єкта з урахуванням як реальних контрольованих, так і неконтрольованих стохастичних збурюючих факторів. Також у процесі синтезу оптимальної структури регулятора необхідно оцінювати і враховувати багатовимірні моделі динаміки як самого об'єкта, його базових частин, контрольованих і неконтрольованих збурюючих факторів, що впливають на об'єкт у його реальному русі.

**Мета роботи.** Метою дослідження, результати якого представлені у цій статті, є отримання та оцінка точності моделі динаміки платформи Стюарта за допомогою обґрунтованого алгоритму ідентифікації динаміки багатовимірного рухомого об'єкта.

**Метод.** У статті використано метод ідентифікації в частотній області багатовимірних стохастичних систем стабілізації рухомих об'єктів з довільною динамікою. Обґрунтований алгоритм ідентифікації моделі динаміки багатовимірного рухомого об'єкта, побудований за допомогою операцій додавання, множення поліноміальних та дробово – раціональних матриць, вінеровської факторизації, вінеровської сепарації дробово – раціональних матриць, знаходження дисперсійних інтегралів.

**Результати.** В результаті проведених досліджень формалізовано задачу ідентифікації моделі динаміки багатовимірного рухомого об'єкта на прикладі дослідного зразка верстата на основі платформи Стюарта. Результати включають ідентифікацію динамічної моделі платформи Стюарта, її передатної функції та передатної функції формуючого фільтра та верифікація результатів ідентифікації яка підтверджує достатню точність отриманих моделей.

**Висновки.** Обґрунтований алгоритм ідентифікації дозволяє знаходити порядок та параметри лінеаризованої системи звичайних диференціальних рівнянь багатовимірного об'єкта та матриці спектральних щільностей збурень, які діють на нього в умовах роботи наближених до реального режиму функціонування дослідного зразка об'єкта. Аналіз результатів ідентифікації динамічних моделей платформи Стюарта показує, що основний вплив на переміщення центру мас рухомої платформи має зміна керуючих впливів. Однак нехтування впливом збурень знижує точність позиціонування платформи. Тому для синтезу системи керування слід застосовувати методи, які дозволяють визначити структуру та параметри багатовимірного регулятора з урахуванням таких впливів.

**КЛЮЧОВІ СЛОВА:** ідентифікація, матриця передавальних функцій, спектральна щільність, функціонал якості, платформа Стюарта.

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