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RADIOPHYSICS

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MINIMUM-DURATION FILTERING

Myriad filtering and meridian filtering are known as robust methods of signal processing. The theory of these methods is based on the generalized Cauchy distribution and maximum-likelihood criterion. Based on the "Principle of Minimum Duration", we present an alternative approach to justify and generalize the myriad and meridian filtering methods. The proposed approach shows that the myriad and meridian filtering methods are special cases of the minimum-duration filtering methods derived from a concept of "signal quasi-duration". Mathematically, this concept is implemented through the concept of a functional (i.e., a function of a function) by using the proposed set of cost functions. On this foundation, a "superfamily" of quasi-duration functional is built, and a general class of minimum-duration filtering methods which depends on the three free-adjustable parameters is introduced. The numerical simulations are performed to compare the proposed and conventional methods for the problem of filtering a constant signal which is distorted by a mixture of Cauchy, Laplacian and Gaussian noise.

Keywords: myriad filtering, meridian filtering, duration.

NOMENCLATURE

PMD Principle of Minimum Duration;

ML maximum-likelihood;

RMSE Root Mean Square Error.

A unknown signal amplitude, or filter output;

b constant in the equalizing procedure;

D functional of strict duration, or "strict duration";

$D(A)$ one-variable objective function corresponded to D;

$D^{(\alpha,\beta,\ldots)}$ functional of quasi-duration, or "quasi-duration" ;

$D^{(\alpha,\beta,\ldots)}(A)$ one-variable objective function corresponded to $D^{(\alpha,\beta,\ldots)}$;

$D_S^{(\alpha,\beta,q)}$ quasi-duration based on the superset of cost functions;

$D_M^{(\alpha,q)}$ quasi-duration based on the generalized Meshalkin cost function;

$f(t)$ shape of the observed signal;

$g(t)$ observed signal;

g_i ith sample of the observed signal;

N number of signal samples, or filter window size;

p tail constant of the generalized Cauchy distribution;

$p(z)$ probability density function;

q free-adjustable parameter called "smoothing degree";

$s(t)$ arbitrary signal;

T time interval;

t time argument;

x argument of cost function;

y intermediate variable in the equalizing procedure;

y_k kth approximation to y;

α – free-adjustable parameter associated with the scale parameter;

β – free-adjustable parameter associated with the shape of data distribution;

σ – scale parameter, or standard deviation of noise;

$\chi(x)$ – ideal cost function;

$\psi(x)$ – arbitrary cost function;

$\psi^{(\alpha,\beta,\ldots)}(x)$ – "quasi-duration" cost function;

$\psi_R^{(\beta)}(x)$ – "root cost function";

$\psi_R^{(\alpha,\beta,q)}(x)$ – "root cost function with smoothing", or "q-smoothed root cost function";

$\psi_{\log}^{(\alpha,q)}(x)$ – "q-smoothed logarithmic cost function";

$\psi_{med}^{(\alpha,q)}(x)$ – "q-smoothed median cost function";

$\psi_D^{(\alpha,\beta,q)}(x)$ – "generalized Demidenko cost function";

$\psi_M^{(\alpha,q)}(x)$ – "generalized Meshalkin cost function";

$\psi_S^{(\alpha,\beta,q)}(x)$ – member of the "superset of cost function";

INTRODUCTION

The principle of maximum likelihood is a mathematical foundation of many filtering methods. To use this principle, it is necessary to specify the joint probability density

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function which should be maximized over all desired parameters. In the case of independent and identically distributed data samples, the mathematical expression for the joint probability density function is considerably simplified with reducing the computational complexity and filter structure.

Based on the maximum-likelihood principle, the myriad filtering [1–2] and the meridian filtering [3–4] have been introduced as robust methods [5–6] of signal processing in impulsive environments. These methods are based on the assumptions that the signal samples are independently Cauchy-distributed and meridian-distributed, respectively. In spite of the common features of these methods, each of them presents an individual class of nonlinear filtering methods with the one free-tunable parameter associated with the scale parameter of noise distribution. Later, a more general class has been proposed [7], where the filtering methods depend on the two free-adjustable parameters associated with the scale parameter and the tail-constant of the generalized Cauchy distribution.

In this paper, we present a larger class of filtering methods. This class depends on the three free parameters that need to be adjusted, where the first two parameters coincide with the two parameters mentioned above, and the third parameter is associated with the shape of data distribution. In contrast to [7], this class is based on the "Principle of Minimum Duration" (PMD) [8–10]. Therefore, it will be referred to as the class of "minimum-duration filtering."

At the beginning of this paper, we describe the problem statement based on the PMD. In general, the PMD states a non-energy criterion, when the signal processing should produce a signal with a minimum duration. In this paper, we restrict the study to the approximation problem with the one amplitude parameter and show that the minimum-duration filtering is derived from the concept of "signal quasi-duration" by the PMD. Mathematically, this concept is based on the concept of a functional (i.e., a function of a function), where the cost function of the function, which describes the signal, is used. We construct a new set of cost functions and build a "superfamily" of quasi-duration. On this basis, for the discrete case we introduce a general class of the minimum-duration filtering methods. At the end of this paper, the performance of the minimum-duration methods is numerically compared to that of the conventional ones for the problem of filtering a constant signal which is distorted by a mixture of Cauchy, Laplacian and Gaussian noise.

1 PROBLEM STATEMENT

The original problem statement is to build the filtering methods by using the PMD. The mathematical problem statement requires a formalization of the concept of "signal duration". In this paper, we use the two concepts, namely "strict duration" and "quasi-duration" [8].

The strict duration, D, of any signal, $s(t)$, is defined as a measure of the nonzero signal values. Mathematically, D is the functional

$$D \equiv D[s(t)] = \int_{-\infty}^{\infty} \chi[s(t)]dt, \qquad (1)$$

where $\chi(x) = \begin{cases} 1, & if\ x \neq 0 \\ 0, & if\ x = 0 \end{cases}$ plays the role of an ideal cost function which we call the "Titchmarsh cost function" (p. 319 in [11]). Since D is the functional, it will be also referred to as the "functional of strict duration." Despite an obvious physical interpretation, the strict duration cannot be constructively applied to the problem formulation [8].

The concept of quasi-duration enables to formulate the problems constructively, although it has a less obvious physical interpretation. The quasi-duration, $D^{(\alpha,\beta,...)}$, is the functional

$$D^{(\alpha,\beta,...)} \equiv D^{(\alpha,\beta,...)}[s(t)] = \int_{-\infty}^{\infty} \psi^{(\alpha,\beta,...)}[s(t)]dt, \qquad (2)$$

where $\alpha,\beta,...$ are the real free-adjustable parameters, $\psi^{(\alpha,\beta,...)}(x)$ is the continuous cost function with the property: $\psi^{(\alpha,\beta,...)}(x) \to \chi(x)$ as parameters $\alpha,\beta,...$ go to their boundary values. This property provides the limiting process from $D^{(\alpha,\beta,...)}$ to D, that makes sense in the noiseless case. Since $D^{(\alpha,\beta,...)}$ is the functional, it will be also referred to as the "quasi-duration functional."

Phenomenologically, the PMD is formulated as: "After processing, the signal duration should be minimal." An applicability domain of the PMD depends on what concept is used. If the concept of the strict duration is used, the PMD can be applied to time-limited and noise-free signal. However, if the concept of the quasi-duration is used, the applicability domain of PMD is significantly extended, since it becomes possible to process both the noisy signal and the almost time-limited signal (e.g., the Gaussian pulse). On the other hand, there are two situations when using and implementing the PMD. In the first one, the signal is considered as a solution, which is either a sum of the known and unknown signal components or an unknown signal of a given class. In this situation, the PMD is implemented either by variation of the unknown signal component or by variation of the signal itself. In the second one, the signal is considered as an error (or residual) signal. Provided that the approximation problem is formulated for a finite number of unknown parameters, the PMD is implemented by variation of these parameters [10].

Further the approximation problem with the one unknown linear parameter which is the signal amplitude is considered. Let $g(t)$ be the signal observed in the time interval T, and $f(t)$ be the shape of this signal. Let A be the unknown signal amplitude. Then the error signal is

$$s(t) = g(t) - Af(t); \quad t \in T\ . \qquad (3)$$

Applying the PMD to (1) with (3) leads to the following problem

$$\arg\min_{A} D(A) = \arg\min_{A} \int_{-\infty}^{\infty} \chi[g(t) - Af(t)]dt\ , \qquad (4)$$

where D becomes a one-variable objective function, $D(A)$, to be minimized in A. The advantage of (4) is that the complete destruction of a large part of $g(t)$ will not change the solution. The disadvantage of (4) is that the negligible noise makes it impossible to solve this problem.

Applying the PMD to (2) with (3) yields

$$\arg\min_{A} D^{(\alpha,\beta,\ldots)}(A) = \arg\min_{A} \int_{-\infty}^{\infty} \psi^{(\alpha,\beta,\ldots)}[g(t) - Af(t)]dt \, , \quad (5)$$

where $D^{(\alpha,\beta,\ldots)}$ becomes the one-variable objective function, $D^{(\alpha,\beta,\ldots)}(A)$, to be minimized in A. The problem (5) defines the minimum-duration estimator of A based on the concept of signal quasi-duration. Provided that $f(t) = \text{const}$, from (5) the mathematical statement of the filtering problem for the constant signal is readily obtained. The advantage of (5) is that by selecting a cost function, which should give the Titchmarsh cost function in the limit, one can eliminate the drawback of (4) and naturally satisfy the two additional and important requirements, namely: 1) robust behavior for large values of the argument when the contribution of the small number of large values is limited; 2) smooth behavior for small values of the argument to perform the optimum processing of a large number of small values. The first one determines the filter behavior with respect to the impulses (outliers), and the second one determines the filter behavior with respect to the noise (inliers). Therefore, the mathematical problem is reduced to finding such a cost function, which satisfies the requirements mentioned above and generalizes the some known filtering methods (in particular, the average, median, myriad, and meridian filtering). The way to solve this problem is to introduce a finite set of free-adjustable parameters which control the cost function behavior, and, consequently, which control the filtering result in a given noise environment.

2 REVIEW OF THE LITERATURE

The cost functions technique is the theoretical basis to build the methods of the optimal data processing and, in particular, of the optimal filtering. For example, using a quadratic cost function $\psi(x) = x^2$ leads to the method of average filtering, which is the optimal in the case of Gaussian noise. The use of an absolute cost function $\psi(x) = |x|$ leads to the median filtering, which is the optimal method in the case of Laplacian noise. An important feature of these methods is that they do not require any adjustment to the noise parameters. However, this is not the case for the other methods. For example, the methods of myriad filtering and meridian filtering are obtained by using the cost functions $\psi(x) = \log(x^2 + \alpha^2)$ and $\psi(x) = \log(|x| + \alpha)$; $\alpha > 0$, respectively, which provide the optimal filtering in the cases of Cauchy noise and "Meridian" noise [1–4]. Unlike previous methods, these methods depend on the noise scale parameter σ and require the optimal tuning of the free parameter as $\alpha = \sigma$. Similarly, the use of the cost function $\psi(x) = \log(|x|^p + \alpha^p)$; $\alpha > 0$; $0 < p \le 2$ results in the construction of the filtering method in the case when the noise has a generalized Cauchy distribution. Here, there are two adjustable parameters, α and p [7]. Unfortunately, the cost functions mentioned above do not provide the limiting process to $\chi(x)$. Therefore, they can not be used as a function $\psi^{(\alpha,\beta,\ldots)}(x)$. However, they can be derived as special (limiting) cases for the given noise environment.

The limiting process from the quasi-duration functional to the functional of strict duration is the theoretical foundation of the proposed approach. This requirement noticeably limits the set of cost functions that can be used in (5). In [8, 10, 13] the following sets were considered: 1) one-parameter set of the "root cost functions"; 2) two two-parameter sets of the root cost functions with the quadratic and absolute smoothing; 3) two three-parameter sets of the "q-smoothed root cost functions" and "generalized Demidenko cost functions".

The use of the one-parameter set of the "root cost functions" leads to the following. Let $\psi^{(\alpha,\beta,\ldots)}(x)$ be the cost function

$$\psi_R^{(\beta)}(x) = |x / \Lambda|^{\beta}; \quad 0 < \beta < 1 \, , \quad (6)$$

where Λ is the constant with the physical dimension of x; further $\Lambda = 1$. Since $\lim_{\beta \to +0} \psi_R^{(\beta)}(x) = \chi(x)$, the function (6) may be used in (2). In addition, the following properties hold: (*i*) $\psi_R^{(\beta)}(x)$ is neither convex nor concave for all real x; (*ii*) $\psi_R^{(\beta)}(x)$ has even symmetry, where $\psi_R^{(\beta)}(0) = 0$ and $\psi_R^{(\beta)}(\pm 1) = 1$; (*iii*) $\lim_{x \to -0} [d\psi_R^{(\beta)}(x)/dx] = -\infty$ and $\lim_{x \to +0} [d\psi_R^{(\beta)}(x)/dx] = \infty$. Due to its typical behavior, the function $\psi_R^{(\beta)}(x)$ was named as the "root cost function" (even when β is an irrational number) [10]. Let the quasi-duration functional based on (6) be denoted as $D^{(\beta)}$. Then substituting (6) into (5) yields [10]

$$\arg\min_{A} D^{(\beta)}(A) = \arg\min_{A} \int_{T} |g(t) - Af(t)|^{\beta} \, dt; \, 0 < \beta < 1 \, , \quad (7)$$

where $D^{(\beta)}(A)$ is the objective function to be minimized in A. The advantage of (7) is that the solution of (7) makes sense in the cases when there is the complete destruction of the large part of data and when the data are distorted by noise. The disadvantages of (7) are the following. First, the noise appearance can lead to the bias of estimator. Second, the noise nature is left out of account. The latter means that the cost function which depends on more free parameters and which takes into account the noise nature can be more efficient than (6).

The two-parameter sets of the root cost functions with the quadratic and absolute smoothing are defined by the similar equations that by introducing a smoothing degree, q, have been summarized as [10]

$$\psi_R^{(\alpha,\beta,q)}(x) = k_R^{(\alpha,\beta,q)}[(1 + |x|^q / \alpha^q)^{\beta/q} - 1] \, , \quad (8)$$

where $\alpha > 0$; $0 < \beta \le 1$; $0 < q < \infty$; $\beta < q$; $k_R^{(\alpha,\beta,q)} = 1/[(1 + |x_1|^q / \alpha^q)^{\beta/q} - 1]$ and $\psi_R^{(\alpha,\beta,q)}(x_1) = 1$. This function represents a three-parameter set of the

"q-smoothed root cost functions". Passing to the limit in (8) as $\beta \to +0$ yields the "q-smoothed logarithmic cost function"

$$\psi_{\log}^{(\alpha,q)}(x) = k_{\log}^{(\alpha,q)} \ln(1+|x|^q/\alpha^q), \qquad (9)$$

where $\alpha > 0$; $0 < q < \infty$; $k_{\log}^{(\alpha,q)} = 1/\ln(1+|x_1|^q/\alpha^q$. This function is the generalization of the myriad and meridian cost functions, which are for $q = 2$ and $q = 1$, respectively. It is easy to see that in the case of generalized Cauchy distribution the "smoothing degree" parameter q coincides with the tail constant p of this distribution.

Contrariwise, passing to the limit in (8) as $\beta \to 1$ yields

$$\psi_{med}^{(\alpha,q)}(x) = k_{med}^{(\alpha,q)}[(1+|x|^q/\alpha^q)^{1/q} - 1]; \quad 0 < q < \infty, \qquad (10)$$

where $k_{med}^{(\alpha,q)} = [(1+|x_1|^q/\alpha^q)^{1/q} - 1]$; $x_1 \neq 0$ is the normalization point, where $\psi_{med}^{(\alpha,q)}(x_1) = 1$. Since the $\psi_{med}^{(\alpha,q)}(x)$ approaches the absolute cost function for $|x|^q/\alpha^q >> 1$, further it will be referred to as the "q-smoothed median cost function" (despite the fact that for $q > 1$ and for $q < 1$ the behavior of (10) is different near the zero value of x). If $q = 2$, the special case of (10) is a pseudo-Huber cost function [12]. But if $q = 1$, from (10) one obtains the absolute cost function.

The analysis of (8) shows that under fixed q the limiting process from $\psi_R^{(\alpha,\beta,q)}(x)$ to $\chi(x)$ is performed by α and β (when their boundary values are zero) just in the one direction (first by α and then by β). This drawback has been eliminated in [13] by generalizing the cost functions proposed by E. Z. Demidenko [14] and L. D. Meshalkin [15].

The function $\psi_D(x) = \dfrac{x^2}{x^2+\alpha^2}$ was suggested by E. Z. Demidenko for the regression problem [14]. By introducing b and q parameters, it can be generalized to [13]

$$\psi_D^{(\alpha,\beta,q)}(x) = k_D^{(\alpha,\beta,q)}\left[1 - \left(1 + \frac{|x|^q}{\alpha^q}\right)^{-\beta/q}\right], \qquad (11)$$

where $\alpha > 0$, $0 < \beta < \infty$, $0 < q < \infty$, and $k_D^{(\alpha,\beta,q)} = \dfrac{(1+|\alpha/x_1|^q)^{\beta/q}}{(1+|\alpha/x_1|^q)^{\beta/q} - (|\alpha/x_1|^q)^{\beta/q}}$ is defined by $\psi_D^{(\alpha,\beta,q)}(x_1) = 1$. It is easy to check, that the Titchmarsh cost function $\chi(x)$ is the limit of (11) as $\beta \to \infty$, and the q-smoothed logarithmic cost function $\psi_{\log}^{(\alpha,q)}(x)$ is the limit of (11) as $\beta \to +0$.

The function $\psi_M(x) = 1 - \exp\left(-x^2/\alpha^2\right)$ was suggested by L. D. Meshalkin for robust estimation [15], [16]. It can be generalized to [13]

$$\psi_M^{(\alpha,q)}(x) = k_M^{(\alpha,q)}\left[1 - \exp\left(-\frac{|x|^q}{\alpha^q}\right)\right], \qquad (12)$$

where $\alpha > 0$, $0 < q < \infty$ and $k_M^{(\alpha,q)} = \left[1 - \exp\left(-|x_1|^q/\alpha^q\right)\right]^{-1}$ is defined by $\psi_M^{(\alpha,q)}(x_1) = 1$. It is obvious that as $\alpha \to 0$ the function $\psi_M^{(\alpha,q)}(x)$ tends to $\chi(x)$. In addition, for fixed $\alpha > 0$ it approaches the $\chi(x)$ as $q \to 0$, and it has a form of the "rectangular hole" as $q \to \infty$ [13].

3 MATERIALS AND METHODS

It is seen that (11) is a continuation of (8) on negative values of β. Hence, there exists a continuous (by α, β and q) set of cost functions which is defined by the common member

$$\psi_S^{(\alpha,\beta,q)}(x) = k_S^{(\alpha,\beta,q)}\left[\left(1 + \frac{|x|^q}{\alpha^q}\right)^{\beta/q} - 1\right], \qquad (13)$$

where $k_S^{(\alpha,\beta,q)} = 1/[(1+|x_1|^q/\alpha^q)^{\beta/q} - 1]$, $-\infty < \beta \leq 1$, $\alpha > 0$, $0 < q < \infty$, $\beta < q$. This set incorporates the following cost functions: 1) q-smoothed median cost functions for $\beta = 1$ (in particular, the pseudo-Huber cost function); 2) q-smoothed root cost functions for $0 < \beta < 1$; 3) q-smoothed logarithmic cost functions for $\beta = 0$ (in particular, the meridian and myriad cost functions); 4) generalized Demidenko cost functions for $-\infty < \beta < 0$; 5) Titchmarsh cost function for $\beta \to -\infty$. Since this set is sufficiently representative, it will be referred to as a "superset" of cost functions. Below, the Meshalkin cost function is also included in this superset.

With regard to the small values of x, the contribution of α is different for each cost function derived from (13). To eliminate this shortcoming, we have proposed to equalize the second-order derivative of the cost functions at zero [13]. Further, the method for producing a modified superset of cost functions is presented for $q = 2$. The similar technique may be obtained for any $0 < q < \infty$.

Let α_{myr}^2 be the fixed value of α^2 for the myriad cost function. Let the equality: $d^2\psi_S^{(\alpha,\beta,2)}(x)/dx^2 = d^2\psi_{\log}^{(\alpha,2)}(x)/dx^2$ be hold at $x = 0$. Then

$$\frac{(\beta/2)}{y\left[\left(1 + \frac{|x_1|^2}{y}\right)^{\beta/2} - 1\right]} = \frac{1}{b}, \qquad (14)$$

where $y = \alpha_S^2$ denotes α^2 for the function $\psi_S^{(\alpha,\beta,2)}(x)$ and $b = \alpha_{myr}^2 \ln\left(1+|x_1|^2/\alpha_{myr}^2\right)$. Equation (14) states the equalizing problem, where y is to be determined, and can be solved by using the Newton's method for finding roots

$$y_{k+1} = y_k - \frac{u(y_k)}{u'(y_k)}, \qquad (15)$$

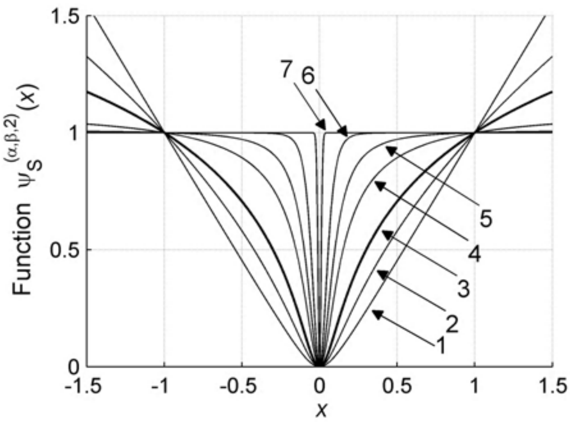
where y_k is the kth approximation to y, and where $u(y_k)$ and $u'(y_k)$ are the reduced function and its first derivative at y_k, respectively. The convergence of (15) is ensured by the following approach. For $-\infty<\beta<0$, the function $u(y)=y\left[\left(1+\frac{|x_1|^2}{y}\right)^{\beta/2}-1\right]-\frac{\beta}{2}b=0$ should be used with its first derivative $u'(y)=\left(1+\frac{|x_1|^2}{y}\right)^{\beta/2}\left[1-\frac{(\beta/2)|x_1|^2}{y+|x_1|^2}\right]-1$ and with the initial guess $y_0=\alpha_{myr}^2$. For $0<\beta<1$, the function $u(y)=\left[\left(1+y|x_1|^2\right)^{\beta/2}-1\right]/y-\frac{\beta}{2}b=0$ should be used with its first derivative $u'(y)=\frac{(\beta/2)(1+|x_1|^2 y)^{\beta/2-1}|x_1|^2 y-(1+|x_1|^2 y)^{\beta/2}+1}{y^2}$ and with the initial guess $y_0=1/\alpha_{myr}^2$. The equalizing procedure based on (15) typically converges to machine precision within 3–5 iterations.

Fig. 1 represents the superset and the modified superset of cost functions for $q=2$ and $x_1=1$. Fig. 1a shows the superset without the equalizing procedure, when all cost functions have the same value $\alpha^2=0.01$. In this figure, there are depicted the pseudo-Huber (curve 1), q-smoothed root with $\beta=1/2$ (curve 2), myriad (curve 3) and generalized Demidenko (the curves 4–7 are for $\beta=-1;-2;-5;-100$ in (13), respectively) cost functions. It is seen that the sequence of these cost functions tends to $\chi(x)$ as $\beta\to-\infty$. Fig. 1b shows the modified superset of cost functions, which is produced by the equalizing procedure for $\alpha_{myr}^2=0.01$. Here, the curve 7 is similar to the graph of the Meshalkin cost function, coinciding visually with it. Thus, in the limit as $\beta\to-\infty$, the Meshalkin cost function is obtained. This fact can readily be proved, since the finiteness of the second-order derivative at zero is provided on condition that $\alpha_S^2=const*\beta$. Substituting this value into (13) with $q=2$ and passing to the limit in (13) as $\beta\to-\infty$ yield the Meshalkin cost function.

Based on (13), the quasi-duration functional can be expressed as

$$D_S^{(\alpha,\beta,q)}\equiv D_S^{(\alpha,\beta,q)}[s(t)]=\int_{-\infty}^{\infty}\psi_S^{(\alpha,\beta,q)}[s(t)]dt=k_S^{(\alpha,\beta,q)}\times$$
$$\times\int_{-\infty}^{\infty}\left[\left(1+\frac{|s(t)|^q}{\alpha^q}\right)^{\beta/q}-1\right]dt, \qquad (16)$$

where $\alpha>0$, $-\infty<\beta\le 1$, $0<q<\infty$, and $\beta<q$. It has the following special cases: 1) q-smoothed median functional



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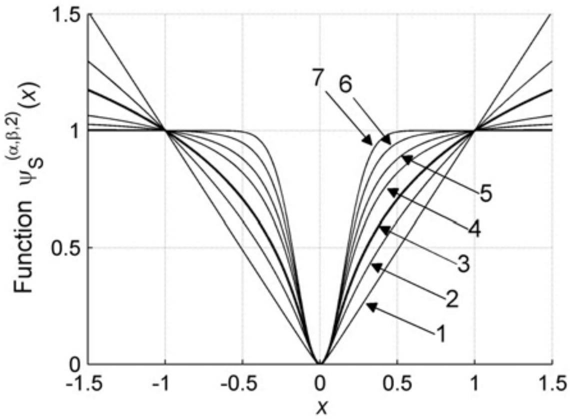

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Figure 1 – Superset of cost functions (a) without and (b) with the equalizing procedure

(when $\beta=1$); 2) pseudo-Huber functional (when $\beta=1$ and $q=2$); 3) q-smoothed root functional (when $0<\beta<1$); 4) q-smoothed logarithmic functional (when $\beta\to 0$); 5) myriad functional (when $\beta\to 0$ and $q=2$); 6) meridian functional (when $\beta\to 0$ and $q=1$); 7) generalized Demidenko functional (when $-\infty<\beta<0$); 8) Demidenko functional (when $\beta=-2$ and $q=2$). The generalized Meshalkin functional defined by

$$D_M^{(\alpha,q)}[s(t)]=k_M^{(\alpha,q)}\int_{-\infty}^{\infty}\left[1-\exp\left(-\frac{|s(t)|^q}{\alpha^q}\right)\right]dt \qquad (17)$$

is also derived from (16) in the limit as $\beta\to-\infty$ after the equalizing procedure. Thus, (16) determines a large family of functionals, which will be referred to as the “superfamily of quasi-duration”. On this basis, the minimum-duration estimate is defined as a solution of the appropriate optimization problem related to minimizing the quasi-duration (16). Assuming $f(t)=const$, below we write the general class of the minimum-duration filtering methods for the discrete case with the following notations: g_i is the sample of observed signal, N is the number of signal samples in filter window, and A denotes the unknown amplitude value, which is the filter output. This class is given by the problem

$$\arg\min_{A}\left\{k_S^{(\alpha,\beta,q)}\sum_{i=1}^{N}\left[\left(1+\frac{|g_i-A|^q}{\alpha^q}\right)^{\beta/q}-1\right]\right\}, \quad (18)$$

where $\alpha>0$, $-\infty<\beta\le 1$, $0<q<\infty$, and $\beta<q$. Note, that $k_S^{(\alpha,\beta,q)}>0$ for $0<\beta\le 1$ and $k_S^{(\alpha,\beta,q)}<0$ for $-\infty<\beta<0$. The filtering consists in finding the optimal value of A. The special cases of (18) can be written more simply without the nonessential constants.

The computational complexity of the optimization problem (18) can be reduced [17], when instead of the optimal value, $A\in R$, where R is the set of real numbers, the quasi-optimal value, $A\in\{g_i\,|\,i=1,...,N\}\subset R$, is computed as the filter output. It is seen that the quasi-optimal value of A makes, at least, one of the terms in the sum (18) equal to zero.

4 EXPERIMENTS AND RESULTS

We have compared the potential capability of the minimum-duration methods in the problem of filtering the constant signal distorted by additive noise. Further, we have selected the q-smoothed root filtering ((18) with $q=2$ and $\beta=1/2$), Demidenko filtering ((18) with $q=2$ and $\beta=-2$), and Meshalkin filtering ((18) with $q=2$ and $\beta\to\infty$). These methods have been compared with the methods of average, median, pseudo-Huber, myriad and maximum-likelihood (ML) filtering. The three cases, when the constant signal with amplitude $A=1$ was additively distorted by the Cauchy noise (case 1), by the mixture of Cauchy and Laplacian noise (case 2), and by the mixture of Cauchy and Gaussian noise (case 3), have been examined with the same value of the scale parameter for each type of noise. The mixture of Cauchy and Laplacian noise was formed with a priori probabilities of 1/2 for each. The mixture of Cauchy and Gaussian noise was formed with a priori probabilities of 2/3 and 1/3, respectively. Thus, in these cases, the probability density function, $p(z);\, z\in R$, of noisy signal is defined by $p(z)=p_C(z)=\frac{1}{\pi}\frac{1}{(z-A)^2+\sigma^2}$ (in the case 1), $p(z)=\frac{1}{2}p_C(z)+\frac{1}{2}p_L(z)$; $p_L(z)=\frac{1}{2\lambda}\exp\left(-\frac{|z-A|}{\lambda}\right)$; $\lambda=\frac{\sigma}{\sqrt{2}}$ (in the case 2) and $p(z)=\frac{2}{3}p_C(z)+\frac{1}{3}p_G(z)$; $p_G(z)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-(z-A)^2/2\sigma^2\right]$ (in the case 3).

For the calculations, the fragment of noisy constant signal was numerically simulated on a set of N=121 discrete samples, where N is the length of filter window. The filtering was to estimate the signal amplitude by finding the location of the global minimum of the corresponding optimization problem with the relative accuracy 0,1%. Further, the estimator's Root Mean Square Error (RMSE) averaged arithmetically over 10000 noise realizations have been calculated as a function of the free-adjustable parameter α that associated with the scale parameter of a given noise distribution. In these calculations, the modified superset of cost functions was used for $q=2$.

Fig. 2 represents the calculated RMSE/σ ratio vs the α/σ ratio, where $\sigma=0{,}1$. The solid curves with heavy dots depict the calculated values for the (1) pseudo-Huber filtering, (2) q-smoothed root filtering with q=2 and $\beta=1/2$, (3) myriad filtering, (4) Demidenko filtering, and (5) Meshalkin filtering; the dotted curve (6) with heavy dots depicts the calculated values for ML filtering; the dotted horizontal line depicts the calculated value for the median filtering.

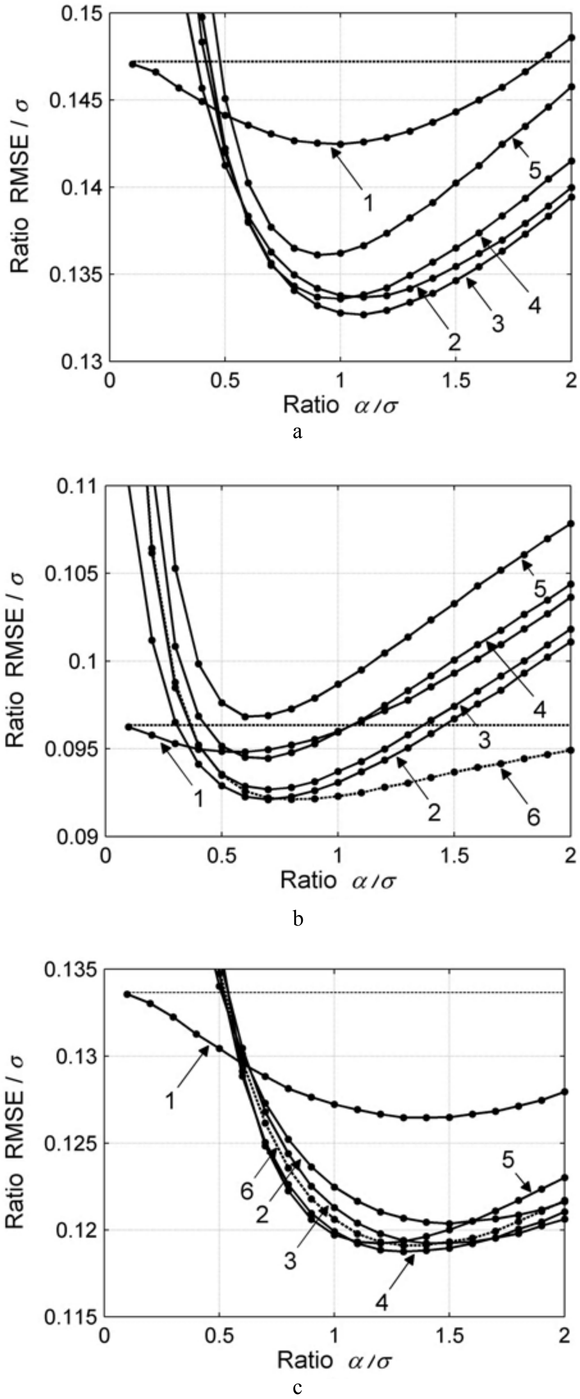

Figure 2 – RMSE/σ ratio vs α/σ ratio for the constant signal distorted by the (a) Cauchy noise, (b) mixture of Cauchy and Laplacian noise, and (c) mixture of Cauchy and Gaussian noise

5 DISCUSSION

As shown in Fig. 2a, in the case 1 the myriad filtering, which is also the ML filtering, provides the best result at $\alpha/\sigma = 1{,}1$ (theoretically, $\alpha/\sigma = 1$). Moreover, it provides the best results within the range $0{,}6 \le \alpha/\sigma \le 2$. However, if $\alpha/\sigma \le 0{,}4$ and α goes to zero, the performance of the myriad filtering (as well as the other minimum-duration filtering with the exception of the pseudo-Huber filtering) dramatically deteriorates; whereas that of the median filtering remains steady throughout. Note also, if the α/σ ratio becomes larger than 2, the RMSE/σ ratio becomes the larger for all these methods (except for the median filtering) and tends to the value which is obtained by the average filtering. In the case 2, Fig. 2b shows that the q-smoothed root filtering (with $q = 2$ and $\beta = 1/2$) provides the best result within the range $0.4 \le \alpha/\sigma \le 0.7$ and achieves the performance of the ML filtering at $\alpha/\sigma = 0.7$. However, for $\alpha/\sigma \ge 0.8$ the ML filtering, which tends to the median filtering as α goes to infinity, is the best. In the case 3, Fig. 2c shows the advantage of the Demidenko filtering, although this advantage is not significant here. This figure shows that within the range $0.6 \le \alpha/\sigma \le 2$ the Demidenko filtering is slightly better than the ML filtering. This can be explained by the fact that the ML estimator has no optimum properties for finite samples. In addition, for all these three cases, the selected minimum-duration filtering methods, used with $q = 2$ (which was close to the optimal value of q) and optimal α, were 5–8 times better than the average filtering. However, the smaller N was, the smaller the advantage.

Thus, these numerical simulations show the following. For the problem of filtering the noisy constant signal, the potential of the minimum-duration filtering exceeds the potential of the median and average filtering. As would be expected, the myriad filtering is the best for the Cauchy noise, the q-smoothed root filtering (with $q = 2$ and $\beta = 1/2$) is the best for the given mixture of Cauchy and Laplacian noise, and the Demidenko filtering is the best for the given mixture of Cauchy and Gaussian noise.

CONCLUSIONS

The goal of signal processing based on the PMD is to produce the signal with the minimum duration. To describe the signal duration in practice, the concept of the signal quasi-duration can be used. This concept is implemented by the quasi-duration functional and, in particular, by the quasi-duration objective function. The superfamily of the quasi-duration functional is proposed. It covers the families that include the q-smoothed median functional, q-smoothed root functional, q-smoothed logarithmic functional, the generalized Demidenko and Meshalkin functionals.

The general class of the minimum-duration filtering methods which depends on the three free-adjustable parameters is introduced. The myriad and meridian filtering methods occupy the intermediate positions in this class.

The potential of the minimum-duration filtering exceeds the potential of the median and average filtering. Theoretically, the minimum-duration filtering methods enable to filter the signal, which is destroyed in more than half length of the filter window, when the median filtering may fail.

By adjusting the free parameters, the proposed approach enables efficient processing of the signal which is distorted by noise of different types. Finding optimal values of α, β and q is the major problem in taking full advantage of the minimum-duration filtering.

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ФИЛЬТРАЦИЯ СИГНАЛОВ НА ОСНОВЕ ПРИНЦИПА МИНИМУМА ДЛИТЕЛЬНОСТИ

Методы мириадной и меридианной фильтрации известны как робастные методы обработки сигналов. Теория этих методов основана на обобщенном распределении Коши и критерии максимального правдоподобия. Мы представляем альтернативный подход, основанный на принципе минимума длительности, к обоснованию и обобщению методов мириадной и меридианной фильтрации. Предлагаемый подход показывает, что мириадная и меридианная фильтрация являются частными случаями методов фильтрации, которые выводятся из концепции «квазидлительность сигнала». Математически эта концепция реализуется через понятие функционала с помощью предложенного множества стоимостных функций. На этом фундаменте построено «суперсемейство» функционала квазидлительности и введен общий класс методов фильтрации, который зависит от трех свободно-настраиваемых параметров. Приводятся результаты числового моделирования для сравнения эффективности предложенных и стандартных методов для задачи фильтрации постоянного сигнала, который искажен смесью шумов Коши, Лапласа и Гаусса.

Ключевые слова: мириадная фильтрация, меридианная фильтрация, длительность.

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ФІЛЬТРАЦІЯ СИГНАЛІВ НА ОСНОВІ ПРИНЦИПУ МІНІМУМУ ТРИВАЛОСТІ

Методи міріадної й меридіанної фільтрації відомі як робастні методи обробки сигналів. Теорія цих методів заснована на узагальненому розподілі Коші та критерії максимальної правдоподібності. Ми надаємо альтернативний підхід, заснований на принципі мінімуму тривалості, до обгрунтування та узагальнення методів міріадної й меридіанної фільтрації. Запропонований підхід показує, що міріадна й меридіанна фільтрації є частинними випадками методів фільтрації, які виводяться з концепції «квазітривалість сигналу». Математично ця концепція реалізується через поняття функционалу за допомогою запропонованої множини вартісних функцій. На цьому фундаменті побудовано «суперсімейство» функціоналу квазітривалість та уведено загальний клас методів фільтрації, який залежить від трьох параметрів, що вільно налаштовуються. Приводяться результати числового моделювання для порівняння ефективності запропонованих та стандартніх методів для задачі фільтрації постійного сигналу, який є спотвореним сумішшю шумів Коші, Лапласа та Гаусса.

Ключові слова: міріадна фільтрація, меридіанна фільтрація, тривалість.

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