# МАТЕМАТИЧНЕ ТА КОМП'ЮТЕРНЕ МОДЕЛЮВАННЯ

# МАТЕМАТИЧЕСКОЕ И КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ

# MATHEMATICAL AND COMPUTER MODELLING

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## OPTIMIZATION OF SOME REINSURANCE STRATEGIES

The basic purpose of the work is a study of existing approaches to reinsurance directed towards modeling of distribution and minimization of risk for an insurance portfolio, and forming a strategy for its optimal reinsurance using developed decision support system. A method for a search of optimal reinsurance strategy is proposed. For this purpose statistical models were selected that correspond to the structure and volume of portfolio losses as well as the number of these losses. The simulation model for the total insurance losses is developed. While finding an optimal reinsurance strategy it was taken into consideration the dependence of the load coefficient on a specific form of reinsurance. A numerical study of the dependence between optimal reinsurance strategy and the varying load coefficient has been performed. It was established that taking into consideration of the variable load coefficient for specific risk capital values for an insurance company the stop-loss strategy provides worse results than other forms considered. An architecture and the functional layout for decision support system are proposed, and appropriate software was developed in C#. The decision support system functioning has been illustrated on simulated example. The system will provide a useful instrument for a business analytic to support decision making while selecting a strategy for insurance portfolio in specific conditions.

**Keywords:** modeling in reinsurance, optimization of reinsurance, load coefficient, decision support system, choice of reinsurance strategy.

### NOMENCLATURE

$Y_i$ is random variables that characterize claim sizes to insurance company;

$N$ is a total number of claims to insurance company;

$N$ is a specific number of claims to insurance company;

$P(x)$ is an unconditional probability of $x$;

$\theta$ is a Poisson distribution parameter;

$E$ is a mathematical expectation operator;

$\mathrm{Var}(\cdot)$ is a variance;

$e$ is a base for natural logarithm;

$F(x)$ is a cumulative distribution function for $x$;

$f(x)$ is a distribution function for $x$;

$\Phi(\cdot)$ is a Laplace function;

$N(\cdot)$ is a normal distribution;

$\mu$ is a mean value of random variable;

$\sigma^2$ is variance;

$LN(\cdot)$ is a lognormal distribution;

$k$ is a lognormal distribution parameter;

$c$ is a constant for logistic distribution;

$\alpha$ is a form parameter for logarithmic logistic distribution;

$b$ is an extra scalar parameter for logarithmic logistic distribution;

$X_i$ are random variables that denote independent identically distributed losses;

$F^{*n}$ is an $n$-th order convolution for distribution $F$;

$S$ is a collective loss;

$c$ is a proportion coefficient;

$q$ is a proportion coefficient;

$\theta$ is a proportion coefficient;

$G$ is a probability of fulfilling the obligations by insurance company;

$B$ is a total netto-premium;



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$C$ is a guarantying capital;
$v$ is an insurance sum;
$QS$ is a quoted sum;
$XL$ is an excedent loss reinsurance;
$SL$ is a stop-loss reinsurance;
$a_0$ is a limiting predetermined value;
$a_1$ is a maximum limiting predetermined value;
$L$ is a predetermined priority sum;
$L_1$ is a fixed priority value;
$R$ is an income for insurance portfolio;
$P$ is a total premium;
VaR is a value-at-risk;
CaR is a capital-at-risk;
RoCaR is a return on capital-at-risk;
DSS is a decision support system;
$\lambda$ is a parameter for Poisson distribution;
IC is an insurance company;
DM is a decision maker;
DSS is a decision support system;
CDF is a cumulative distribution function;
PDF is a probability density function;

**INTRODUCTION**

Modern development of insurance market in Ukraine requires from its participants of appropriate financial stability that is a corner stone for the future successful operation. Taking into consideration existing financial instability of the state as a whole, the solvency indicators for each insurance company (IC) represent a key criterion for their competition ability. This is especially true with respect to the situations taking place in sophisticated quickly changing environment. To smooth the insurance premiums with respect to the risks accepted by an insurer and to balance this way the insurance portfolio as well as to take into consideration potential financial possibilities of an insurer there exist reinsurance institutes. Practical application of the modern reinsurance strategies increases financial stability of insurance operations and their profitability, shifting a part of risks, accepted for insurance, to other insurers.

To develop a new reinsurance program for selected risks it is necessary to find an optimal solution regarding the form and volumes of reinsurance. In other words the problem arises how to take correctly into consideration appropriate number of influencing factors, such as a model of business, financial stability of an enterprise, the enterprise ability to accepting or rejecting the risks, current market conditions and possibilities regarding the ability for risk management solutions. In spite of the fact that insurer and his client both take part in the reinsurance process most of the existing theoretical studies on the subject are devoted to the search of optimal reinsurance form from the insurer (cedent) point of view and stick to some specific reinsurance form taking into consideration selected optimality criterion and calculation of insurance premium principle.

As far as the decisions directed towards reinsurance problems solving are closely linked to financial risks, an appropriate problem statement must be formulated as one that minimizes the risks. That is why modern solutions based on mathematical risk and decision theory should involve some innovations into information processing and provide a decision maker (DM) with convenient criteria for their usage in the risk management environment.

The search for optimal in various senses insurance strategies is an important direction of research for several decades and it preserves its importance as of today. The first studies regarding the optimal reinsurance were directed towards the search of acceptable loss distribution function between insurer and reinsurer. It was shown that stop-loss reinsurance helps to minimize the variance of respective payments and to maximize expected insurance company income comparing to any other form of reinsurance (loss distribution function) [1–4]. In [5] it was also shown that the quoted reinsurance is optimal in the sense that this is the cheapest way to restrict the variance of non-distributed risks under condition that the load coefficient is increasing with growth of the reinsured part of the loss variance.

The purpose of this work is in studying of some existing approaches to reinsurance directed towards modeling of distribution and risk minimization for an insurance portfolio, and forming a strategy for its optimal reinsurance using developed specialized decision support system (DSS).

**1 PROBLEM STATEMENT**

Let we have the data in the form of identically distributed random variables $Y_i$, $i=1,...,N$ characterizing claim sizes to insurance company; and $X_i$, $i=1,...,N$ are independent identically distributed losses of insurance company. Then the problem of searching for optimal reinsurance strategy for an insurance portfolio is to find maximum of expected income value $E(R)$, with minimum of CaR such, that goal function $\text{RoCaR} = E(R)/\text{CaR}$ is maximized. The problem is to be solved under restrictions on the predetermined value of $a_0$ (the priority sum), and the predetermined maximum value equal to $a_1$.

The problem statement also includes statistical models selection for losses of an insurance portfolio, and development of decision support system as a handy computational instrument for decision maker searching for optimal reinsurance strategy.

**2 REVIEW OF THE LITERATURE**

The problem of reinsurance optimization is of interest for insurance companies and researchers what causes development of various solutions directed towards solving this problem. Most of the research works are of theoretical nature. It is assumed in [6] that the expected value premium principle could be applied and it was shown that the stop-loss reinsurance maximizes the expected utility of terminal wealth. In [7] the authors are concerned with the problem of purchasing the best risk protection from insurance company. The question of choosing the risk measure is discussed and several choices of non-symmetric risk measures are examined. Also sufficient conditions for optimality of reinsurance contract are provided within restricted class of admissible contracts. Some explicit forms for optimal contracts are derived in the case of absolute deviation and truncated risk measure. The article [8] is devoted to theoretical extension of results relevant to introduction of mean-variance premium principles approach. The authors of the research [9] showed that the limited stop-loss and the truncated stop-loss approaches may lead to optimal contracts under fulfilling some criteria. These criteria include maximization of expected utility, stability of insurance company functioning, and the survival probability of a cedent. In research [10] the authors also assume the expected value principle and introduce two classes of optimal reinsurance models hiring minimization of well-known criteria based on value-at-risk (VaR) computing approach, and conditional VaR (CVaR) describing the total risk exposure of an insurer.

All the papers mentioned above made a substantial contribution to the study and development of new approaches for solving the problem of optimal reinsurance. However, this problem cannot be classified as a solved one, many studies in this direction are being continued today. Analysis of the publications shows that most of them have restrictions in the sense that they consider some specific criteria and preselected particular premium payment policy. Some of the criteria maximize expected utility of the final income, and some minimize preselected risk measure in form of loss variance, ruin probability, VaR, CVaR or some others. A cedent is usually concerned about the question: how would optimal reinsurance change from one reinsurance form to another. He needs to determine two following assumptions: – the risk measure that is used to find optimal solution; – the premium principle that is used for calculating the reinsurance premium. These choices lead to alternative optimal solutions based on different strategies.

It our study we consider first the possibilities for statistical modeling of insurance portfolio losses using various distribution types. Also alternative possibilities for selecting the reinsurance strategy are studied including proportional and non-proportional forms. As far as the purpose of decision maker working on reinsurance problems always tries to reduce the value of CaR and to maximize income $E(R)$, it will be reasonable to include into optimization criterion the variables mentioned. They are related to each other by another variable called RoCaR. RoCaR maximization provides maximization of expected income $E(R)$, and minimization of CaR. Thus, the RoCaR value is suitable for its use as a criterion for searching optimal reinsurance parameter. Finally, a specialized decision support system architecture and software are developed to solve the problem of searching for optimal reinsurance strategy.

## 3 MATERIALS AND METHODS

To analyze the distribution of risk for some insurance portfolio between insurer and reinsurer it is necessary to construct (or select) a model for its total loss. An integrated policy portfolio of risky insurance is as a rule rather heterogeneous even in the case when it can be divided into separate homogeneous parts. Thus here is the problem of approximating the integrated loss for an arbitrary heterogeneous portfolio. An effective solution for the problem can be known collective risk model. The idea of the collective model is in considering policy portfolio as a source of loss only without taking into consideration the risks related to respective losses. The matter is that initial distributions for a collective model – i.e. distributions of counts and sizes of losses – can be estimated much better (with smaller errors) than the loss distributions for separate homogeneous risk groups.

The collective risk model constructing is based on the following limitations: – the insurance premium is paid at the very beginning of the insurance term, and any other payments are not performed within the period; the claims $Y_1, Y_2,...$, that are issued to insurance company, are not linked to specific contracts, they are considered as an integrated total risk; the random variables $Y_1, Y_2,...$, are independent and identically distributed; the total number of claims $N$ to insurance company and the random variables $Y_1, Y_2,...$ are mutually independent. Very often (e.g. in auto insurance) $N$ has Poisson distribution

$$P(N=n)=\frac{1}{n!}\,\theta^n e^{-\theta}, \quad n=0,1,2,...,$$

with the parameter $\theta=E(N)=Var(N)$. The necessary conditions for the use of this distribution are as follows: – the random variables that characterize number of loss cases in two non-intersecting time intervals are independent; – two and more insurance cases cannot take place simultaneously; – generally the insurance cases can occur at arbitrary moments of time. Usually it is accepted in practice that these conditions are fulfilled for a separate risk and for a portfolio as a whole. For example, in a case of car crash an insurance case is touching for one driver only, and possible accumulation of events we can avoid by applying appropriate insurance strategy that could help to integrate several policies into one risk.

The use of a collective model supposes that during the time period when external factors (say inflation) are changed slightly. In such situations the losses in a specific insurance portfolio are independent and identically distributed. If the assumptions regarding independence of separate losses are usually fulfilled, the assumption relevant to identical distribution does not look realistic because of divergence of insurance premiums. On the other side the losses in a collective model are considered jointly (as an integrated value) on a definite time interval. This fact allows for taking into consideration that these random values belong to the same sample representing a mixture of different distributions for separate losses. In practice each insurance strategy and each portfolio type is characterized with its own (mixed) loss distribution that depends on the sizes of insurance premiums for separate risks and on a type of insurance events. For example, a mean loss caused by industrial enterprise fire is much higher than a loss caused by the fire in a living house. These two cases are distinguished from the mean loss in auto insurance, and the losses in auto insurance are also different for different types of insurance. However, the practice of insurance shows that the loss structure in all types of insurance are very similar. Say, the number of small losses is usually much larger than the number of large ones. In a strict sense the «concentration of losses» is decreasing with growing of their sizes. The smallest losses are very often also rare and are not substantial from economic point of view. And the quantitative relations between large and small losses as well as the borders between them are different for different insurance types.

In the majority of practical cases the most important point is adequacy of mathematical model to the distribution of losses size in the area of large losses because just large losses make noticeable economic influence. It is reasonable to perform the model search in the family of distributions that have a scalar parameter and contain together with the CDF $F(x)$ all the distributions of the following type: $F(x\,|\,b)$, $b>0$. This is convenient approach in cases when different types of currency are used. The transition to new currency requires changing the scalar parameter only; all other parameters and the PDF $f(x)=F'(x)$ remain the same.

Very often statistical data is asymmetric and the distributions should be transformed to normal. In fact for asymmetric data with positive asymmetry there exist many examples where application of natural logarithm results in quite acceptable for practical use normal distribution. For example, if $X$ is a claim size and it is necessary to construct a model for $Y=\ln(X)$ using the normal distribution: $Y \sim N(\mu,\sigma^2)$, then we could use the distribution to build the model for $X$. This can be done by the change of variables: $X=e^Y$. Thus, we come to the lognormal distribution $LN(\mu,\sigma^2)$ with scalar parameter $\mu$, and the form parameter $\sigma^2$:

$$f(x)=\frac{1}{x\sqrt{2\pi\sigma^2}}\, e^{(\ln x-\mu)^2/2\sigma^2};$$

$$F(x)=\Phi\left(\frac{\ln x-\mu}{\sigma}\right);\quad E(X^k)=\exp(k\mu+k^2\sigma^2/2).$$

The lognormal distribution can be hired as a model for describing loss size in a separate insurance case. Less popular though very similar to normal is logistic distribution that is described by the density function:

$$f(x)=\frac{1}{c\sigma}\left(1+\exp((y-\mu)/c\sigma)\right)^{-2}\exp((y-\mu)/c\sigma),$$

and the cumulative distribution function is as follows:

$$F(y)=\left(1+\exp((y-\mu)/c\sigma)\right)^{-1}.$$

As a result of logarithmic transform we get the logarithmic logistic distribution:

$$f(x)=\frac{\alpha(x/b)^{\alpha-1}}{b(1+(x/b)^{\alpha})^2};\quad F(x)=1-\frac{1}{1+x/b^{\alpha}};$$

$$E(X^k)=b^k B\left(1+\frac{k}{\alpha},1-\frac{k}{\alpha}\right)=b^k\frac{k\pi/\alpha}{\sin(k\pi/\alpha)},$$

where $b=e^{\mu}$; $\alpha=1/(c\sigma)$. The drawback of this distribution is in complexity of its parameters estimation.

Symmetric and defined over all values of real arguments density has Laplace distribution:

$$f(y)=0.5\,\alpha\,\exp(-\alpha|y-\mu|).$$

The cumulative distribution function consists of the two symmetric with respect to $\mu$ exponential distributions as follows:

$$F(x)=\begin{cases}0.5\alpha\exp(\alpha(y-\mu)), & y\le\mu;\\ 1-0.5\alpha\exp(-\alpha(y-\mu)), & y>\mu.\end{cases}$$

After the transform: $X=e^Y$ we get logarithmic Laplace distribution with form parameter $\alpha$ and scalar parameter $b=e^{\mu}$:

$$F(x)=\begin{cases}0.5\,(x/b)^{\alpha}, & 0<x\le b;\\ 1-0.5\,(x/b)^{-\alpha}, & x>b.\end{cases}$$

PDF for the logarithmic Laplace distribution is defined as follows:

$$f(x)=\begin{cases}\alpha(x/b)^{\alpha-1}/(2b), & 0<x\le b;\\ \alpha(x/b)^{-\alpha-1}/(2b), & x>b,\end{cases}$$

$$E(X^k)=b^k\alpha^2/(\alpha^2-k^2).$$

Usually the areas of small and medium losses are approximated with two straight lines not well enough, though in the area of high losses such approximation is quite is quite possible even in the cases when the frequency of large losses is somewhat overestimated. The left side ($x\le b$) of the logarithmic Laplace distribution could be replaced by more appropriate distributions for small losses, say gamma or inverse Gaussian.

Another candidate for the losses approximation on the interval $(0,b)$ is Pareto distribution with the following CDF: $F(x)=1-(x/b)^{-\alpha}$, $x\ge b$, where $b$ is a shift parameter. This function is transformed into zero point distribution defined for all $x\ge 0$: $F(x)=1-((b+x)/b)^{-\alpha}$, with PDF of the form:

$$f(x)=\frac{\alpha}{b}\left(1+\frac{x}{b}\right)^{-\alpha-1}.$$

In some forms of insurance the zero point Pareto distribution has a tendency to overestimation of frequency for the largest losses. In such cases it recommended to replace the transform: $X = e^{Y}$, that allows the transition from exponential to Pareto distribution with the «weaker transform» $X = Y^{Z}$, $z > 1$.

The unbiased exponential distribution $F(y) = 1 - e^{-\beta y}$ leads to the Weibull distribution in the form: $F(x) = 1 - \exp(-(x/b)^{\alpha})$, $x > 0$, where $\alpha = 1/z$, with the following PDF:

$$f(x) = \frac{\alpha}{b}(x/b)^{\alpha-1}\exp(-(x/b)^{\alpha}).$$

The given above short review of some possible CDFs and respective PDFs could be used for the losses description in separate insurance cases.

The model describing collective losses is based on a suggestion that random losses in the insurance portfolio are independent for separate insurance cases, belong to the same distribution, and do not depend on their random count on a given time interval under consideration. The last requirement means independence of average size of loss on the number (count) of losses observed. This condition can be violated in some cases, for example, in auto insurance: on icy roads the number of car body damages is growing fast what results in a large number of small losses with the average loss decreasing simultaneously.

Such situations are inevitable in conditions of influence of external factors (climate conditions, demand variations) that influence simultaneously number of cases as well as size of respective losses. If the insurance covers the losses caused by climate factors then the numbers and size of losses cannot be considered as independent. However, the reason for this is in an attempt of performing simultaneous insurance for several possible loss (risk) factors with a single policy. When the risk factors (reasons) are considered separately the number of losses almost never depends on their sizes. Thanks to this fact collective model in most insurance cases is practically applicable. Moreover it was just collective model that initiated the theory of risk and made substantial impact on its further development and success.

Let $N$ be the number (count) of losses for a given portfolio on a definite time interval (usually one year), and let $X_1, X_2, ..., X_N$ are independent identically distributed losses the distribution of which does not depend on $N$. Now the integrated losses can be represented in the form:

$$S = X_1 + ... + X_N.$$

The moments of random variable $S$ can be found using respective values for $N$ and $X$ (the values of $X_1, X_2, ..., X_N$ have the same distribution as $X$):

$$E(S) = E_N \left( E\left( \sum_{n=1}^{N} X_n \mid N \right) \right) =$$

$$= E_N \left( \sum_{n=1}^{N} E(X_n \mid N) \right) = E(N)E(X);$$

More difficulties are encountered when we need to determine the distribution $G$ for the total loss $S$ using the distributions for $N$, and $X$. However, practically there is no other way to find this distribution. Usually it is impossible to perform direct adjustment of the distribution form because of limited data volumes (short samples of annual data). The distribution $G$ can be expressed via distribution $p_n = P(N = n)$ for counts $N$ and the distribution $F(x) = P(X \le x)$ for loss $X$:

$$G(s) = P(S \le s) \sum_{n=0}^{\infty} p_n \, P(S \le s \mid N = n) = \sum_{n=0}^{\infty} p_n \, F^{*n}(s),$$

where $F^{*n}$ means $n$-th order convolution for distribution $F$ ($F^{*0}(x) = 0$ with $x < 0$, and $F^{*0}(x) = 1$ with $x > 0$). However, such explicit computing of the infinite sum is possible only in very rare cases say when $N$ has geometric distribution (e.g. negative binomial with the parameter $\alpha = 1$), and $X$ has exponential distribution. Another possible approach to solving the problem is hiring recursive Panger technique that also requires substantial computational resources. Quite acceptable approach to computing the distribution $G$ is in simultaneous modeling of the counts number and average loss and generation of appropriate distribution with Monte Carlo technique.

Quite popular is proportional approach to reinsurance when reinsurer takes predetermined part of risk coordinated with insurer. In this case the whole loss $X$ (loss size for one insurance case or total annual loss) is divided into two parts: $qX$ and $(1-q)X$ according to the rule: $X = qX + (1-q)X$, $0 < q < 1$. There are different reasons for the risk distribution. Formally the probability of fulfilling the obligations by insurance company $G(B+C)$ is determined by the total netto-premium $B$, distribution of total loss $S$, and guarantying capital $C$. The client may consider reliability of fulfilling the obligation $G(B+C)$ as inadequate and can apply for extra protection from some other IC. Consider in short the forms of proportional and non-proportional reinsurance.

Proportional reinsurance forms:

– quoted sum (QS) reinsurance. In such case reinsurer accepts a fixed part of all insurance policies say 50%, i.e. receives 50% of which premium (except for reinsurance commission) and pays 50% for each loss;

– excedent sums reinsurance. With the quoted reinsurance all the risks are split between insurer and reinsurer in some proportion: $c : (1-c)$. With the excedent sums reinsurance parts of risks depend on insurance sum $v$ according to the rule: $c = c(v) = \min(v_0/v, 1)$. In other words insurer takes all the risks with the insurance sum $v \le v_0$ and splits with reinsurer the risks with the insurance sum that exceeds $v_0$. The reinsurer accepts the part of risk that corresponds to the difference between the insurance sum and $v_0$. All other details correspond to the quoted reinsurance: for the risk with the insurance sum $v$ the premium and losses are split between insurer and reinsurer according to the relation: $c(v)/1-c(v)$.

Non-proportional reinsurance forms:

– excedent loss (XL) reinsurance. In this case for each loss $X$ insurer pays the sum $\min(X, a_0)$ that is limited by the predetermined value of $a_0$ (the priority sum), and reinsurer pays the following sum: $\max(X - a_0, 0)$. Sometimes reinsurance is predetermined in the limits of some maximum value equal to $a_1$, i.e. $\min(\max(X - a_0, a_1), 0)$. Insurer again is responsible for the part of loss $\max(X - a_1 - a_0, 0)$ until there exists another reinsurance agreement with priority sum: $a_1 + a_0$. Size of the premium that belongs to reinsurer depends on expected count and loss size of that exceeds $a_0$, and on the value of $a_1$;

– excedent cumulative loss reinsurance. This form of reinsurance is distinguished from the previous one with the priority sum that depends on the total loss caused by one insurance event (say hurricane or earthquake). This reinsurance form takes into account the possibility for simultaneous occurrence of a large number of small losses that create together substantial sum;

– stop-loss (SL) reinsurance. The stop-loss reinsurance results from development of the excedent loss reinsurance from a separate loss through cumulative to the annual. If the total annual loss $S$ of insurer (for one insurance form) exceeds predetermined priority sum $L$, then reinsurer accepts a part of loss over this priority. Usually not more than some fixed value $L_1$. In other words reinsurer takes the loss $\min(S, L)$ or $\min(S, L) + \max(S - L - L_1, 0)$, and reinsurer $\max(S - L, 0)$ or $\min(\max(S - L_0, 0), L_1)$. The stop-loss form provides insurer with maximum protection when the probability of overriding the limit $(L_0 + L_1)$ by total loss is low. Limiting the potential loss of insurer by the value of $L$, reinsurer accepts insurance almost completely.

To solve the optimization problem regarding selection of reinsurance form it is necessary to select optimization criterion. Selection the reinsurance form insurer is mostly interested in the following: how much he can reduce insurance portfolio risk and what income he will get from it. Thus, these two variables should be used to construct the optimization criterion. The measure of value-at-risk (VaR) is used very often for determining maximum possible loss that may take place with predetermined probability on a given time interval. This is a popular approach thanks to availability of a set of rather simple estimation techniques. To compute VaR it is necessary to determine the distribution quantile for the total loss. The possibility of determining the data distribution was considered above.

The insurance portfolio income is determined by the formula [11]: $R = P - S$, where $S$ is total loss for insurance portfolio; $P$ is premium that can be found as follows: $P = (1 + \theta)\, E(S)$, where $\theta$ is extra value for the insurance portfolio risk; $E$ is a symbol for mathematical expectation. It is evident that income is a stochastic variable far as it depends on the random loss $S$. That is why we should consider expected income: $E(R) = P - E(S) = \theta\, E(S)$.

When the size of insurance premium is known then it is possible to determine the size of insurance company risky capital that it can lose with given probability. This capital is called capital-at-risk (CaR): CaR = VaR $- P$. Obviously this value is of substantial importance for insurance company because it shows what capital should the company possess so that to avoid bankruptcy with given probability. As far as the purpose of DM working on reinsurance problems always tries to reduce the value of CaR and to maximize income $E(R)$, it will be reasonable to include into optimization criterion the variables mentioned. They are related to each other by another variable called return on CaR (RoCaR):

$$\text{RoCaR} = \frac{E(R)}{\text{CaR}}.$$

RoCaR maximization provides maximization of expected income $E(R)$, and minimization of CaR. Thus, the RoCaR value is suitable for its use as a criterion for searching optimal reinsurance parameter. The reinsurance parameters can take different values for different reinsurance forms but comparing their RoCaR values we can find optimal reinsurance strategy (RoCaR will take maximum value). To perform necessary computing experiments a decision support system was developed that is based on the models and criteria considered.

## 4 EXPERIMENTS

The DSS should provide DM at insurance company for selection of optimal reinsurance strategy for a given insurance portfolio. The system is based on simulation ideas that provide a possibility for searching optimal solutions for reinsurance problems. DSS includes the following three basic modules: loss estimation, reinsurance module, and final results module (all implemented on the C# programming platform). The simplified system architecture, that corresponds to hierarchical system construction approach, is given in Fig. 1. The DSS like this one are constructed according to the general system analysis principle that suppose availability of functionality completeness, taking into consideration possible uncertainties (like incomplete and noisy data, unknown forms of distributions, parametric uncertainties etc), and control of all computing stages with appropriate statistical criteria.

The loss estimation module computes total loss for an insurance portfolio. To illustrate the DSS functionality (to find the loss value) we used appropriate Monte Carlo simulation technique. The key problem for simulation was in selection of the most suitable probability distribution for the average losses and for the frequency of insurance cases. Decision regarding the choice of the distribution type should be based on the preliminary results of testing available statistical data what has been done in advance.

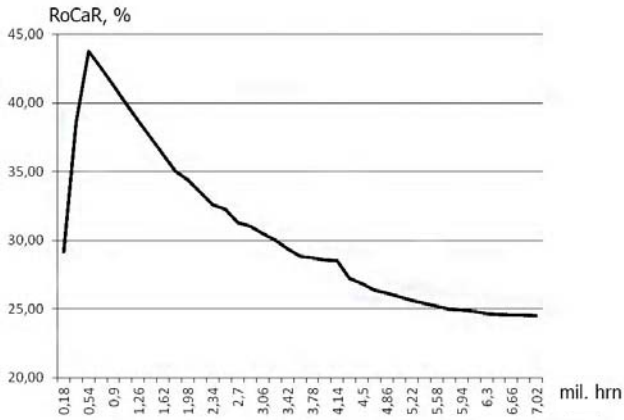


Figure 3 – RoCaR (%) plot versus $a_0$

The module of results performs comparison of different reinsurance forms as dependent on the company's risk capital and expected income. The final result is the choice of optimal reinsurance form with optimal value of respective parameter using RoCaR as optimality criterion. It was established that the value of risk capital CaR, acceptable for insurance company, can be different for alternative reinsurance forms. It means that optimal reinsurance form should be found in each specific reinsurance case. A substantial help in the search for the optimal strategy provides decision support system as far as it reduces efforts of decision maker regarding sophisticated computations, comparison of results, including their visual representation, and selection of the best one for specific case.

## CONCLUSIONS

Thus, the problem has been solved of determining optimal reinsurance strategy based on application of statistical models that correspond to the structure, size and the number of loss cases for an insurance portfolio. To solve this task appropriate optimization problem statement was formulated.

This is a new solution thanks to the fact that the load coefficient was taken into consideration that depends on the form of reinsurance and influences the premium size. Varying the load coefficient it was established that the stop-loss strategy results in lower quality than the others strategies considered.

The results of computing experiments are useful for practitioners because they help to distinguish between alternative reinsurance strategies and to select an appropriate one in a particular case. Practically useful developments also refer to creating the specialized decision support system architecture, functional layout and software, in C#, for solving the problem of searching for the optimal reinsurance strategy. After adjustment to the special needs of an insurance company the system proposed could serve as a handy instrument for a decision maker when searching for acceptable reinsurance form.

The future studies should be directed towards expanding the set of statistical and probabilistic models acceptable for the type of DSS considered. More specifically, the models could be in the form of multivariate conditional distributions (say copulas), Bayesian networks as well as alternative optimization procedures. It is easy to expand the DSS developed with new useful functions including automatic (or semiautomatic) comparison of computed alternatives.

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**ОПТИМИЗАЦИЯ НЕКОТОРИХ СТРАТЕГИЙ ПЕРЕСТРАХОВАНИЯ**

Цель работы заключается в исследовании существующих подходов к перестрахованию, направленному на моделирование распределения и минимизацию риска страхового портфеля, а также на формирование стратегии его оптимального перестрахования с использованием системы поддержки принятия решений. Предложен метод определения оптимальной стратегии перестрахования. Для этого выбраны статистические модели, которые соответствуют структуре, объему и количеству убытков страхового портфеля. При определении оптимального варианта перестрахования учтена зависимость коэффициента нагрузки от вида перестрахования. Коэффициент нагрузки учтен при расчете премии, а при сравнении различных форм перестрахования использованы одинаковые значения этого коэффициента. Выполнено численное исследование зависимости оптимальной формы перестрахования от переменного коэффициента нагрузки. Установлено, что учет переменного коэффициента нагрузки при определенных значениях капитала, которым готова рискнуть страховая компания, вариант stop-loss дает худшие результаты, чем другие формы перестрахования. Разработаны архитектура, функциональная схема, а также программное обеспечение системы поддержки принятия решений для решения задачи оптимизации перестрахования (программная платформа C#). Проиллюстрировано функционирование системы поддержки принятия решений, которая обеспечивает бизнес-аналитика критериями для руководства при принятии решений касательно выбора формы перестрахования страхового портфеля для конкретных условий.

**Ключевые слова:** моделирование в перестраховании, оптимизация перестрахования, коэффициент нагрузки, система поддержки принятия решений, выбор стратегии перестрахования.

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**ОПТИМІЗАЦІЯ ДЕЯКИХ СТРАТЕГІЙ ПЕРЕСТРАХУВАННЯ**

Мета роботи полягає у дослідженні існуючих підходів до перестрахування, спрямованому на моделювання розподілу і мінімізацію ризику страхового портфеля, а також формування стратегії його оптимального перестрахування з використанням системи підтримки прийняття рішень. Запропоновано метод знаходження оптимальної стратегії перестрахування. Для цього вибрано статистичні моделі, що відповідають структурі, розміру та кількості збитків страхового портфеля, а також побудовано імітаційну модель сукупного страхового збитку. При знаходженні варіанту оптимального перестрахування враховано залежність коефіцієнта навантаження від форми перестрахування. Коефіцієнт навантаження враховано при розрахунку премії, а при порівнянні різних форм перестрахування використано однакові значення цього коефіцієнта. Виконано чисельне дослідження залежності оптимальної форми перестрахування від змінного коефіцієнта навантаження. Встановлено, що врахування змінного коефіцієнта навантаження при певних значеннях капіталу, яким готова ризикнути страхова компанія, варіант stop-loss дає гірші результати ніж інші форми перестрахування. Розроблено архітектуру, функціональну схему, а також програмне забезпечення системи підтримки прийняття рішень для розв’язання задачі оптимізації перестрахування (програмна платформа C#). Проілюстровано функціонування системи підтримки прийняття рішень, яка забезпечує бізнес-аналітика критеріями вибору при прийнятті рішення стосовно вибору форми  перестрахування страхового портфеля для конкретних умов.

**Ключові слова:** моделювання у перестрахуванні, оптимізація перестрахування, коефіцієнт навантаження, система підтримки прийняття рішень, вибір стратегії перестрахування.

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